

# ASTR:3771, 2018 Fall, Lecture Outline

- Introduction (slides)
  - Self-introduction
  - Syllabus
  - Python setup homework
  - 15 min left to start chapter 1
- Celestial Sphere I (slides)
  - Equatorial coordinates
    - RA, Dec
    - $HA = LST - RA$ ,  $LST = RA(\text{Meridian}) = HA(\text{Ram})$ 
      - Precession & Julian Day
      - ICRS reference system
- Celestial Sphere II
  - Celestial sphere demo
  - How to estimate the LST at 9PM on Sep 1?
    - Estimate the LST at 9 PM and 12 AM on Sep 1
    - Fall equinox 2018: Sep 22.
    - $LST(12\text{AM}, \text{Sep } 1) = 24\text{h} - 20 \times 4\text{m} = 22\text{h}40\text{m}$
    - $LST(9\text{PM}, \text{Sep } 1) = 22\text{h}40\text{m} - 3\text{h} = 19\text{h}40\text{m}$
  - Alt-Az coordinates
    - Airmass =  $\sec(\text{Zenith Angle})$
    - Plane-parallel atmosphere
  - Spherical trigonometry
    - laws of sine & cosine
    - Applications
      - angular distance between two coordinates
      - PA of one coordinate relative to another
      - RA, Dec  $\rightarrow$  Alt, Az conversion
- Celestial Mechanics I

- History (1546-1727): Tycho, Kepler, Galileo, Newton
- Kepler's first two laws are mostly based on data of Mars: elliptical orbits and equal-area law
  - A line connecting a planet to the Sun sweeps out equal areas in equal time intervals
- Kepler's 3rd law came 10 years after the first two laws in 1619; it used data from 6 known planets:
  - Mercury, Venus, Earth, Mars, Jupiter, Saturn
  - Neptune, Uranus, and Pluto were yet to be discovered.
- Definition of Ellipse (demo) and the **geometric equations of ellipse**
- Newton's laws of motion and law of gravity
- Derivation of **Kepler's laws** using classical mechanics
  - Lagrangian of spherical symmetric system (the simplest geometric system)
  - Definition of Potential, Phi
  - 2 equations of motion
  - IoM 2 = conservation of angular momentum L (Kepler's 2nd law)
- Celestial Mechanics II
  - Derivation of **Kepler's laws** using classical mechanics (continued)
    - IoM 1 = conservation of energy E
    - EoM 1 + 1/r Potential => Kepler's 1st law
    - K1 + K2 => K3, as Ellipse Area/P = L/2
    - IoM 2 = energy conservation
- Celestial Mechanics III
  - The **Vis Viva Equation** (energy conservation)
    - derivation by evaluating the energy at perihelion
    - Derivation using virial theorem:
      - $E = K+U \sim 1/a$
      - $2K+U = 0 \Rightarrow K+U = U/2 = -GMm/2a$

- Application of Vis Viva Equation
  - **Hohmann Transfer orbit**
  - Why Hohmann transfer orbit is the most economical?
    - because it's the transfer orbit with the smallest semimajor axis
- Gravity and potential of extended masses
  - Write down the general forms of  $\mathbf{g}$  and  $\Phi$  as integrals:  
 $\mathbf{g} = -\nabla\phi$ .
  - **Poisson Eq:**  $\nabla^2\phi = 4\pi G\rho$ .
  - Derivation of **Newton's Theorems:** spherical shells of constant density
- Celestial Mechanics IV
  - Application of Newton's Theorems
    - **Homogeneous sphere:** Potential, Lagrangian, EoM, orbits
  - Derivation of the **Virial Theorem**
    - **moment of inertia, virial Clausius**
  - Application of the Virial Theorem
    - Dynamical mass - discovery of dark matter
  - Summary of the chapter
    - Concepts
      - Ellipse
      - Kepler's laws
      - Newton's law of gravity:  $\mathbf{g}$  and  $\Phi$
    - Point mass system
      - derivation of Kepler's laws from Newton's law of gravity
      - orbital dynamics: Vis Viva Equation (Energy conservation) & Hohmann transfer orbit
    - Spherical symmetric systems
      - Newton's theorems - derivation and application on

- homogeneous sphere
  - Any extended systems
      - Virial Theorem - derivation and application
- Light I
  - Outline:
    - light as a tool for astronomy
      - **parallax, magnitude, distance modulus, color index**
    - light as EM wave
      - speed of light measurement (Roemer 1675)
      - polarization
      - radiation pressure
      - speed of light effect: **Doppler shift**
    - light as quantization of energy
      - blackbody radiation - **Planck function and Stefan-Boltzmann law**
  - Parallax and proper motion
    - distance to the moon
    - implicit assumption: the Sun's parallax is zero
    - how to define reference direction
    - definition of parsec with AU
    - ecliptic latitude determines the ellipticity of parallax ellipse
  - Magnitude system
    - monochromatic magnitude
      - $m = -2.5 \log(f/f_{\text{ref}})$
      - e.g., combined magnitude of a triple star system, each is 15th mag
      - $\log(2) = 0.3$ ,  $\log(3) = 0.5$
      - $dm = 2.5/\ln(10) * df/f = 1.1 * df/f$
    - absolute magnitude := apparent mag at 10 pc
      - Distance Modulus:  $m-M = 5*\log(d) - 5$

- filter magnitude
      - Sensitivity Function & Filter transmission function
    - Color Index
- Light II
  - Light as EM wave
    - Diffraction of light (Poisson Spot demo)
    - Polarization of light (Polarized glass demo)
      - Stokes parameters
    - Speed of light: Roemer's timing of Io
    - Propagation of energy and momentum
      - $E^2 = (m c^2)^2 + p^2 c^2$
      - $E = h \nu$
      - Specific intensity,  $I_\nu = dE/dt dA \cos\theta d\Omega d\nu$
      - Specific flux,  $f_\nu = \text{Integrate}(I \cos\theta d\Omega, \Omega)$
      - Radiation pressure:  $P = \text{Force/Area} = dp/dt / dA$
      - Radiation energy density:  $u = dE / (dA c dt)$
  - The conservation of the specific intensity
  - The relation between specific intensity ( $I_\nu$ ) and specific flux density ( $F_\nu$ )
- Light III
  - Blackbody radiation
    - **Boltzmann statistics**
      - Boltzmann factor
      - Thermodynamic equilibrium
    - Planck function derivation from 2-level atoms
      - **Einstein coefficients**
      - Solve for the radiation field that allows  $n_2/n_1 = \text{Boltzmann factor}$
  - Stefan-Boltzmann equation derivation (on textbook)
  - Radiative cooling through blackbody radiation (assigned bonus point question)
    - Fe:  $\rho = 8 \text{ g/cc}$ ,  $S = 0.45 \text{ J/g/K}$ ,  $r = 10 \text{ cm}$ ,  $T = 500\text{K} =$

$$227 \text{ C} = 440 \text{ F}$$

- $t = H/L = 4.7 \text{ hr} (r/10\text{cm}) (500 \text{ K/T})^3$
- quite inefficient -- conduction and convection cools much faster
- Interaction of Light and Matter I
  - **Bohr's semi-classic model** of H atoms
    - Historical development
      - 1890, discovery of electrons, Thomson
      - 1911, discovery of nuclei, Rutherford
      - 1885, Balmer's empirical formula to reproduce wavelengths of H lines
    - Derivation
      - Quantization of total angular momentum
      - Bohr radius of electron's orbit
    - Application
      - Explains Balmer's formula
  - Atomic processes
    - **Bound-bound**
      - photo-excitation, spontaneous emission, stimulated emission
      - collisional (de)-excitation
    - **$q_{ij}$ : velocity-integrated collisional cross-section**
      - $q_{21}/q_{12} = n_2/n_1 = \text{Boltzmann factor}$
- Interaction of Light and Matter II
  - Two-level atom - collision-dominated case
    - critical density
  - Maxwell-Boltzmann velocity distribution
    - Derivation
    - Properties
- Interaction of Light and Matter III
  - Line broadening processes
    - **natural broadening**

- **thermal broadening**
  - **rotation / turbulence Doppler broadening**
  - **pressure broadening**
  - Zeeman splitting
- Spectroscopic notation system
  - quantum numbers of individual e-:
    - n - principle QN -> energy
    - l - angular momentum QN
    - m\_l - magnetic quantum number
    - m\_s - spin QN
  - quantum numbers of multiple e-:
    - $L = \text{Sum}(l) = l_1+l_2, l_1+l_2-1, |l_1-l_2| = 2, 1, 0$
    - $S = \text{Sum}(m_s) = m_{s1}+m_{s2}, m_{s1}-m_{s2} = 1, 0$  for electrons ( $s = 1/2$ )
    - $J = L+S, L+S-1, \dots, |L-S| \implies$  fine structure lines (e.g., Na I doublet)
    - $F = J-I, J-I+1, \dots, J+I-1, J+I \implies$  hyperfine structure lines (e.g., H I 21cm line)
    - I = the spin of nucleus
  - configuration: e.g., Ne: 1s<sup>2</sup> 2s<sup>2</sup> 2p<sup>6</sup>
  - term & level: e.g., O III: 1s<sup>2</sup> 2s<sup>2</sup> 2p<sup>2</sup>
    - 2P<sub>3/2</sub> & 2P<sub>1/2</sub>
- Interaction of Light and Matter IV
  - Review of Radiative Cooling Problem
    - 3/2 kT for gas vs 3kT for solid (vibrational degree of freedom)
    - specific heat vs. equipartition energy
    - where did 3/2 kT come from for gas?
    - how to define a timescale when the process takes forever to complete?
  - Summarize the chapter
  - Introduce other atomic processes

- Zeeman Splitting
  - Bound-free
    - photo-ionization (photoelectric effect)
    - collisional ionization
  - Free-bound
    - recombination
  - Scattering
    - Thomson scattering (elastic scattering, energy of incident particle preserved. low-energy limit of Compton, particle kinetic energy and photon energy do not change)
    - Compton scattering (inelastic scattering, energy of incident particle not preserved)
    - Inverse Compton scattering
- Stellar Spectra I (Slides, showing latest Gaia data)
  - Spectral Classes and Luminosity Classes
    - Harvard classification scheme: **OBAFGKM** LT Annie Cannon (1901)
    - Why Balmer lines are strongest in A-type stars ( $T = 9000$  K)?
    - Why Ca II lines are strongest in G/K type stars ( $T < 5800$  K)?
    - Morgan-Keenan Luminosity classes: I, II, III, IV, V, VI, VII
  - **Hertzsprung-Russell Diagram**
    - Luminosity vs. Temperature
    - Absolute magnitude vs. color index
    - The key is to measure distances
    - Gaia data visualization
  - A position on the HR diagram  $\Leftrightarrow$  spectral class + luminosity class
- Stellar Spectra II
  - Why we need the combination of Boltzmann and Saha

- equations to understand line strengths?
    - Break down the expression of  $N_2/N_{\text{total}}$  and take two limits ( $T \rightarrow 0$ ,  $T \rightarrow \text{infinity}$ )
  - Understanding stellar spectra
    - Maxwell-Boltzmann velocity distribution: PDF of particle velocities
    - Boltzmann Equation: density ratio between energy levels
    - Saha Equation: density ratio between adjacent ions
  - **Saha Equation Derivation** (this takes most of the lecture)
  - Does it make sense?
    - Why H is fully ionized when  $kT$  is much less than 13.6 eV?  
photoionization & collisional ionization vs. recombination
  - Saha Equation Application
    - pure Hydrogen atmosphere
    - Ca II lines
- Stellar Atmosphere I
  - The conservation of specific intensity  $I_{\lambda}$ 
    - Why we use  $I_{\lambda}$  instead  $F_{\lambda}$  for the transfer equation?
  - **Equivalent widths** - definition (positive for absorption, negative for emission)
  - The **radiative transfer equation**
    - Why the absorption term is dependent on  $I_{\lambda}$ ?
    - The solution of the equation when there is no emission
    - The definition of optical depth
    - How to estimate the optical depth of Saturn's ring based on an image of the reflected light?
  - Dominant processes for opacity (**BB, BF, FF, ES**)
- Stellar Atmosphere II
  - The solution of the transfer equation with absorption only
    - Application: How to estimate and correct for

## atmosphere extinction?

- The solution of the transfer equation with source function
  - Result:  $I_\lambda$  approaches  $S_\lambda$  at high optical depths
  - Application 1: Absorption line formation
  - Application 2: **Limb darkening of solar photosphere**
- Stellar Atmosphere III
  - An intuitive understanding of the radiative transfer solution of limb darkening
    - $I(s=0) = S(\tau_\nu = \cos \theta) = S(\tau = 1) = \langle I \rangle(\tau = 1)$
    - mean free path of photons =  $1/(n \sigma)$
  - **Curve of growth** — calculation and application
    - **EW**, oscillator strength (resonant lines = ground state transitions)
    - $f + \text{broadening} \rightarrow \sigma(\nu) \rightarrow \tau(\nu) = N \cdot \sigma(\nu) \rightarrow e^{-\tau(\nu)} \rightarrow \text{EW}$   
discuss two PDFs for  $\sigma(\nu)$ : Gaussian vs. Lorentz
- **Midterm Exam due (Oct 24)**
- A Review of Stellar Spectra and Stellar Atmosphere
  - Concepts / Definitions
    - spectral classes and luminosity classes
    - partition functions,  $Z_i(T)$
    - quantities defining a radiation field:  $\langle I \rangle$ ,  $F_{\text{rad}}$ ,  $P_{\text{rad}}$ ,  $u_\nu$
    - optical depth =  $\kappa \rho s = \sigma n s$
    - plane-parallel atmosphere: vertical optical depth
    - gray atmosphere
    - Eddington approximation
    - LTE: source function is a Planck function,  $S_\nu = B_\nu(T)$
    - mean free path
    - equivalent width
  - Equations of Temperatures

- Planck function:  $T_b$  - brightness temperature
  - Boltzmann equation:  $T_x$  - excitation temperature
  - Maxwell-Boltzmann velocity distribution:  $T_k$  - kinetic temperature
  - Saha equation:  $T_i$  - ionization temperature
  - Radiative transfer equation
    - macroscopic opacity/emissivity vs. microscopic quantities (Einstein coefficients & oscillator strength)
    - source function
    - general solution of RT equation
    - absorption only solution of RT equation
  - Key diagnostic tools
    - HR diagram  $\Leftrightarrow$  classification of stars
    - Curve of Growth  $\Rightarrow$  column density of elements
- Stellar Interior I
  - An overview
    - **hydrostatic equilibrium**, ideal gas law, continuity equation
    - energy transport & generation equations
    - constitutive solutions
  - Focus on the top three equations
    - continuity equation - spherical symmetry
    - ideal gas law - define  $\mu$
    - hydrostatic equilibrium - derivation
  - Polytropic models and Lane-Emden Equation
    - introduce polytropic pressure equation of state
    - examples: e- degenerate gas in white dwarf
    - derivation of **Lane-Emden Equation**
      - define  $n$ ,  $D_n$ , and  $\lambda$
      - make it dimensionless
    - solutions of Lane-Emden Equation

- Stellar Interior II
  - Polytropic Model: **Eddington Standard Model**
    - $n = 3$  case when there is a constant ratio between thermal energy density and photon energy density
    - Mean molecular mass calculations
      - number fractions vs. mass fractions
      - neutral vs. ionized gas
      - e.g., pure H and He gas, neutral vs. ionized
  - Temperature gradient equation
    - derivation for radiative transport
    - application: estimate luminosity
  - main-sequence evolution of the Sun
    - Two consequences of nuclear fusion
      - H  $\rightarrow$  He, so  $\mu$  increases  $\rightarrow$   $T_c$  increases
      - $e^+$  &  $e^- \rightarrow \gamma$ , so that  $e^-$  are destroyed  $\rightarrow$   $\kappa$  decreases
    - Two ways to explain the MS luminosity evolution
      - increase of nuclear reaction rate
        - intuitive understanding of the Gamov peak
        - Coulomb barrier vs. decreasing cross section
        - importance of QM tunneling
      - decrease of opacity from  $e^-$
    - Faint Young Sun Paradox
      - a frozen earth predicted by astronomers vs. sedimentary rocks dated  $\sim 4$  Gyrs.
- Stellar Interior III
  - Equation of energy generation (aka, the luminosity gradient equation)
  - Derive the formula form for  $\epsilon_{\text{gravity}} = L/M = dU/dt / M$
  - Nuclear reaction rate (Gamov peak)
    - pp chain
    - CNO cycle

- triple alpha process
    - alpha capturing process
  - Temperature gradient equations
    - radiative energy transport
    - the condition for convection:  $|dT/dr|_{\text{actual}} > |dT/dr|_{\text{adiabatic}}$
    - convective energy transport
- ISM and Star Formation I (Tracers of cold ISM)
  - ISM - Star Formation - Stellar Evolution - ISM cycle
  - Dust: extinction and polarization
    - $A = \tau = \text{cross\_section} * \text{column\_density}$
    - **Mie scattering:**
      - $\text{cross\_section} \sim 1/\lambda$
      - The Mie solution to Maxwell's equations describes the scattering of an electromagnetic plane wave by a homogeneous sphere.
    - Deviations from the Mie theory and the composition of ISM dust
  - **HI 21 cm line:** traces atomic gas
    - timescale for spontaneous decay
    - emission line: traces N, not T
    - absorption line: traces N/T
    - N<sub>HI</sub> vs N<sub>dust</sub> relation
    - Dust-to-Gas ratio  $\sim 0.01$
- ISM and Star Formation II
  - Seminar question: the polarization angle of dust re-emission vs. dust extincted background light
  - **CO rotational lines**
    - H<sub>2</sub> is lack of an electric dipole
    - **electric dipole moment**
    - moment of inertia
    - quantization of angular momentum

- the CO ladder and CO isotopes
  - Jeans criterion: the condition for gravitational collapse
    - Violation of the virial theorem ( $2K + U = 0$ )
- **Thanksgiving break (Nov 19-23, 2018)**
- ISM and Star Formation III (Driving mechanisms of SF)
  - **Jeans criterion for gravitational collapse:**
    - $2K + U < 0$  (collapse),  $2K + U > 0$  (expansion)
    - $t_{\text{ff}} < R/c_s =$  sound cross timescale
  - homologous collapse:
    - ~~formal derivation using the equation of motion and assume no shell crossing~~
    - Simplified derivation of the free-fall timescale with Kepler's 3rd law (for an orbit w/  $e \rightarrow 1$ )
  - Cloud fragmentation
    - isothermal  $t_{\text{ff}} > t_{\text{KH}}$
    - adiabatic  $t_{\text{ff}} < t_{\text{KH}}$
    - the minimum Jeans mass
  - The initial mass function (IMF)
- ISM and Star Formation IV (Suppressing mechanisms of SF)
  - Hierarchical structure of Molecular Clouds
    - GMC, Complexes, Clumps, Cores
  - Mechanism that suppresses free fall
    - star formation proceeds on **Kelvin-Helmholtz timescale**, not on free-fall timescale
    - turbulence
    - rotational support, conservation of total angular momentum
    - magnetic field
    - momentum and energy feedback from SNe
  - Understanding the low SFR of the Milky Way galaxy
    - $\text{SFR} = \# \text{ of clouds} * \text{SFR}_{\text{per\_cloud}} = (\text{epsilon } M_{\text{H}2} / M_{\text{J}}) * (M_{\text{J}} / t_{\text{SF}}) = \text{epsilon } M_{\text{H}2} / t_{\text{SF}}$

- $\epsilon = 0.01$  and  $t_{\text{SF}} \sim 1e7 \sim t_{\text{KH}}$  can explain why the MW has a total SFR of only 1 Msun/yr given  $M_{\text{H2}} = 1e9 \text{ Msun}$
- why  $\epsilon \sim 0.01$ , i.e., only 1% of the total  $M_{\text{H2}}$  participates in active star formation?
  - this is unexpected because clouds at all scales are more massive than the Jeans mass
    - $M_{\text{J}} = 8 \text{ Msun} (T/10\text{K})^{1.5} (1e10 \text{ m}^{-3} / n)^{0.5}$  where  $T$  is the excitation temperature
    - clumps w/  $n \sim 1e9$  and  $T = 10 \text{ K}$  have  $M_{\text{J}} = 24 \text{ Msun} < 30 \text{ Msun}$ .
    - complexes w/  $n \sim 5e8$  and  $T = 10 \text{ K}$  have  $M_{\text{J}} = 34 \text{ Msun} \ll 1e4 \text{ Msun}$
    - GMCs w/  $n \sim 1e8$  and  $T = 15 \text{ K}$  have  $M_{\text{J}} = 140 \text{ Msun} \ll 1e5 \text{ Msun}$
  - something must have increased the Jeans mass for clouds larger than cores.
    - turbulence support -> **Larson's relation**
    - The origin of interstellar turbulence?
    - Galactic dynamics (orbit crossing) & Supernovae

- Review Session
- Q&A Session