
Chap 2: Our Star - The Sun
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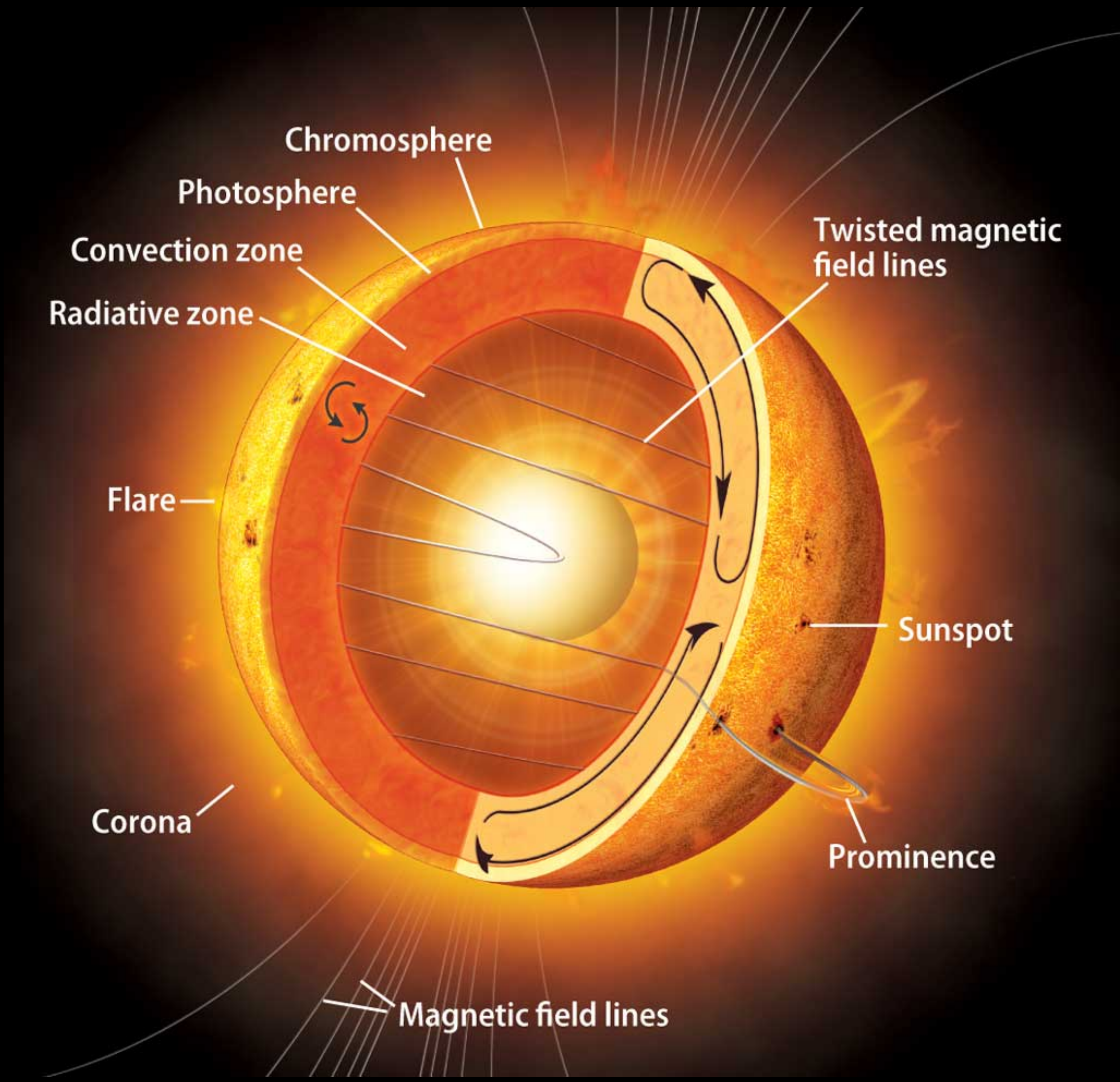
Chap 2: Our Star - The Sun: Key Concepts

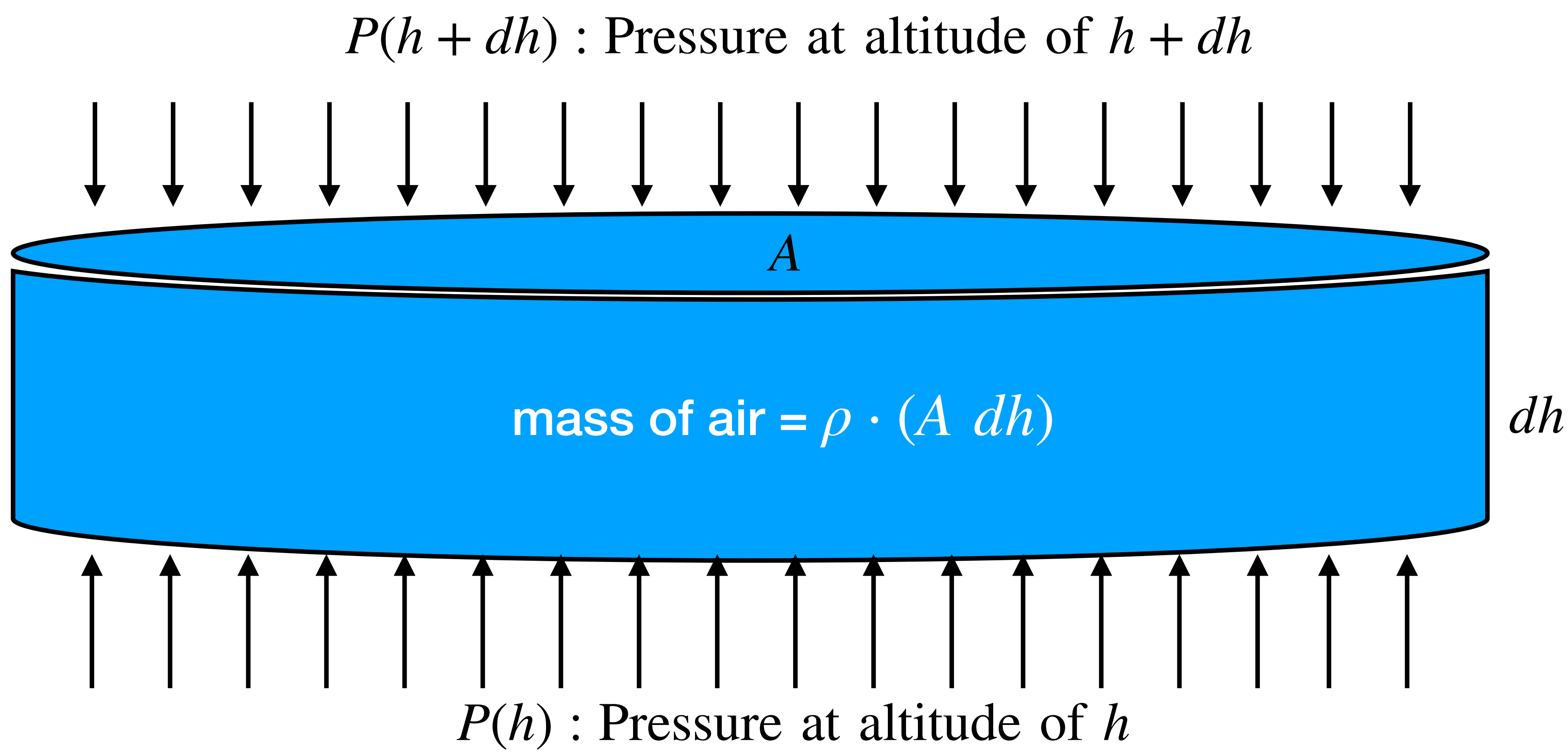
- The sheer mass of the Sun and hydrostatic equilibrium creates the necessary conditions for fusion: dense and hot gas
- Fusion can maintain Solar luminosity over billions of years
- How energy is transported out?
- Fusion model can be tested by neutrino detectors
- Interaction cross section and mean free path
- Last scattering surface
- How limb darkening & absorption lines are produced?
- How temperature determines line strength?
- Solar activities



How a self-gravitating sphere of gas stay stable in size?

Hydrostatic Equilibrium

Hydrostatic Equilibrium Equation for thin Atmospheres



- consider the force balance in a packet of air at an altitude of **h**. The packet has a cylindrical shape with an area of **A** and an infinitesimal height of **dh**
- this packet of air can stay stationary because of a force balance
  - upward force from pressure = $[P(h) - P(h + dh)] \cdot A = - dP \cdot A$
  - downward gravitational force = $\rho \cdot (A \; dh) \cdot g$

Hydrostatic Equilibrium of Thin Plane-Parallel Atmospheres

- consider the force balance in a packet of air at an altitude of **h**. The packet has a cylindrical shape with an area of **A** and an infinitesimal height of **dh**
- this packet of air can stay stationary because of a **force balance**
  - upward force from pressure = $[P(h) - P(h+dh)] \cdot A = -\, dP \cdot A$
  - downward gravitational force = $\rho \cdot (A\ dh) \cdot g$
- Equating the two forces means **hydrostatic equilibrium**:

$$-dP = \rho\ g\ dh \Rightarrow$$

$$-d(nkT) = (\mu m_p n)\ g\ dh$$

where we have applied the **ideal gas law** and expressed mass density as number density. If the temperature is constant, i.e., the **isothermal condition**, we can take *kT* out of the differentiation, and we can also move n to the left side (assuming also **thin atmosphere** so that *g* is constant):

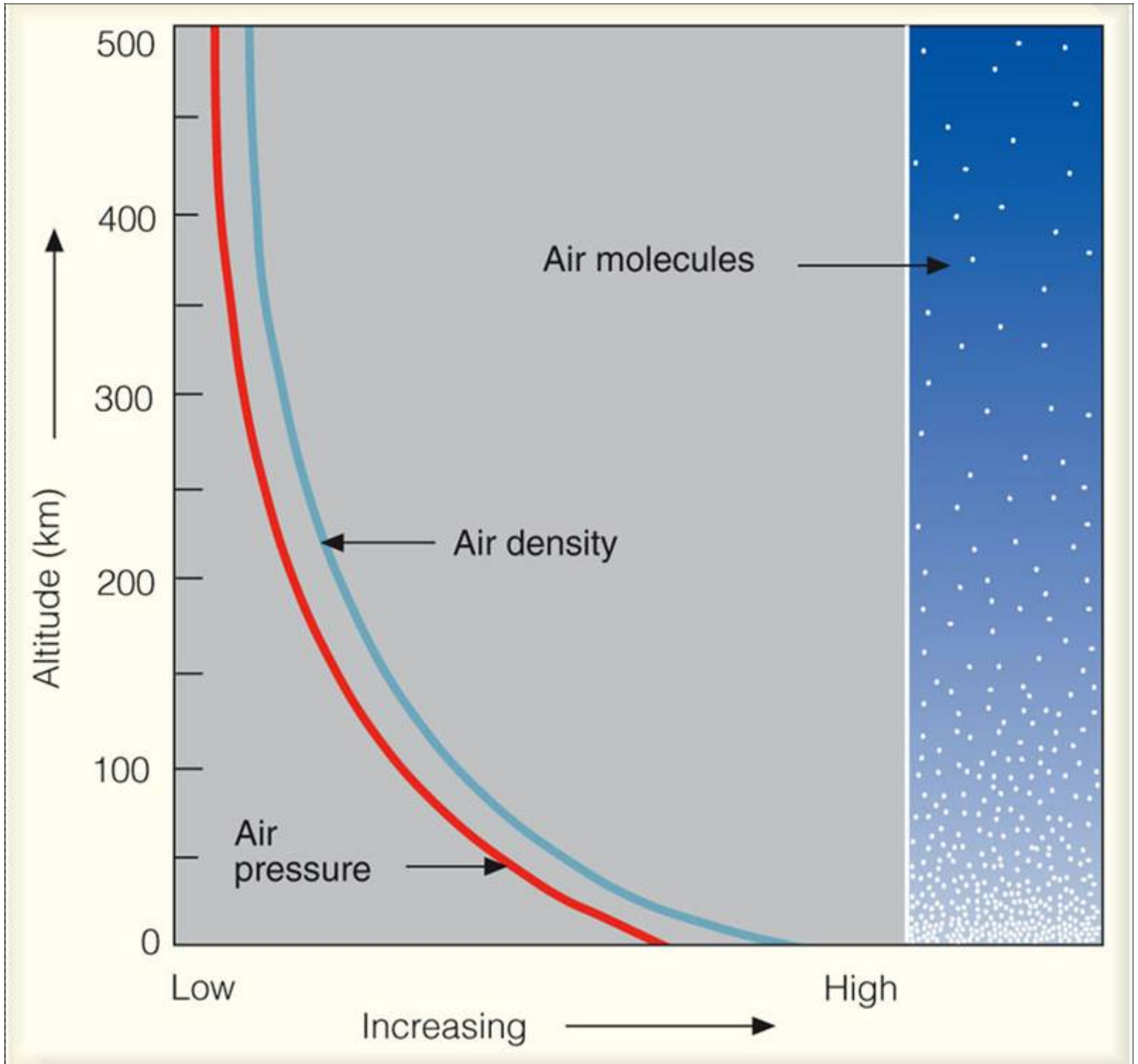
$$\frac{dn}{n} = -\frac{\mu m_p g}{kT}\ dh$$

- Integrating both side from altitude of *0* to altitude of *h*, we have a solution:

$$n(h) = n_0 \exp\left(-\frac{h}{h_S}\right) \text{ where } h_S = \frac{kT}{\mu m_p g} \text{ is the } \textbf{scale height}.$$

Pressure/density profile of roughly isothermal, thin atmosphere

$n(h) = n_0 \exp\left(-\frac{h}{h_S}\right)$ where $h_S = \frac{kT}{\mu m_p g}$ is the **scale height**.



exp(x) = e^x, *where e* is Euler's number 2.71828

Practice: Calculate the scale height of N_2 atmosphere

$$n(h) = n_0 \exp\left(-\frac{h}{h_S}\right) \text{ where } h_S = \frac{kT}{\mu m_p g} \text{ is the **scale height**.}$$

For an 100% N_2 atmosphere, assume a constant temperature of 280 K (+7 C).

Constants: k = 1.38e-23 J/K, m_p = 1.67e-27 kg, g = 9.8 m/s^2, e = 2.718

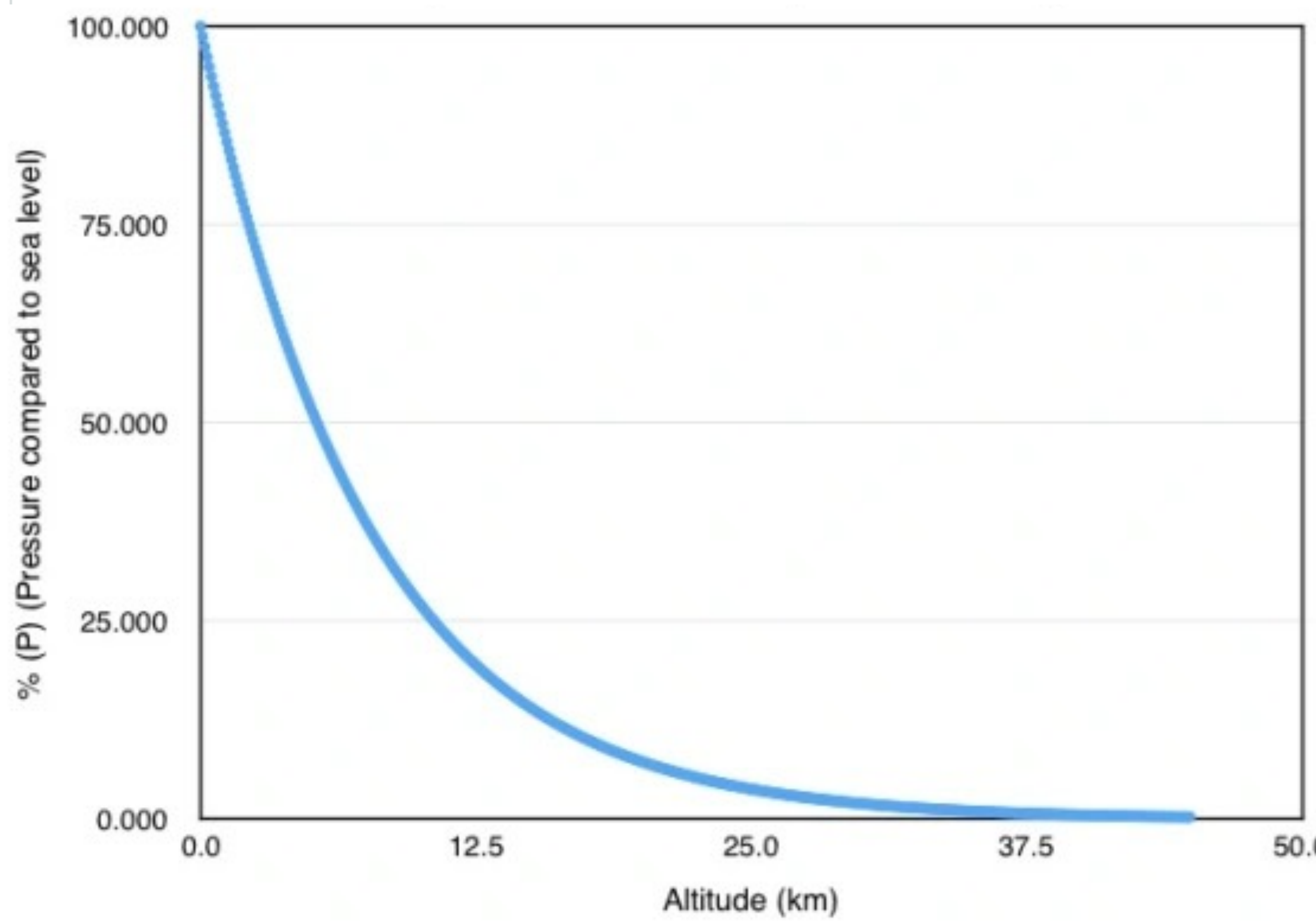
1. What is the mean molecular weight?

2. What's the scale height?

2. What are the N_2 densities at 1x and 3x the scale height if its density at sea level (n_0) is set to be at 1 unit (~10^{25} m^{-3}).

(Boltzmann constant * 280 kelvin) / (28 * proton mass * (9.8 * (m / (s^2)))) =

8.42285689 kilometers

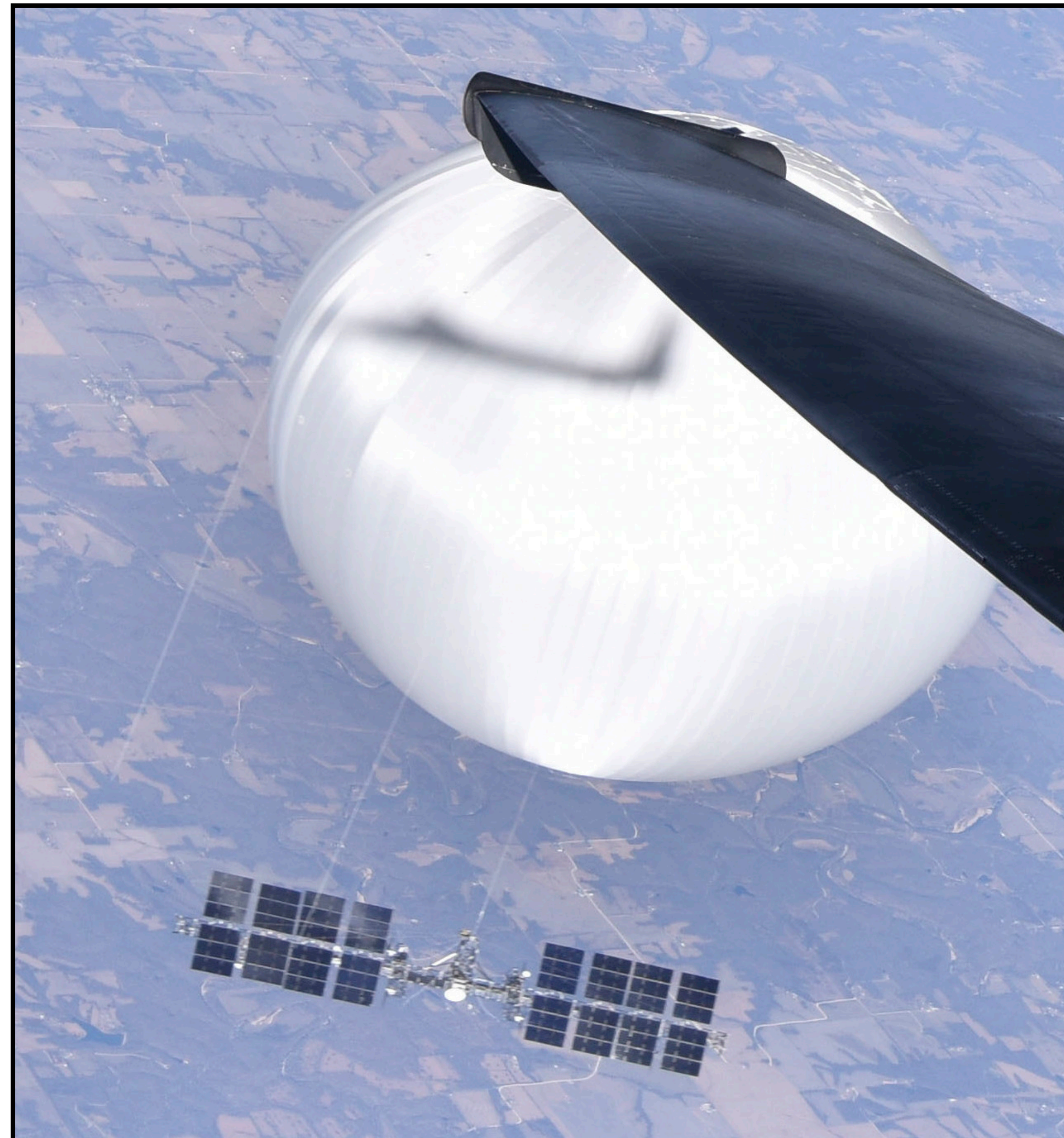


Answer: 8.4 km, 0.37 unit at 1 h_s, 0.05 unit at 3 h_s

Practice: Weight capacity of high altitude balloons

$$\rho(h) = \rho_0 \exp\left(-\frac{h}{h_S}\right) \text{ where } h_S = \frac{kT}{\mu m_p g} \text{ is the } \textbf{scale height}.$$

- Suppose we have measured:
  - the scale height of the Earth's atmosphere: 8 km
  - the air density at the sea level: 1.2 kg/m^3
  - the altitude of the balloon: 18 km (60,000 ft)
  - the diameter of the balloon: 60 m (200 ft)
- How much mass the balloon has to carry to stay at the fixed altitude? i.e., neither fall downward nor float upwards



1.2 * exp(-18/8) * (4 * PI/3 * 30^3) = 14304 kg = 14 metric tons

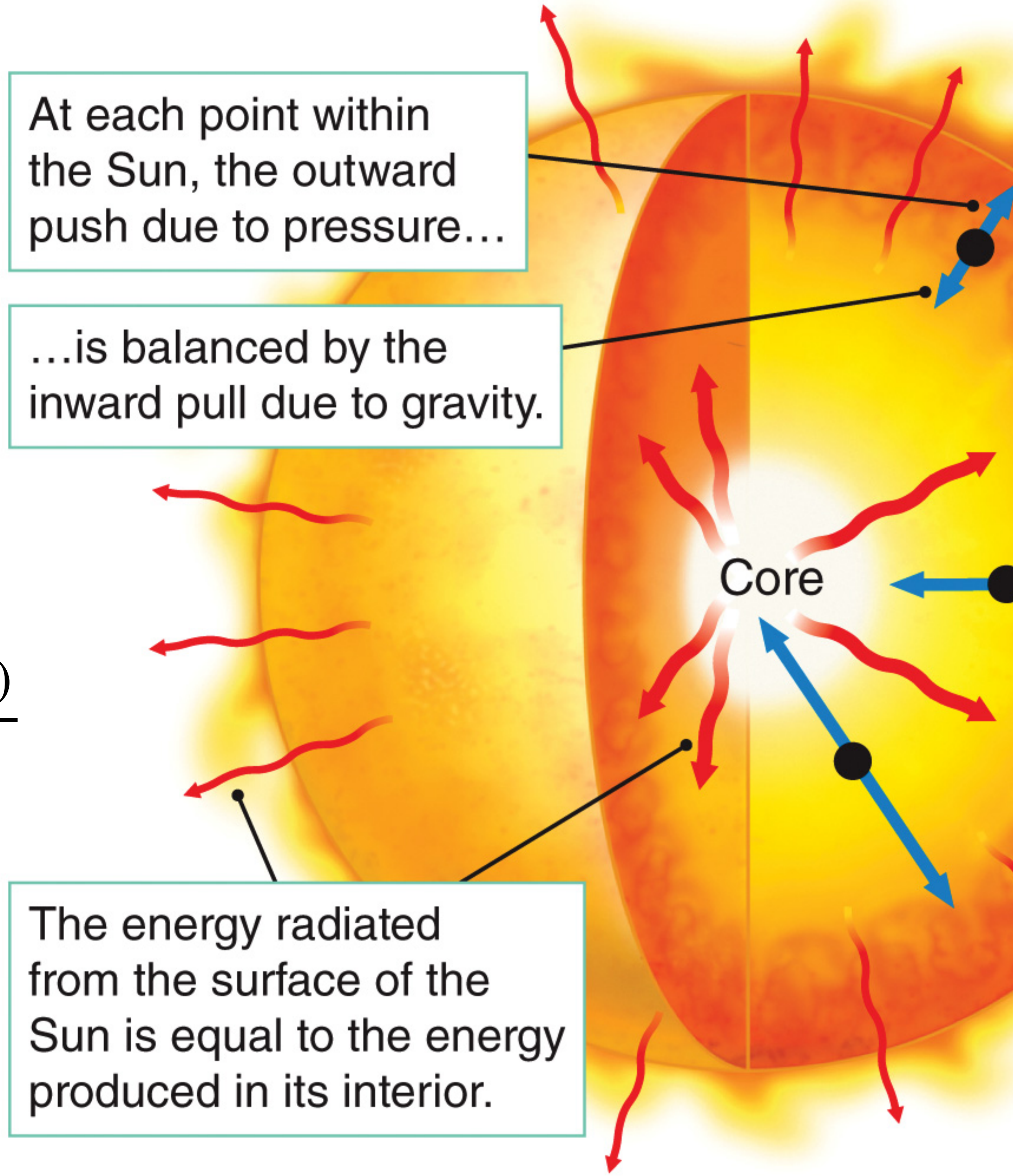
The Sun is Spherical Object in Hydrostatic Equilibrium

- Outward pressure gradient balances inward force of gravity at each layer in the Sun, maintaining **Hydrostatic Equilibrium.**
- For **spherical symmetry**, the equation for the equilibrium is:

$$\frac{dP(r)}{dr} = -\rho(r)\ g(r) = -\rho(r)\frac{GM(r)}{r^2}$$

- Compared to **plane-parallel** atmosphere:

$$\frac{dP(h)}{dh} = -\rho(h)g(h)$$



Given hydrostatic equilibrium,
infer interior conditions of
the Sun

Stellar Structure Models - Basic Equations

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}$$

HYDROSTATIC EQUILIBRIUM

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

MASS CONSERVATION

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

ENERGY EQUATION

$$\left.\frac{dT}{dr}\right|_{rad} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

RADIATIVE TRANSPORT

$$\left.\frac{dT}{dr}\right|_{\mathrm{ad}} = -\left(1-\frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2}$$

ADIABATIC CONVECTION

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Stellar Structure Models - Constitutive Relations

CONSTITUTIVE RELATIONS (CR)

$$P = \frac{\rho k T}{\mu m_{\rm H}} + \frac{1}{3} a T^4$$

$$\overline{\kappa} = \left\{ \begin{array}{l} \overline{\kappa}_{bf} = \text{bound-free} \\ \overline{\kappa}_{ff} = \text{free-free} \\ \overline{\kappa}_{es} = \text{electron scattering} \end{array} \right\}$$

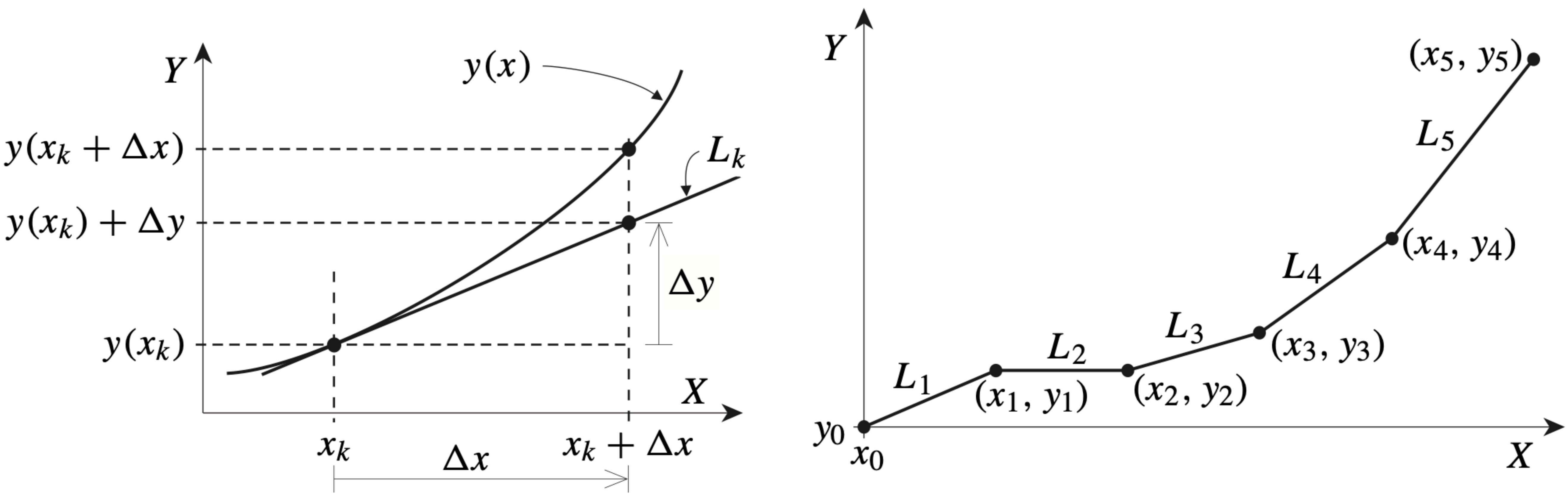
FROM TABLES OR FITTED TO A FUNCTION

$$\epsilon = \left\{ \begin{array}{l} \epsilon_{\text{pp-chain}} \\ \epsilon_{\text{CNO cycle}} \\ \epsilon_{3\alpha} \end{array} \right.$$

Euler Method: a numerical procedure to solve differential equations

$$\frac{dy}{dx} = f(x, y)$$

$$y(x_k + \Delta x) \approx y_k + \Delta x \cdot f(x_k, y_k)$$



https://www.uah.edu/images/people/faculty/howellkb/DEText-Ch9.pdf

$$r = R^* \begin{cases} T \to 0 \\ P \to 0 \\ \rho \to 0 \end{cases}$$

Euler Method: a numerical procedure to solve differential equations

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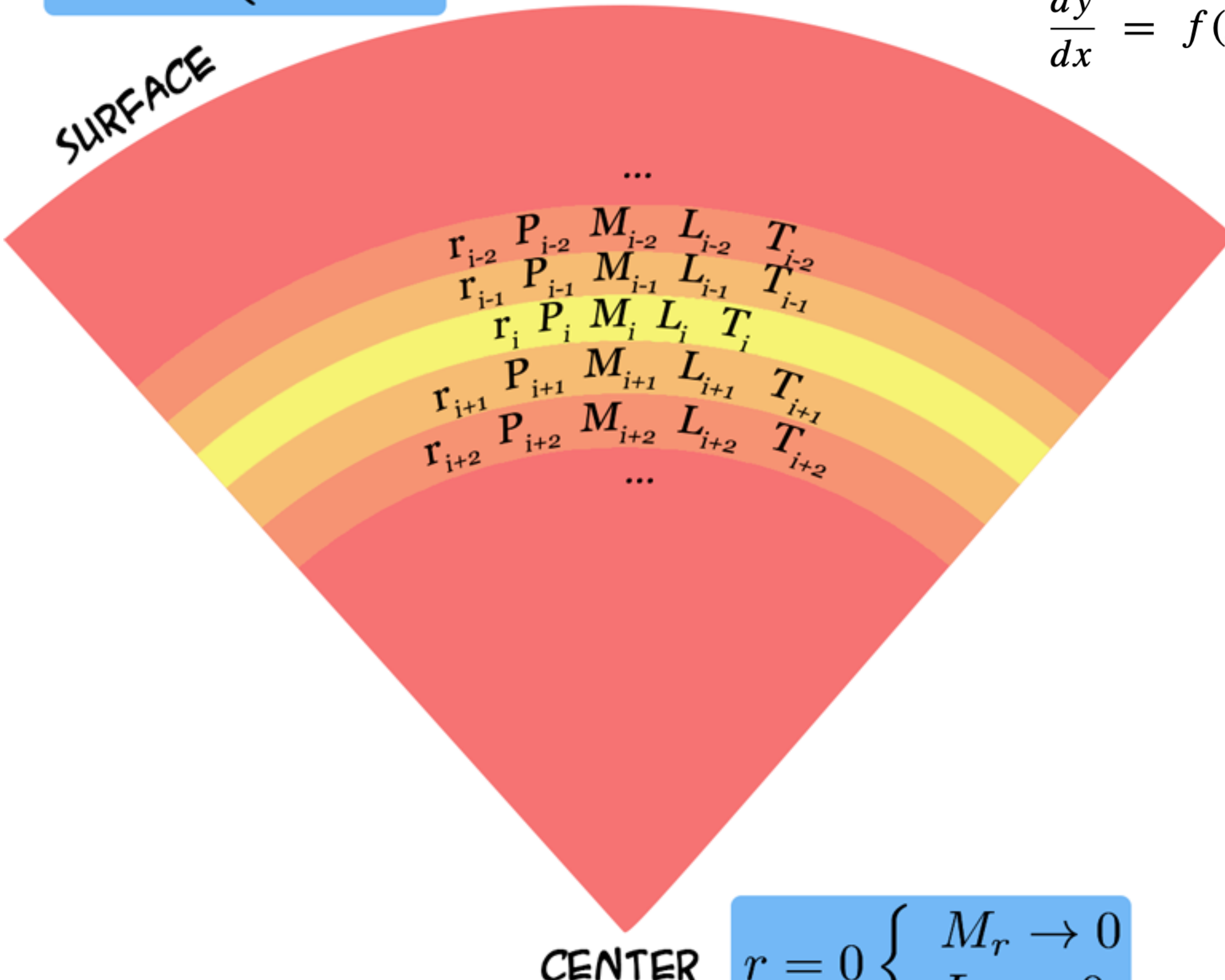
$$\frac{dy}{dx} = f(x, y)$$



$$r = 0 \begin{cases} M_r \to 0 \\ L_r \to 0 \end{cases}$$

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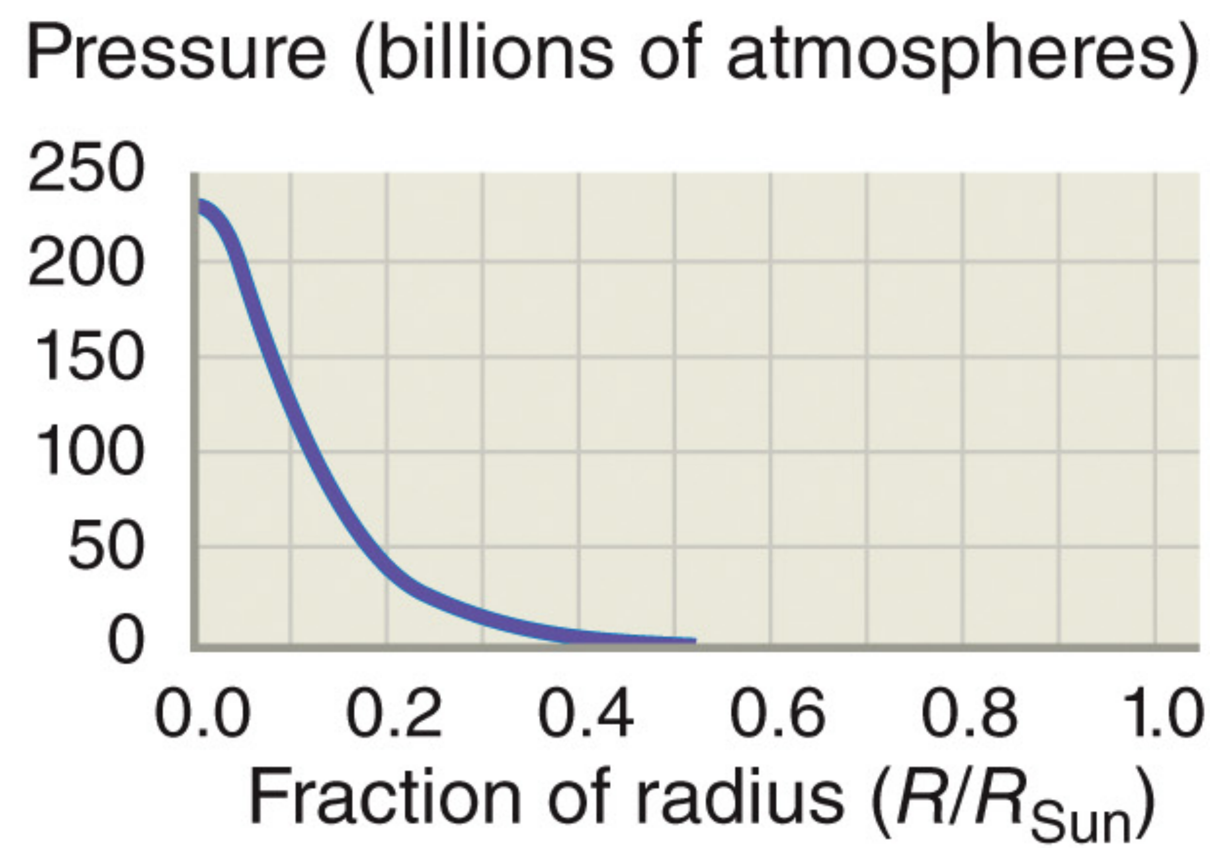
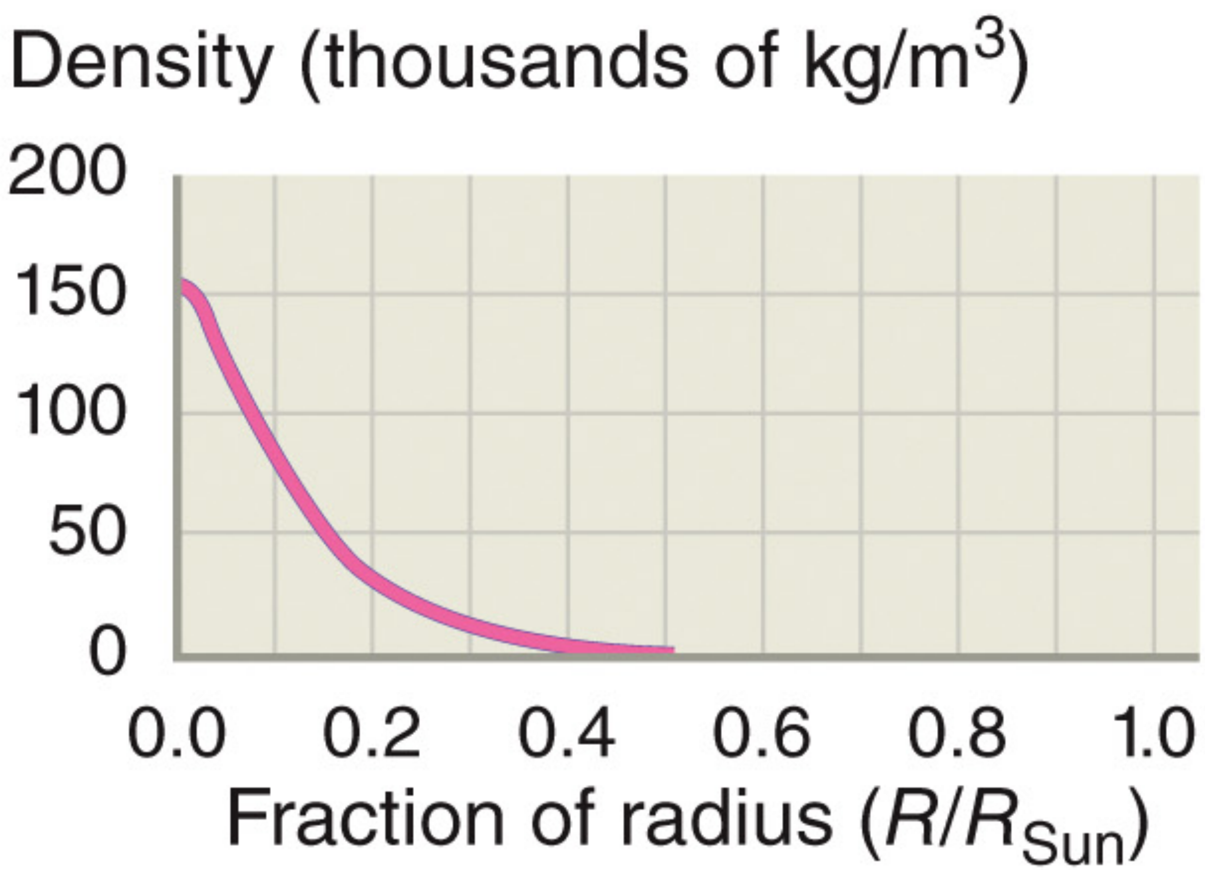
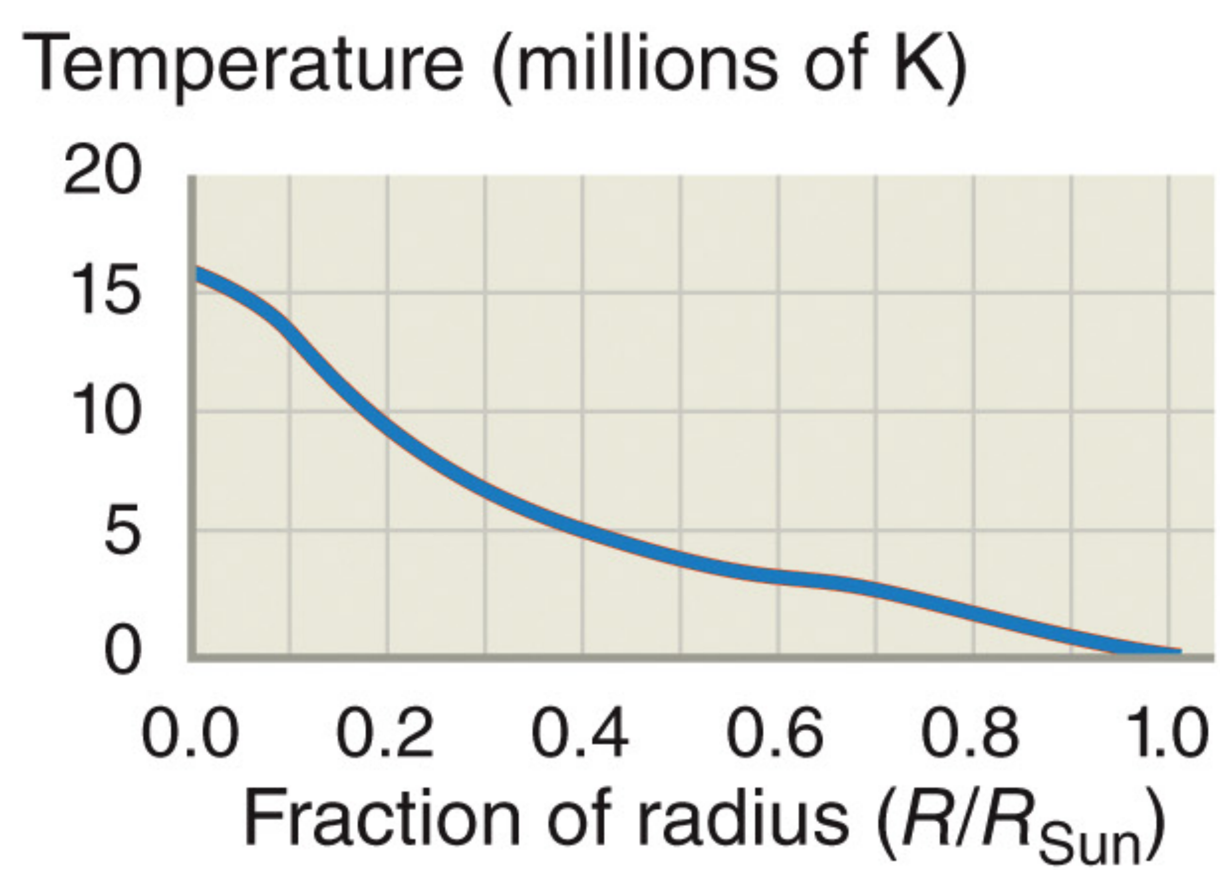
Conditions in the Center of the Sun

- Density, temperature, and pressure increase toward the center of the Sun, thus creating the necessary conditions for **nuclear fusion**.
- 15 million K
- 150 metric ton/m^3







Increasing Pressure and Temperature as You Are Buried Deeper

How the Sun generates its energy?

Nuclear Fusion

Energy Balance: Energy Generation Rate = Energy Loss Rate

- As measured by radiometric dating of meteorites, the Sun has existed for at least 4.6 billion years;
- The Sun releases 3.8e26 Joules per second through electromagnetic radiation; For 4.6 billion years, this amounts to a total energy of 5.5e43 Joules.

- For comparison, the total gravitational potential energy lost during the gravitational collapse of the solar nebula is only:

$$E_G = 0 - (-GM^2/R) = 3.8 \times 10^{41} J$$

which could only last the Sun for 32 Myrs.

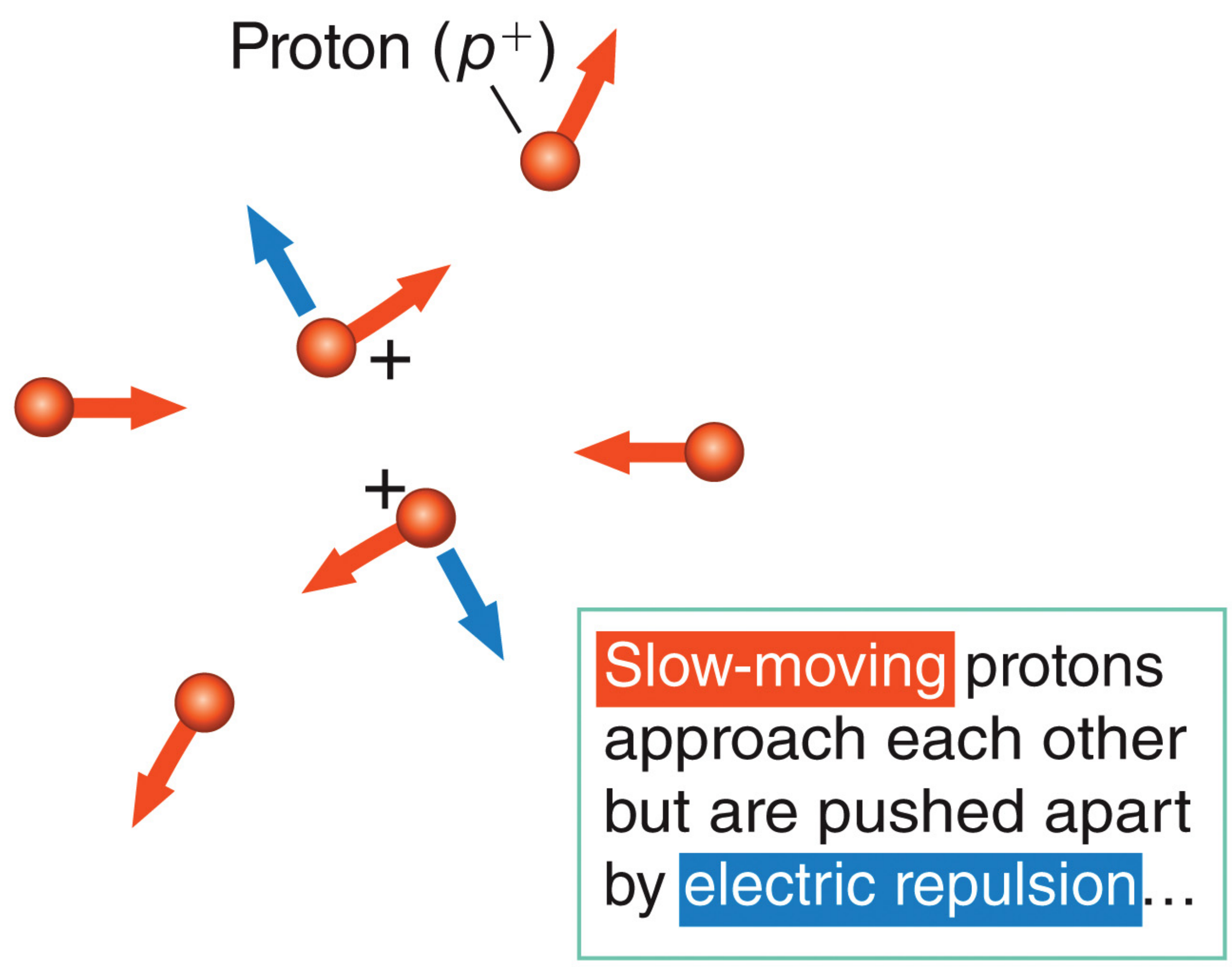
- Nuclear fusion is the only viable source of energy capable of powering the Sun for this long. Fusion creates more massive nuclei from less massive ones but loses total mass in the process.

Nuclear Fusion: the Coulomb barrier

- **Nuclear fusion** involves the fusing of atomic nuclei to form heavier elements.
- Nuclei consist of protons (positively charged) and neutrons (no charge). So electrostatic force push nuclei apart following Coulomb's law:

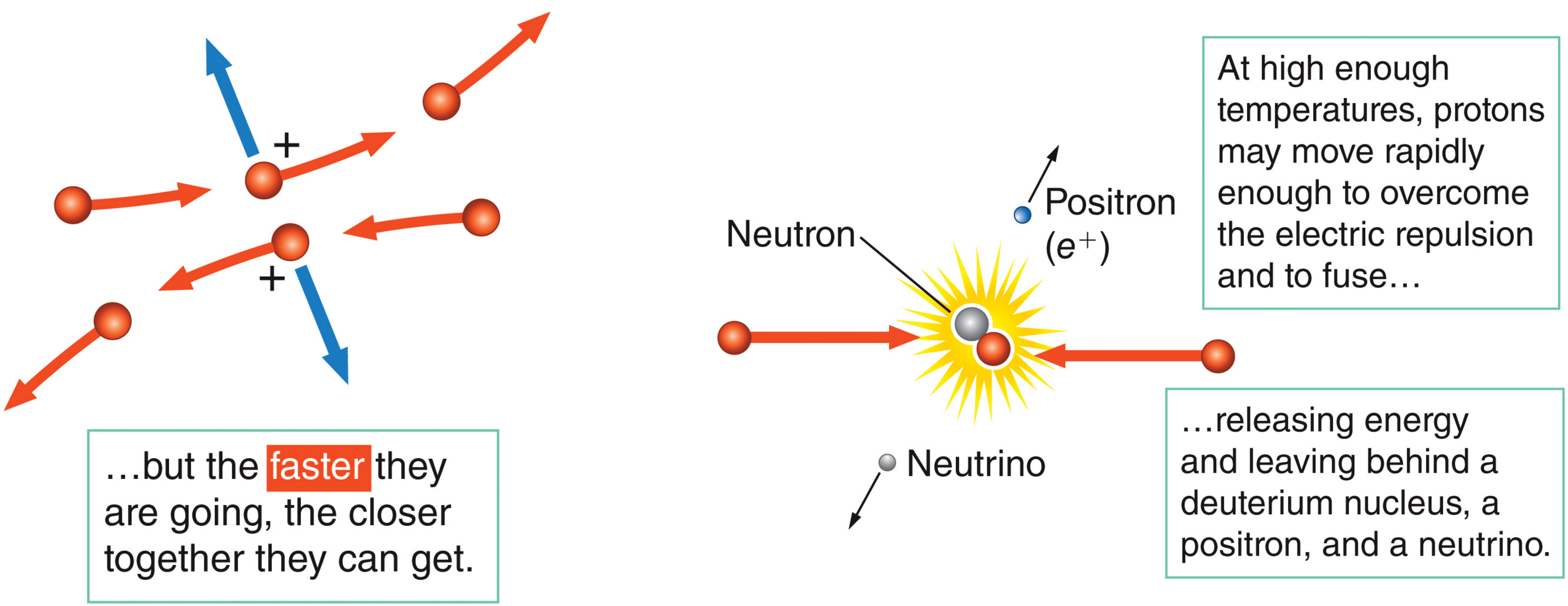
$$F = K\frac{q_1 q_2}{r^2}$$

- Eventually, the strong nuclear force can overcome the push of the electrostatic force and bind protons together. But to enable fusion, this Coulomb barrier must be overcome first so that the nuclei are brought into extreme proximity.



Nuclear Fusion: Required Conditions

- Fusion requires slamming protons together at high speed (i.e., at high temperature) to overcome the Coulomb barrier.
- Sufficient frequency of collisions can be sustained only when the nuclei are densely packed together (i.e., at high density).



Conditions in the Center of the Sun

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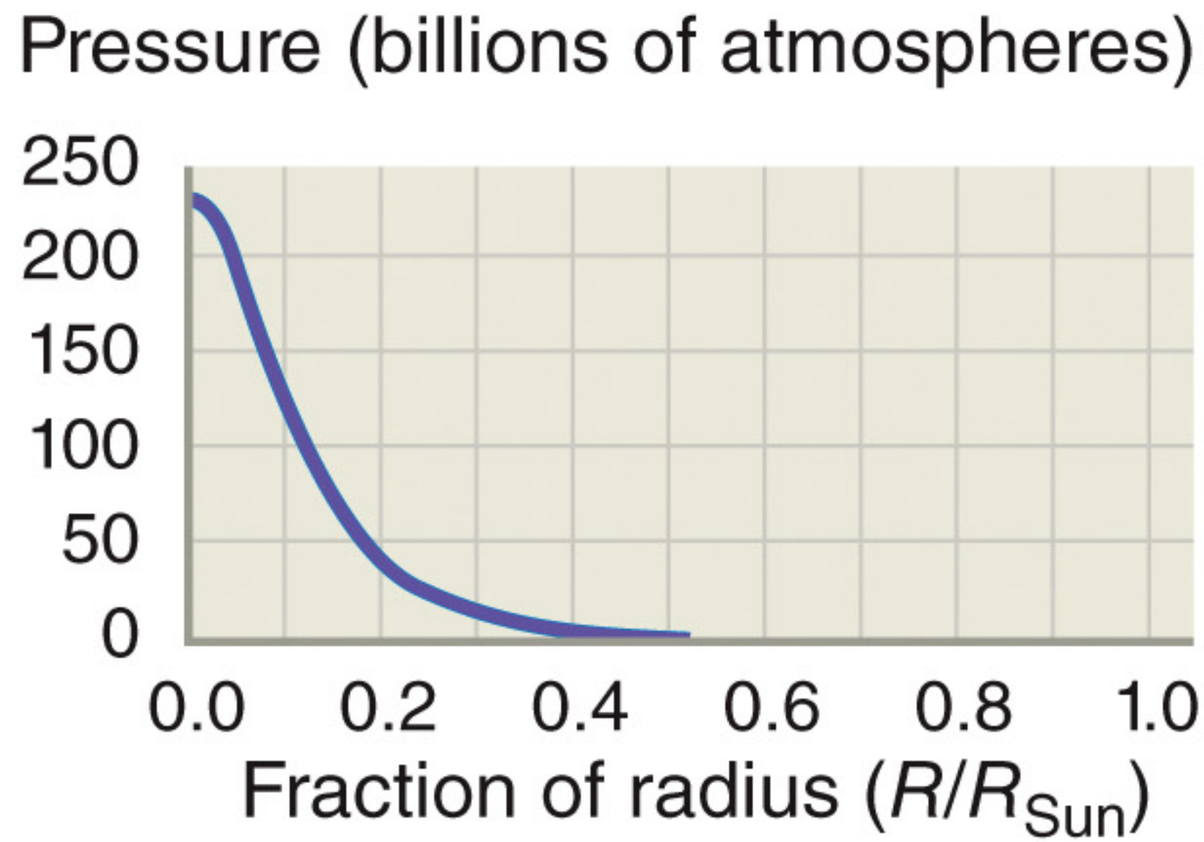
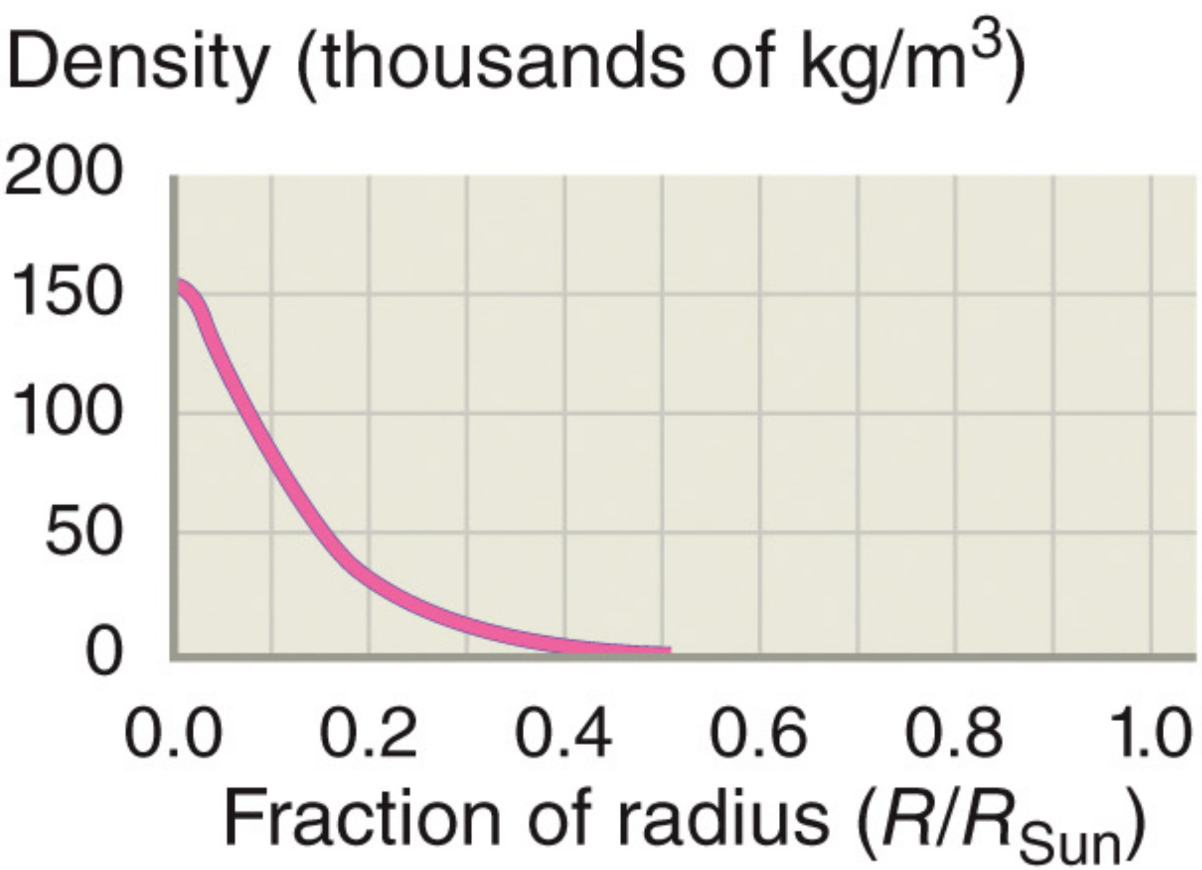
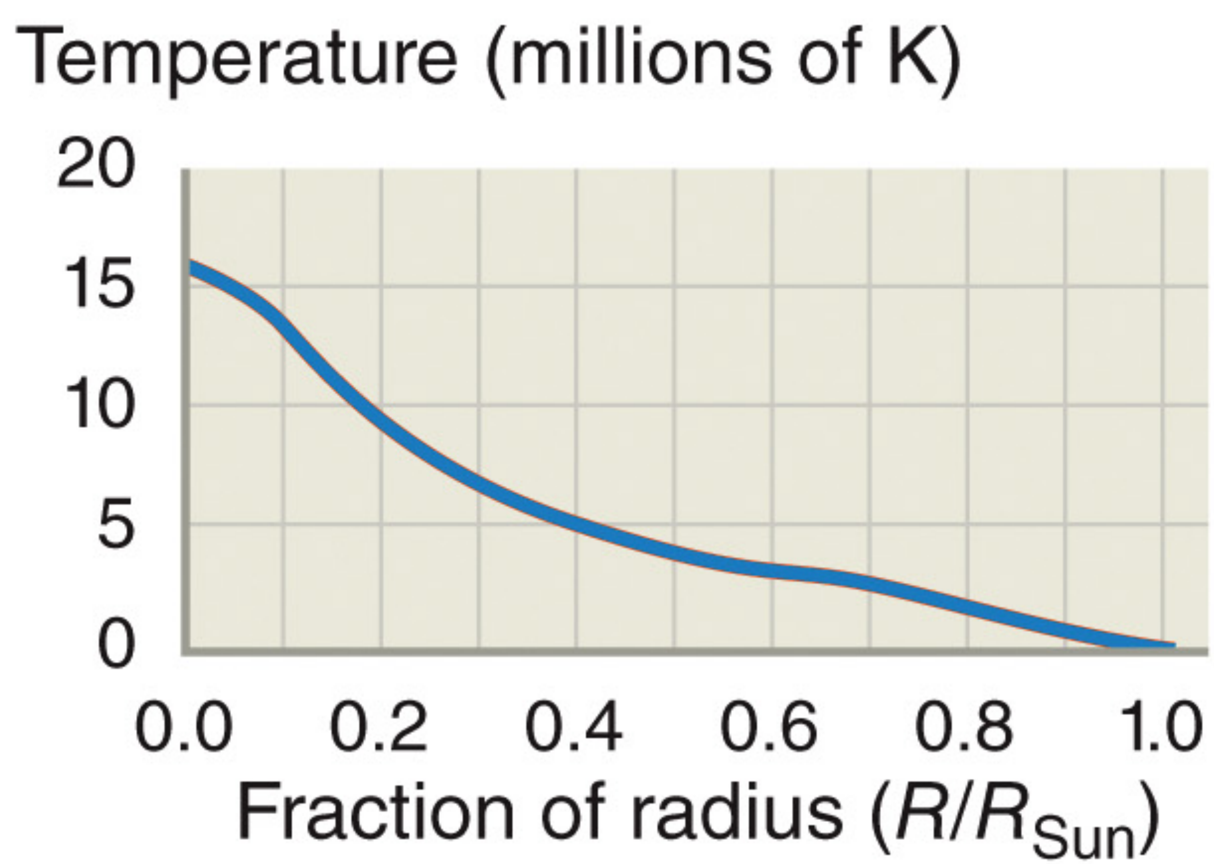




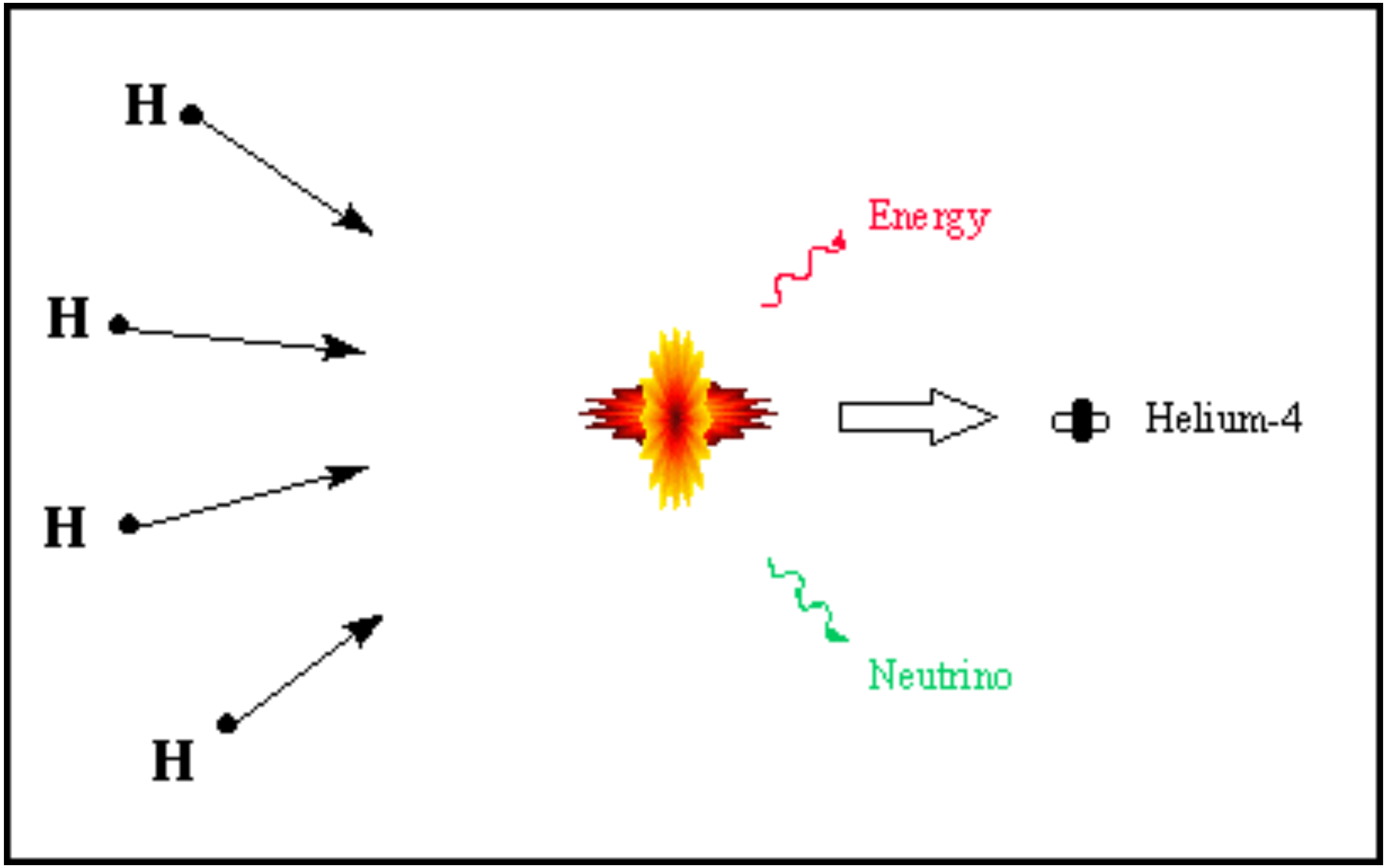


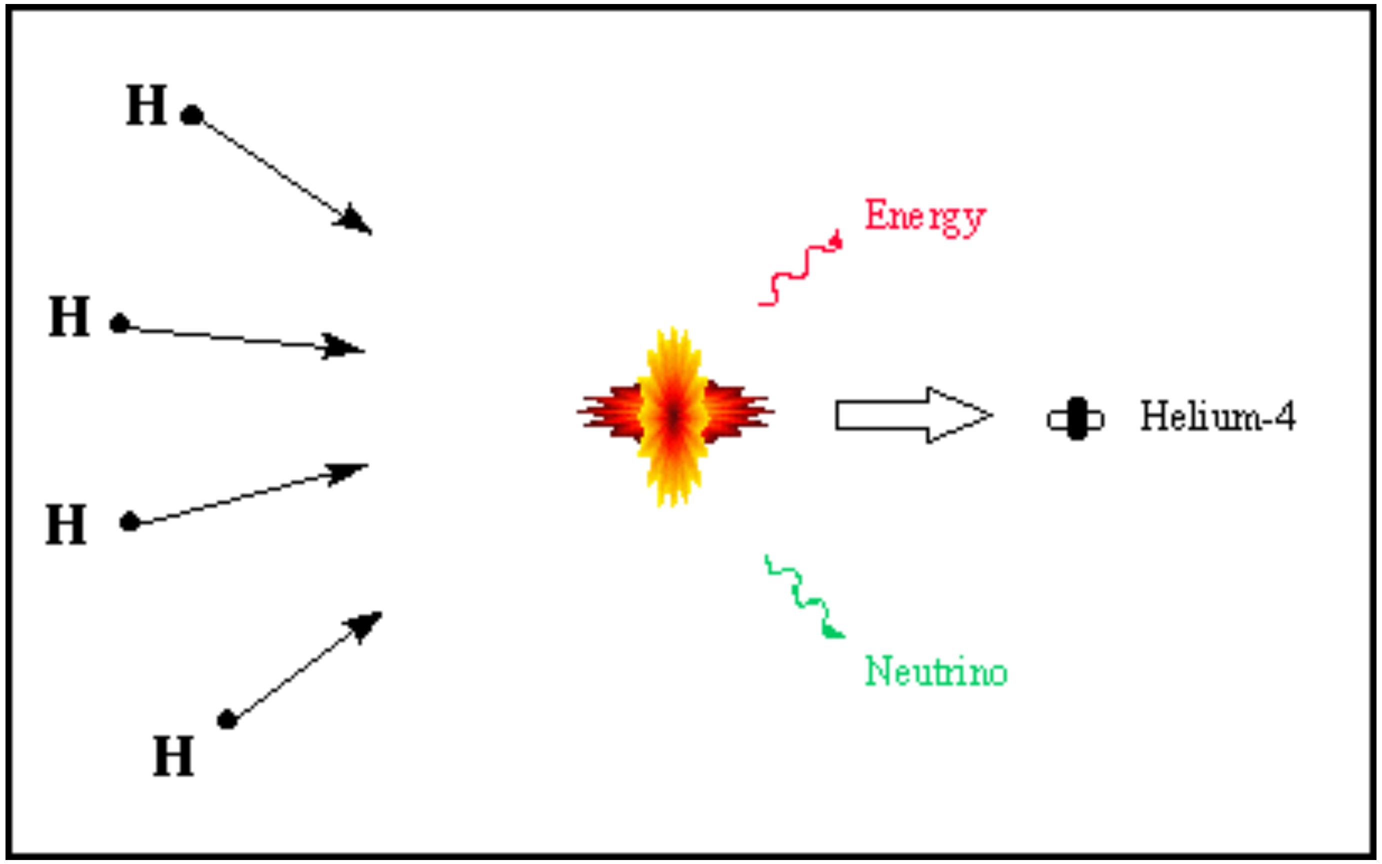
How the Sun generates its energy?

The proton-proton chain

Hydrogen Burning in the Solar Core

- Hydrogen is the most abundant element in the Universe (~75% of the baryonic mass).
- Fusing Hydrogen to the next element (Helium) requires four Hydrogen atoms
- How does this process actually happen? Can we simply slam 4 protons?





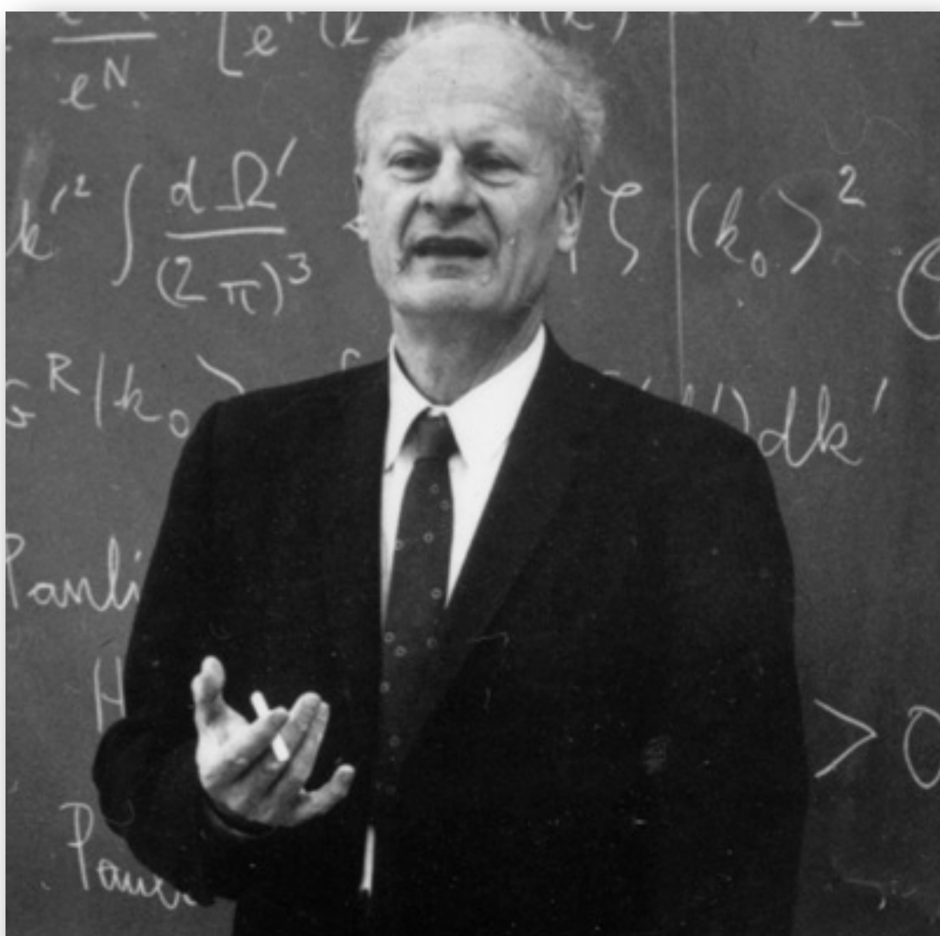
But the probability of 4 hydrogen atoms all colliding at the same time is extremely small.

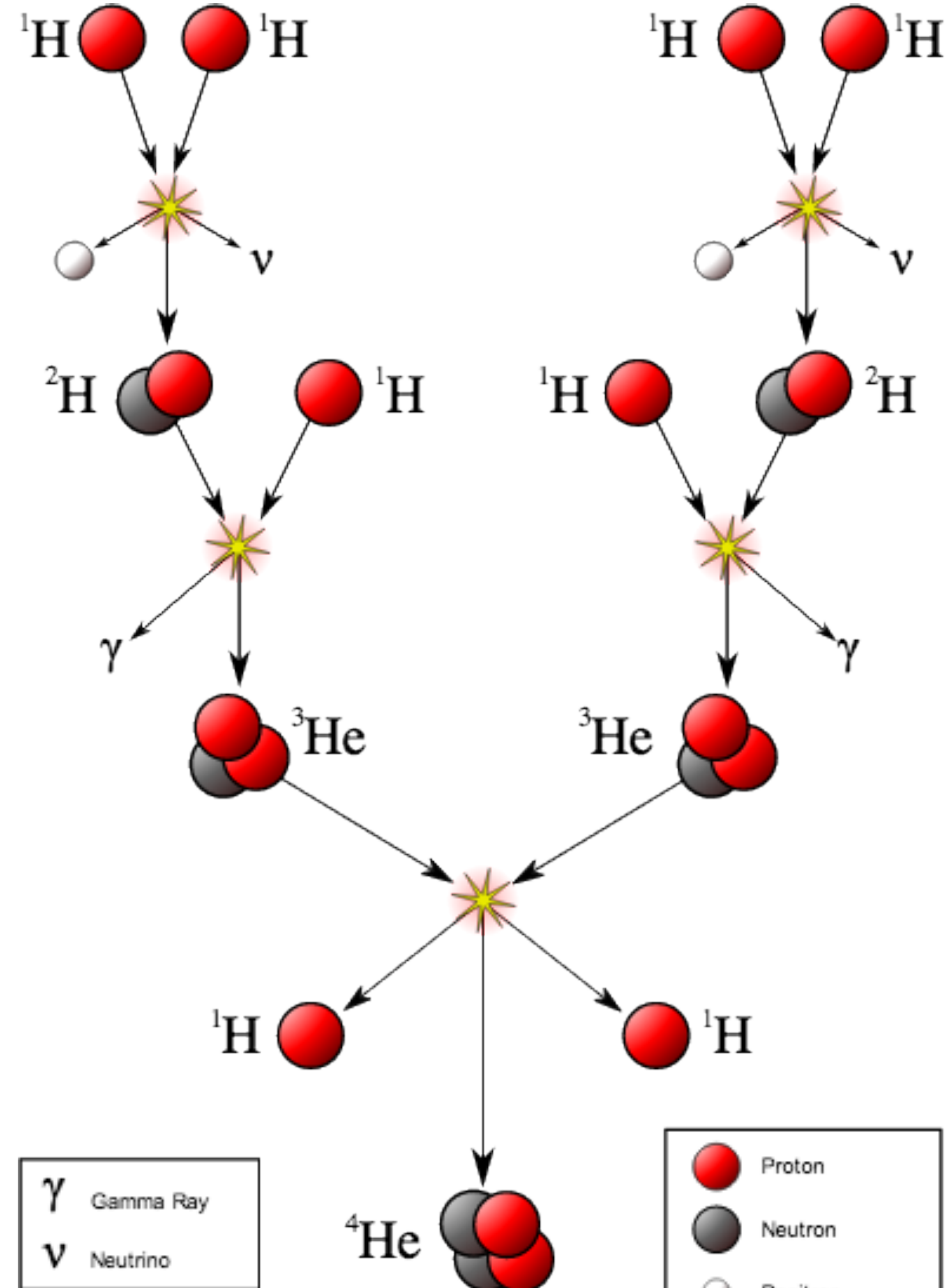
Think about four protons trying to set up a committee meeting ...

So what actually happens?

p-p chain: a more detailed description (binary collisions)

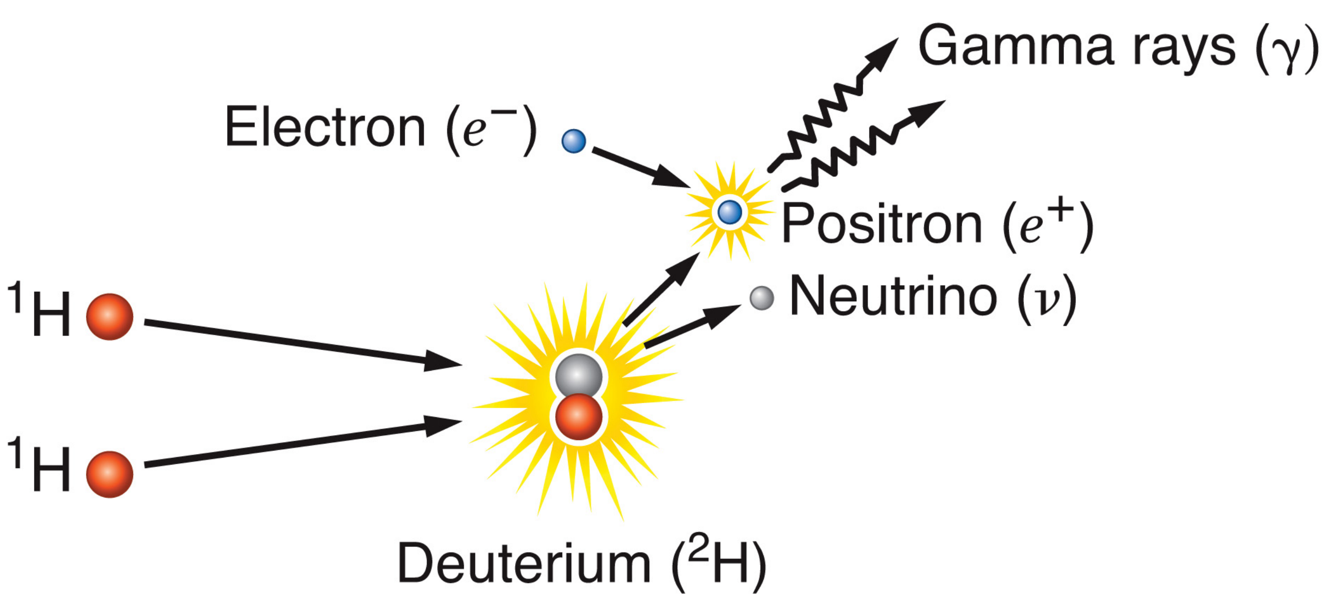
first worked out by Hans Bethe in 1938





Proton-Proton Chain, Step 1

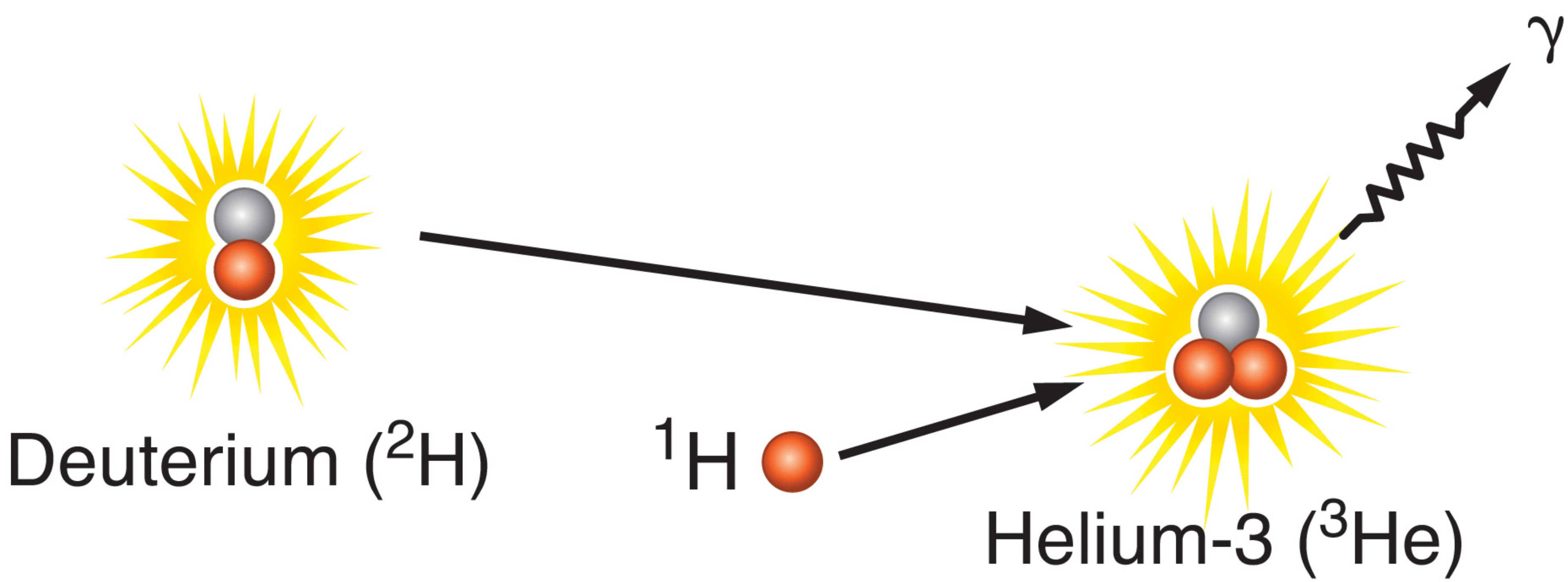
- Hydrogen nuclei are really just single protons, so the hydrogen fusion process is called the **proton-proton chain**.
- Step 1: Two hydrogen nuclei (protons) fuse to make a deuterium nucleus (^2H).



- Two protons collide. One of them emits a **positron** and **neutrino**, which makes it become a neutron.
- A **positron** is the **antimatter** counterpart of an electron. The positron meets an electron and they annihilate each other.
- The mass of both is converted into energy in the form of gamma-ray photons.
- This step must happen twice in order to make one helium nucleus (^4He).

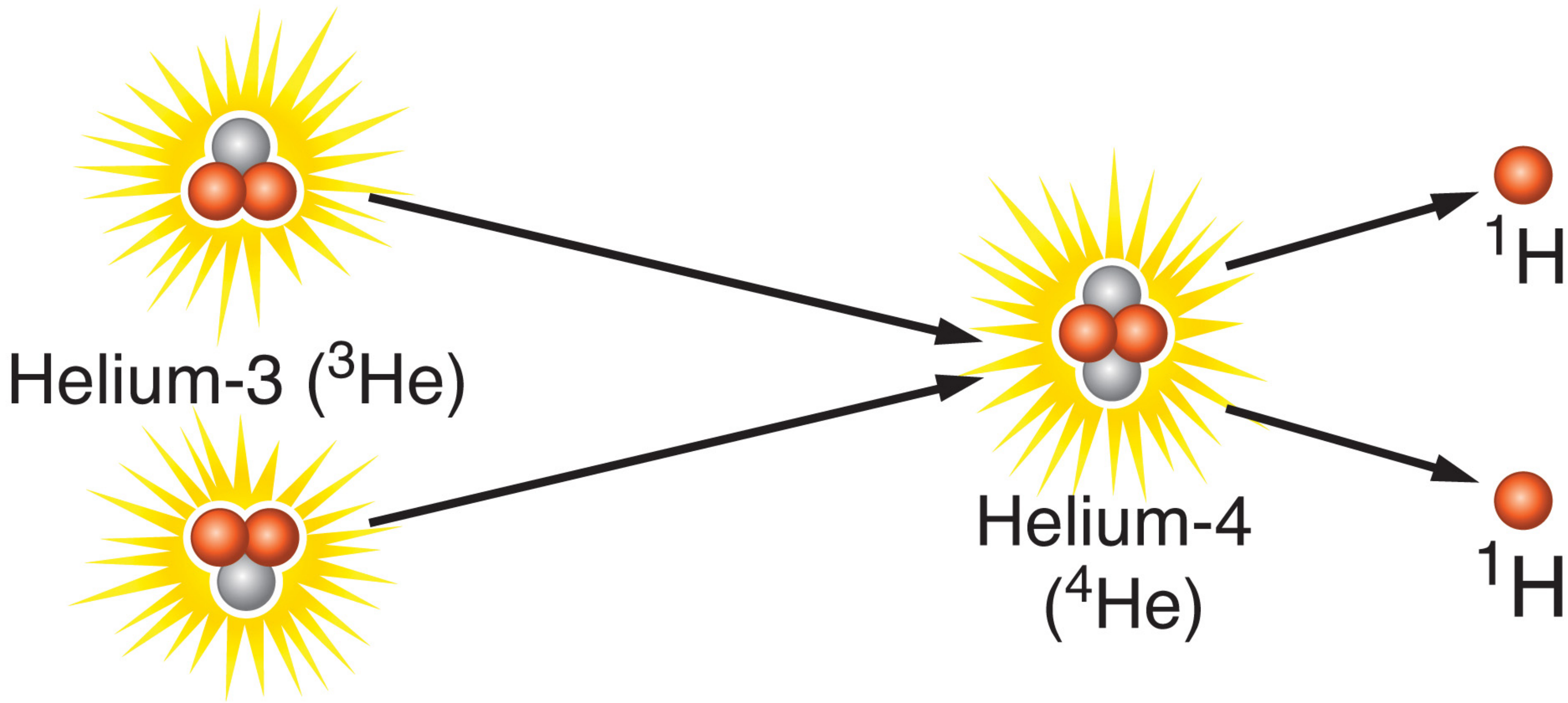
Proton-Proton Chain, Step 2

- Step 2: The **deuterium** nucleus collides with another proton, producing a **helium-3** nucleus (^3He).
  - ^3He is an isotope of helium. It has the same number of protons but has one fewer neutron than a normal helium nucleus (^4He).
  - This fusion reaction directly produces another gamma-ray photon.
- These gamma-ray photons and the ones produced in Step 1 leave the core and eventually make their way to the surface.
- This step must also happen twice to produce one helium nucleus (^4He).

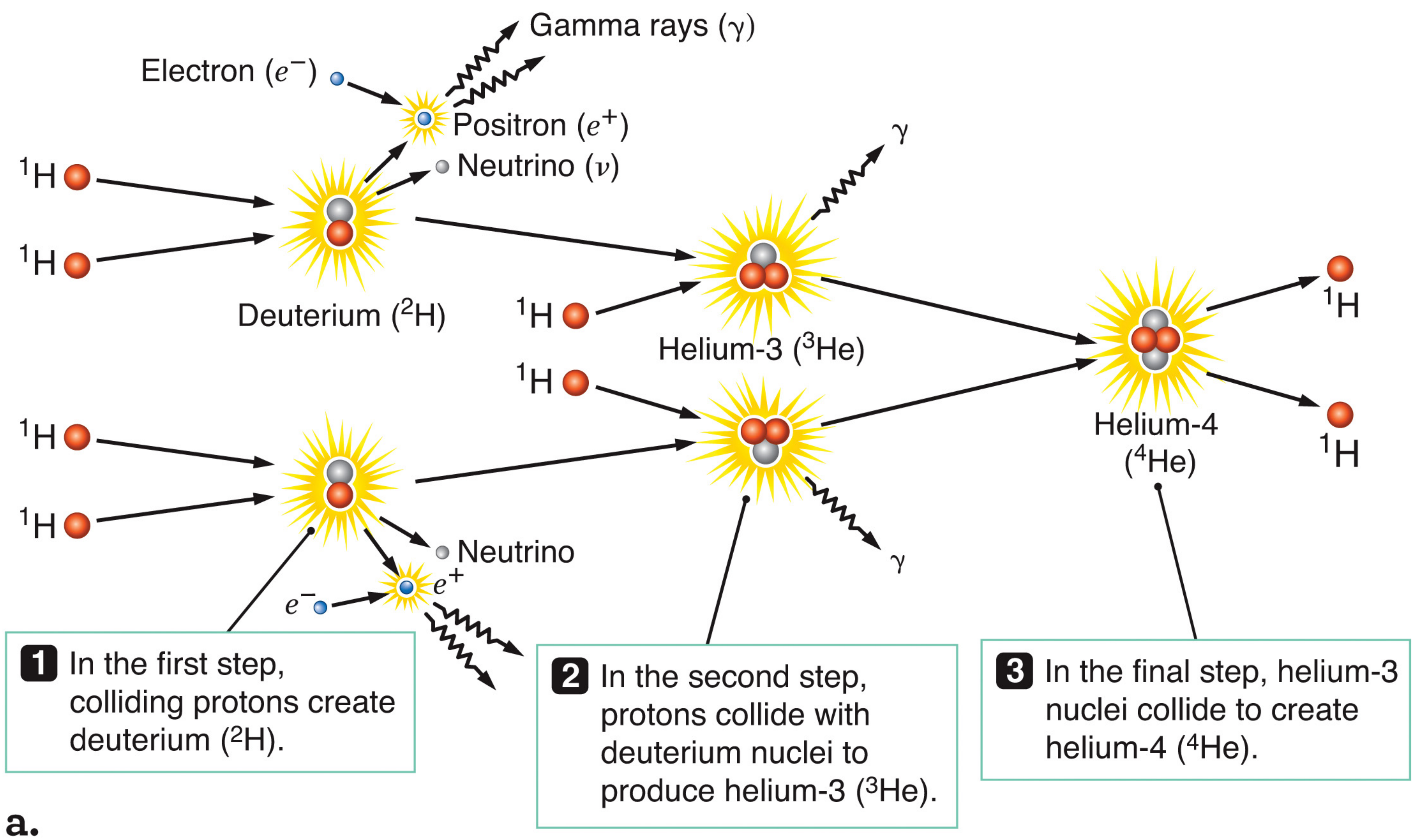


Proton-Proton Chain, Step 3

- Step 3: Two **helium-3** nuclei fuse to create one normal **helium-4** nucleus (^4He).
  - Two protons (H nuclei) are ejected during the collision.
  - The energy produced in this step makes the ^4He and protons move faster than they were before, ensuring more collisions.
- The process is now complete. It started with four protons (H nuclei) and results in one helium nucleus (^4He) and a lot of energy released.

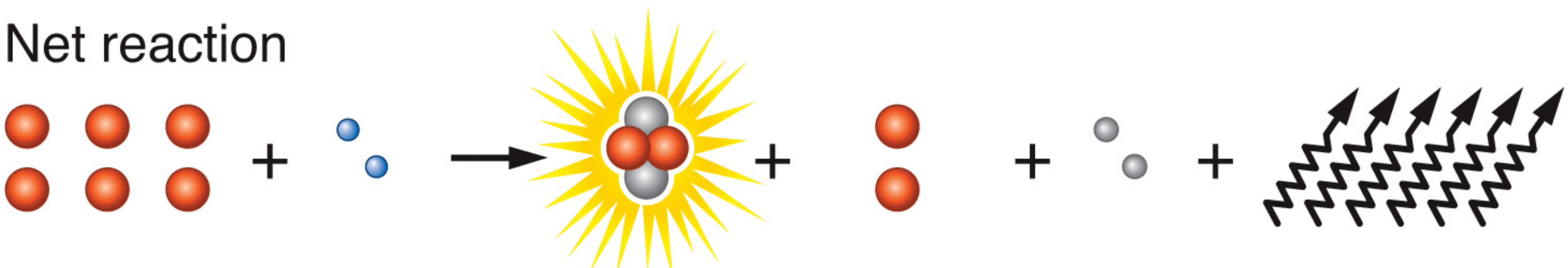


proton-proton chain - detailed procedure of binary collisions



proton-proton chain: net reaction (you'll need this for homework)

Each pp chain reaction generates 4×10^{-12} Joules of energy as photons, along with 2 neutrinos



$$6\,{}^{1}H + 2\,e^{-} \longrightarrow {}^{4}He + 2\,{}^{1}H + 2\,\nu + 6\,\gamma$$

- The mass of 4 hydrogen nuclei ($4 \times 1.6726 \times 10^{-27}$kg) is larger than 1 helium nucleus (6.6447×10^{-27}kg) by 4.57×10^{-29}kg.
- The energy associated with that **mass loss of 0.7%** is:

$$E = mc^2 = \left(4.57 \times 10^{-29}\text{kg}\right) \times (3.00 \times 10^{8}\text{m/s})^2 = 4.11 \times 10^{-12} J$$

Energy Generation Rate —> Fuel Consumption Rate —> Lifetime

1. Energy Production per Unit Mass

- **Process**: pp-chain (Proton-Proton chain)
- **Energy Product**: $\frac{\Delta mc^2}{4m_p}$
- **Unit**: J/kg

2. Energy Generation Rate

- **Current Rate**: $L_\odot$
- **Unit**: J/s = Watt

3. Fuel Consumption

- **Fuel Consumption Rate**: $\frac{L_\odot}{\Delta mc^2/4m_p}$
- **Unit**: kg/s
- **Total Available Fuel**: $f \cdot M_\odot$

4. Stellar Lifetime Calculation

- **Lifetime (τ)**: $\frac{f \cdot M_\odot}{L_\odot/(\Delta mc^2/4m_p)}$
- **Simplified Expression**: $f\left(\frac{M_\odot}{4m_p}\right)\left(\frac{\Delta mc^2}{L_\odot}\right)$
- **Proportionality**: $\tau \sim \frac{M}{L}$
- **Unit**: s (seconds)

$$\tau_{\rm MS} = \left(\frac{fM}{4m_p}\right)\left(\frac{\Delta mc^2}{L}\right) \propto \frac{M}{L}$$

Energy Generation Rate —> Fuel Consumption Rate —> Lifetime

- Each single PP-chain reaction produces $4.11 \times 10^{-12} J$, and converts 4 hydrogen nuclei to 1 helium nucleus, **costing 4 m_p of H fuel** although most of the mass is transferred into He.

- The Sun emits energy at a rate of 3.8e26 Watt (which is *J/s*), how much hydrogen fuel does it burn every second (in kg of hydrogen)?

- Fuel consumption rate: (3.8e26 J/s / 4.11e-12 J) x (4 m_p) = 620 billion kg/s

- The Sun has **a limited amount of hydrogen** available to fuse into helium, and this determines the **main-sequence lifetime** of the Sun (recall HR diagram).

The Lifetime of the Sun

- The Sun consumes hydrogen at a rate of 620 billion kilograms per second, so each year the Sun consumes:

$$M_{\text{year}} = \left(6.2 \times 10^{11}\ \text{kg/s}\right) \times \left(3.16 \times 10^{7}\ \text{s/yr}\right) \approx 2 \times 10^{19}\ \text{kg/yr}$$

- The mass of the Sun is $2 \times 10^{30}\ \text{kg}$, but only 10% is hot and dense enough for fusion to occur:

$$0.1 \times \left(2 \times 10^{30}\right)\ \text{kg} = 2 \times 10^{29}\ \text{kg}.$$

- If we know how much fuel the Sun has and how much fuel the Sun fuses in one year, we can find the lifetime of the Sun:

$$\text{Lifetime} = \frac{M_{\text{fuel}}}{M_{\text{year}}} = \frac{2 \times 10^{29}\ \text{kg}}{2 \times 10^{19}\ \text{kg/yr}} = 10^{10}\ \text{yr}$$

The Sun has a lifetime of 10 billion years!

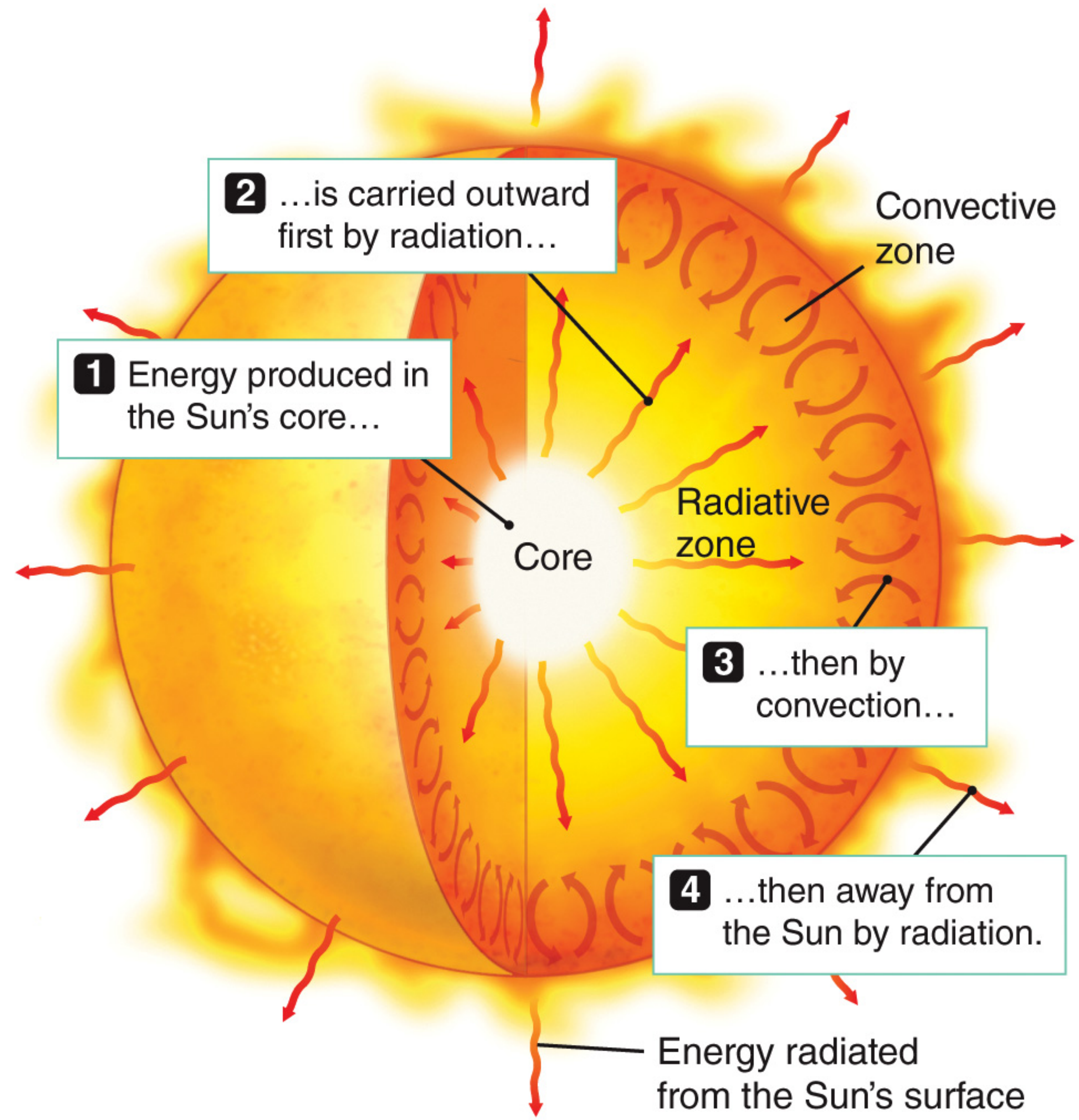
- *After 10 billion years, what will be the mass of the Sun?*

How does the energy get out?

first radiative diffusion,
then convection

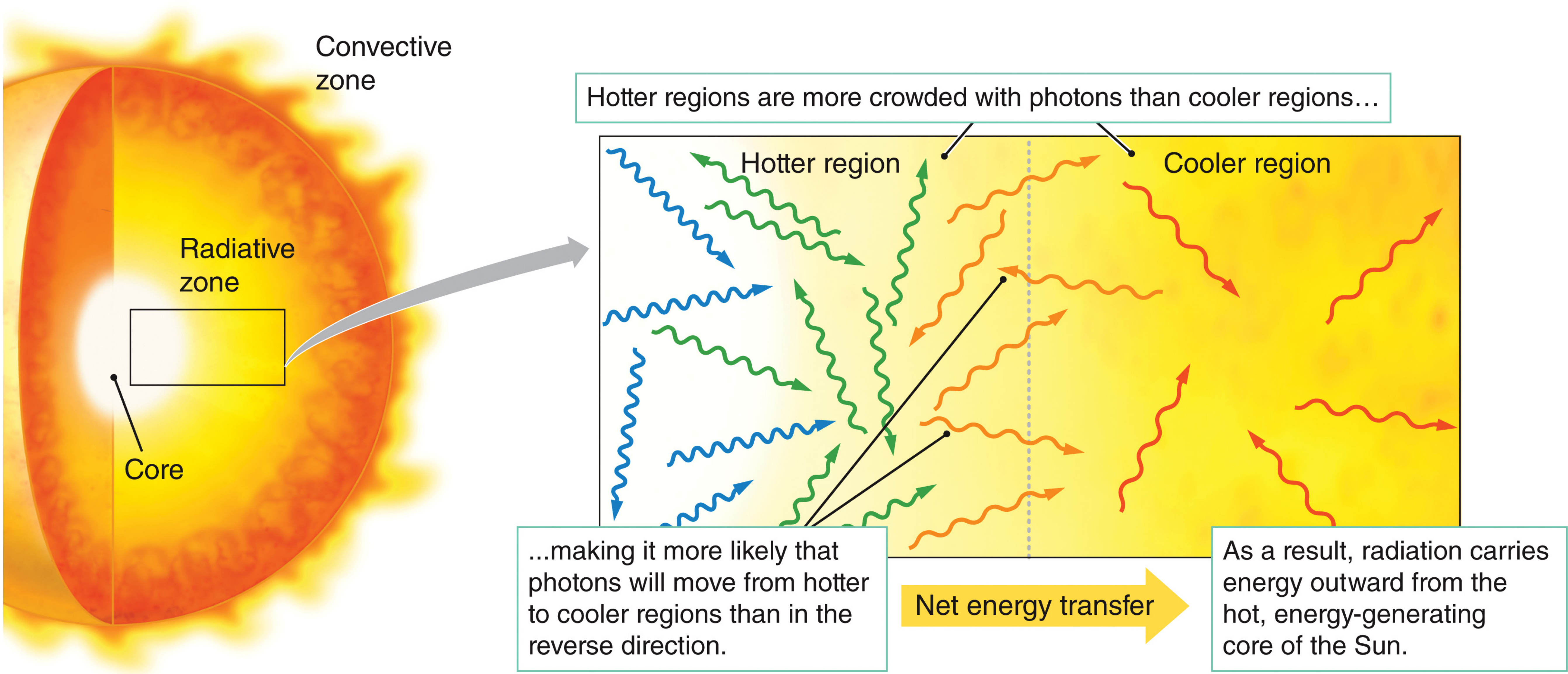
Energy Transport in the Sun: First Radiation then Convection

- Energy produced in the core must get out.
- Escaping energy passes through two different layers, defined by their temperature and density.
- In the inner layer, **radiation** transfers energy via photons.
- In the outer layer, **convection** carries energy by moving hot gas up and cool gas down.



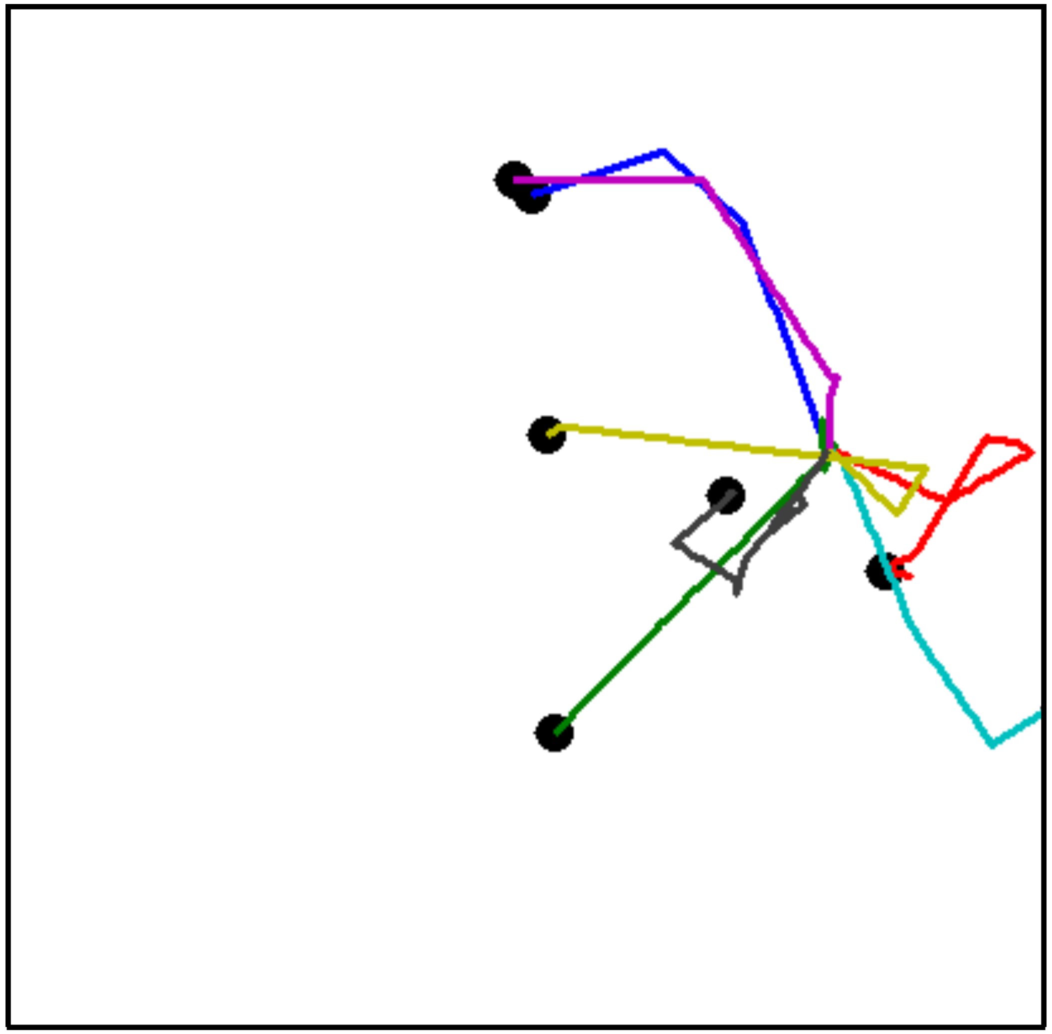
Radiative Zone: Lower 70% of the Sun

- **Radiative transfer**: Energy escapes in the form of high energy photons, radiating outward from the core. **The radiative zone extends to about 70 percent of the way to the surface.**
- Because temperature decreases from the core to the surface, photons tend to diffuse outwards, as illustrated below.

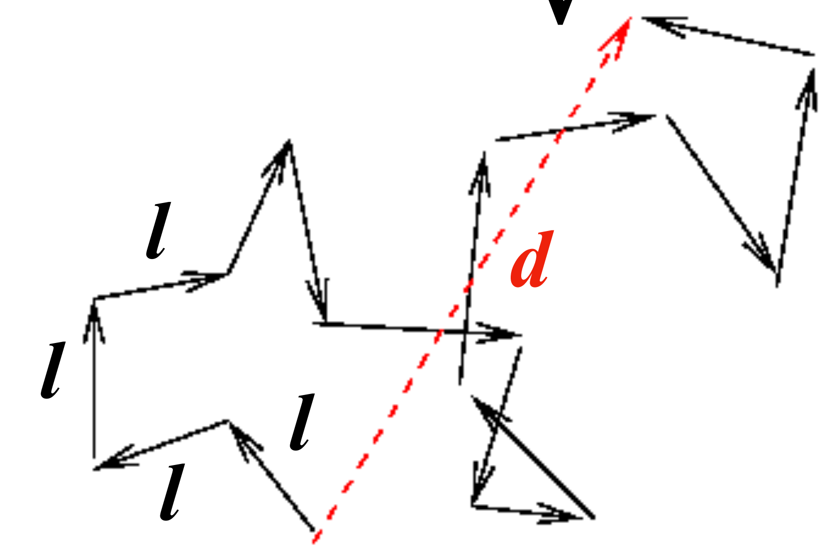


Radiative Zone: Random Walk of Photons

- Because of the **opacity of the matter**, each photon can only travel a short distance before it is either (a) absorbed and re-emitted or (b) scattered. The **mean free path** it travels is $l = 1/(n\sigma)$, where σ is the interaction cross section, and n the density of the matter.
- As a result, photons take random walks to slowly diffuse from the core to the surface. The **random-walk process** forces the photons to travel much greater distances than one solar radius, delaying their escape by thousands of years.



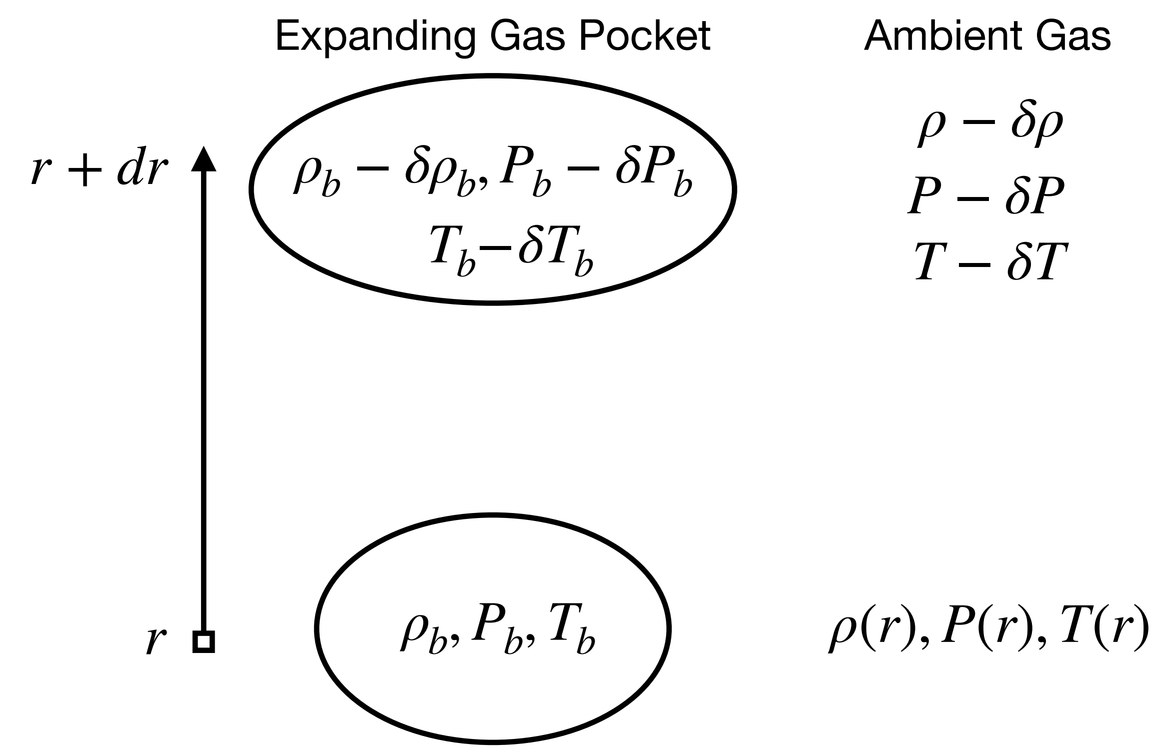
$$d = l\sqrt{N}$$

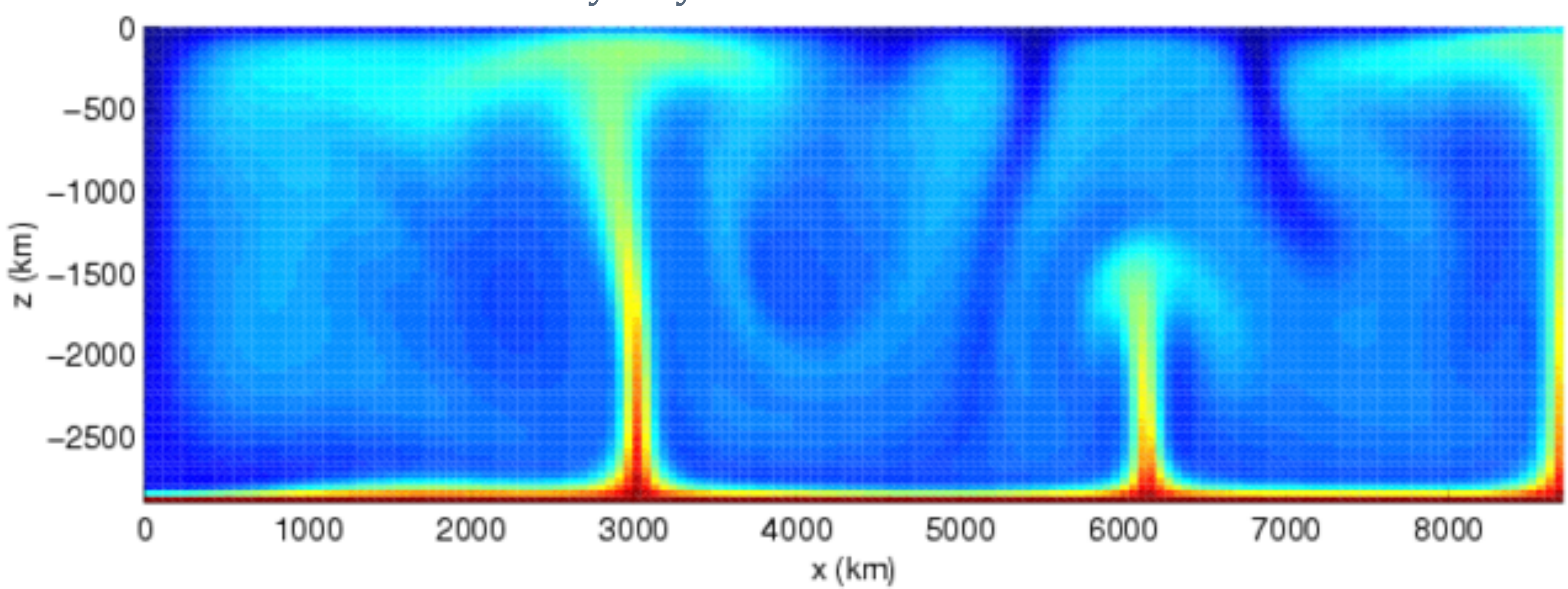


where d is the overall displacement, l is the mean free path, and N is the number of steps

Physical Conditions for Convection to Happen

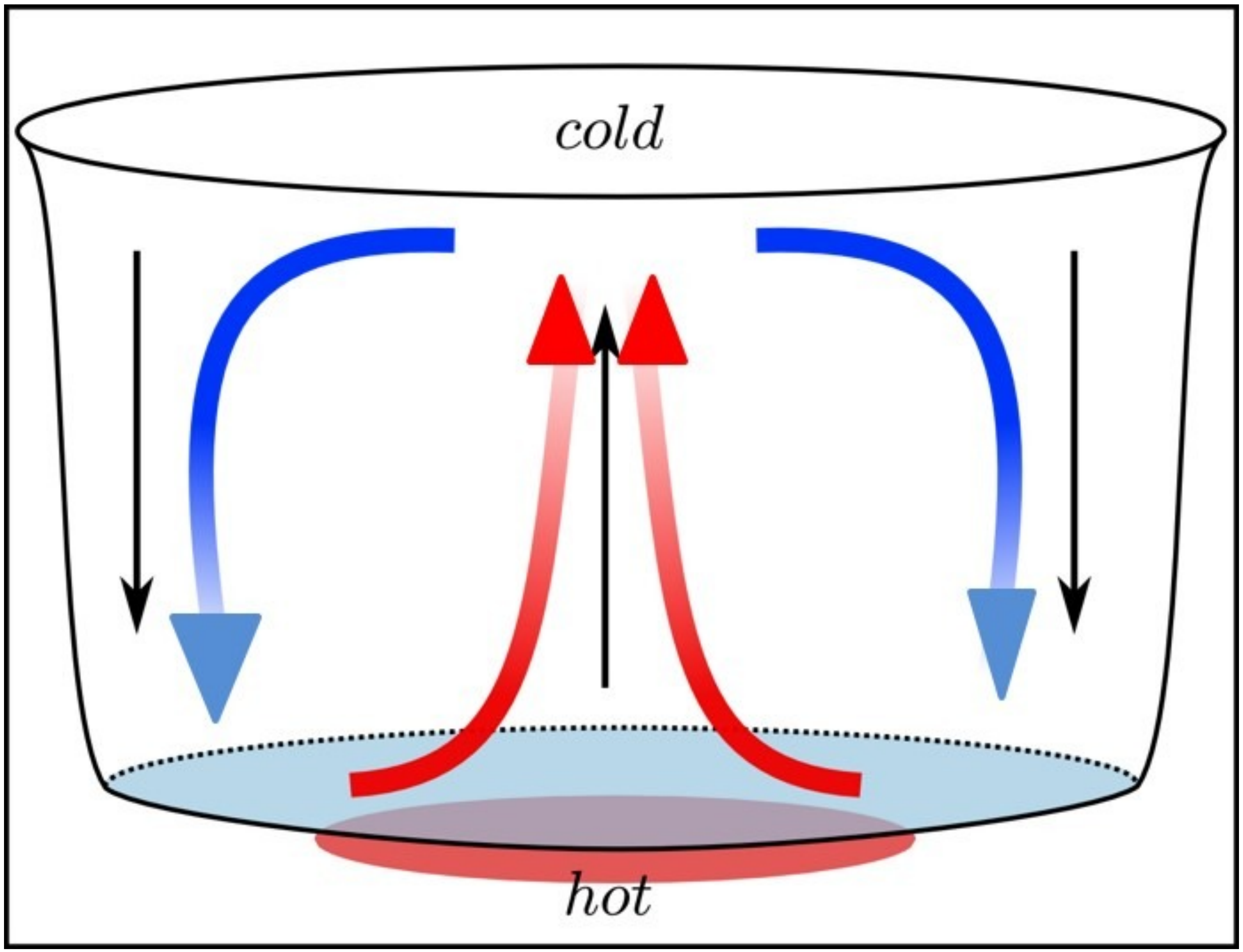
- When **adiabatic expansion** of a gas pocket causes its temperature to **drop *less*** than the **temperature gradient** of the ambient gas, **convection** ensues:
$$-(1-\frac{1}{\gamma})\frac{T}{P}\frac{dP}{dr} < -\frac{dT}{dr}$$
- **Why**? $P = nkT$, warmer gas at the same pressure as colder gas will have lower density. So the pocket will continue to rise due to buoyancy





Convective Zone: Outer 30% of the Sun

- The hot gas rises, and when it reaches the top of the convective zone (the Sun's surface), it releases its energy as photons that radiate into space.
- The gas at the surface is now cold and it sinks, which allows another bubble of hot gas to rise up in its place.



How do we know convection is important in the Sun?

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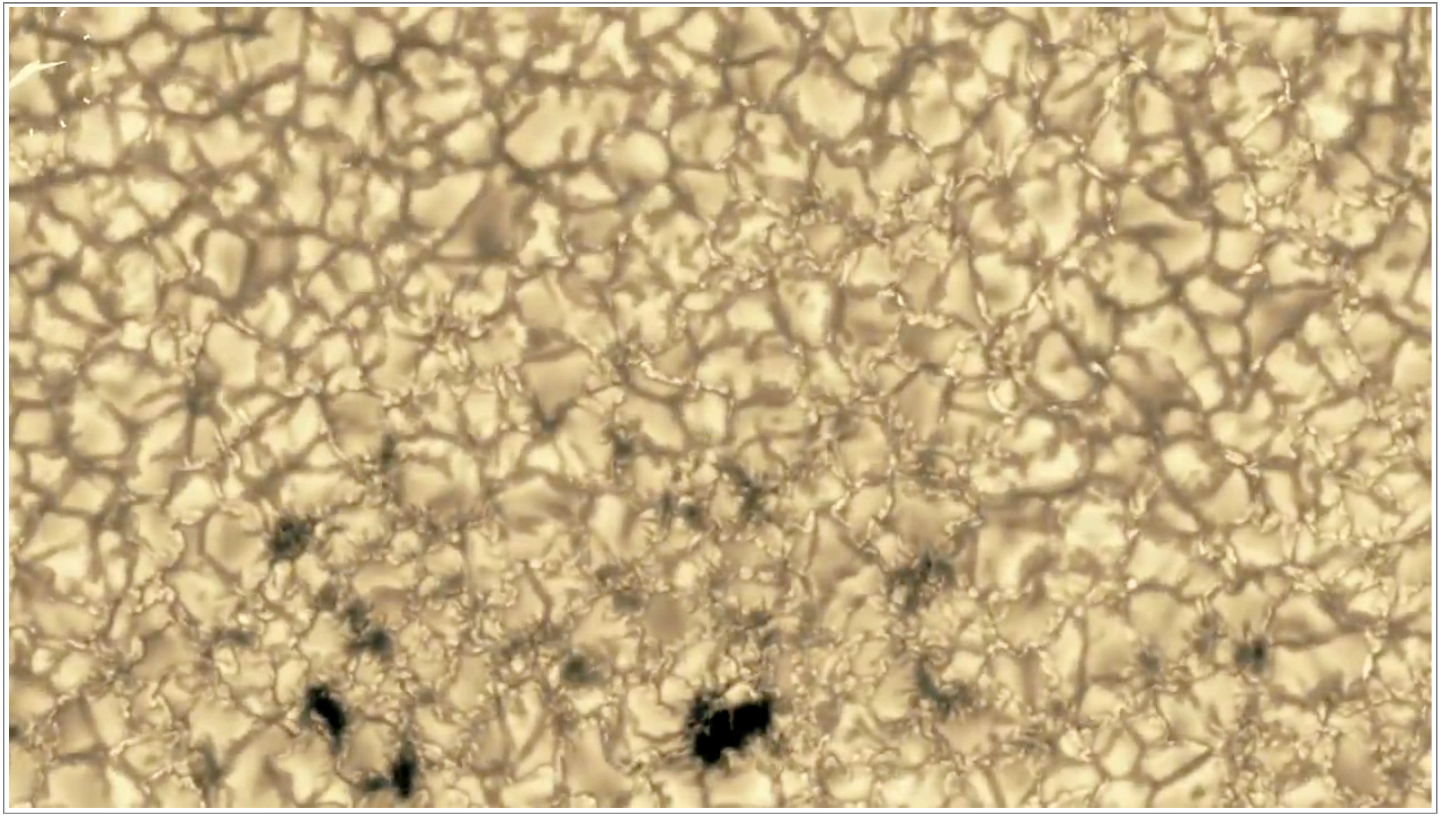
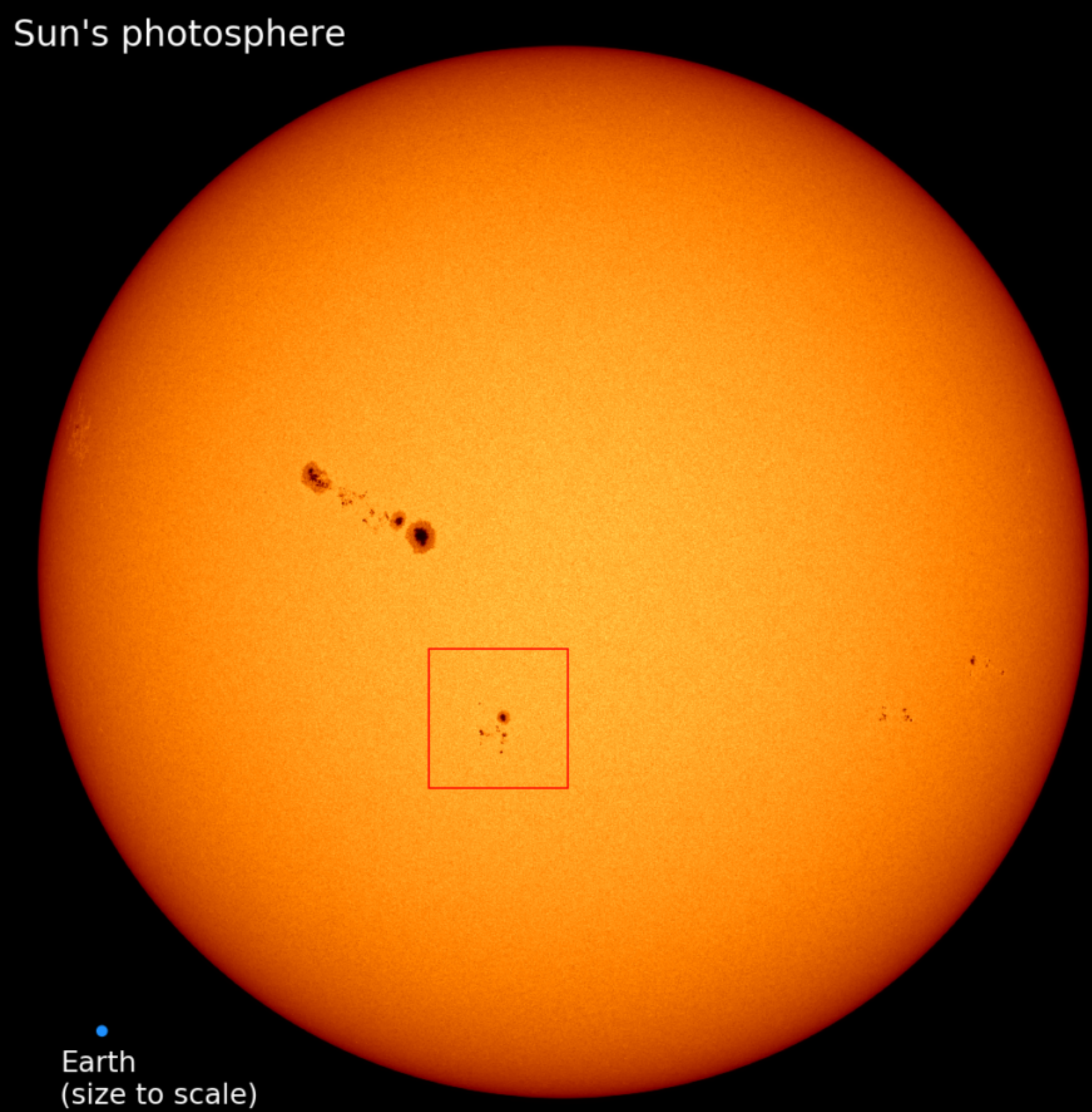
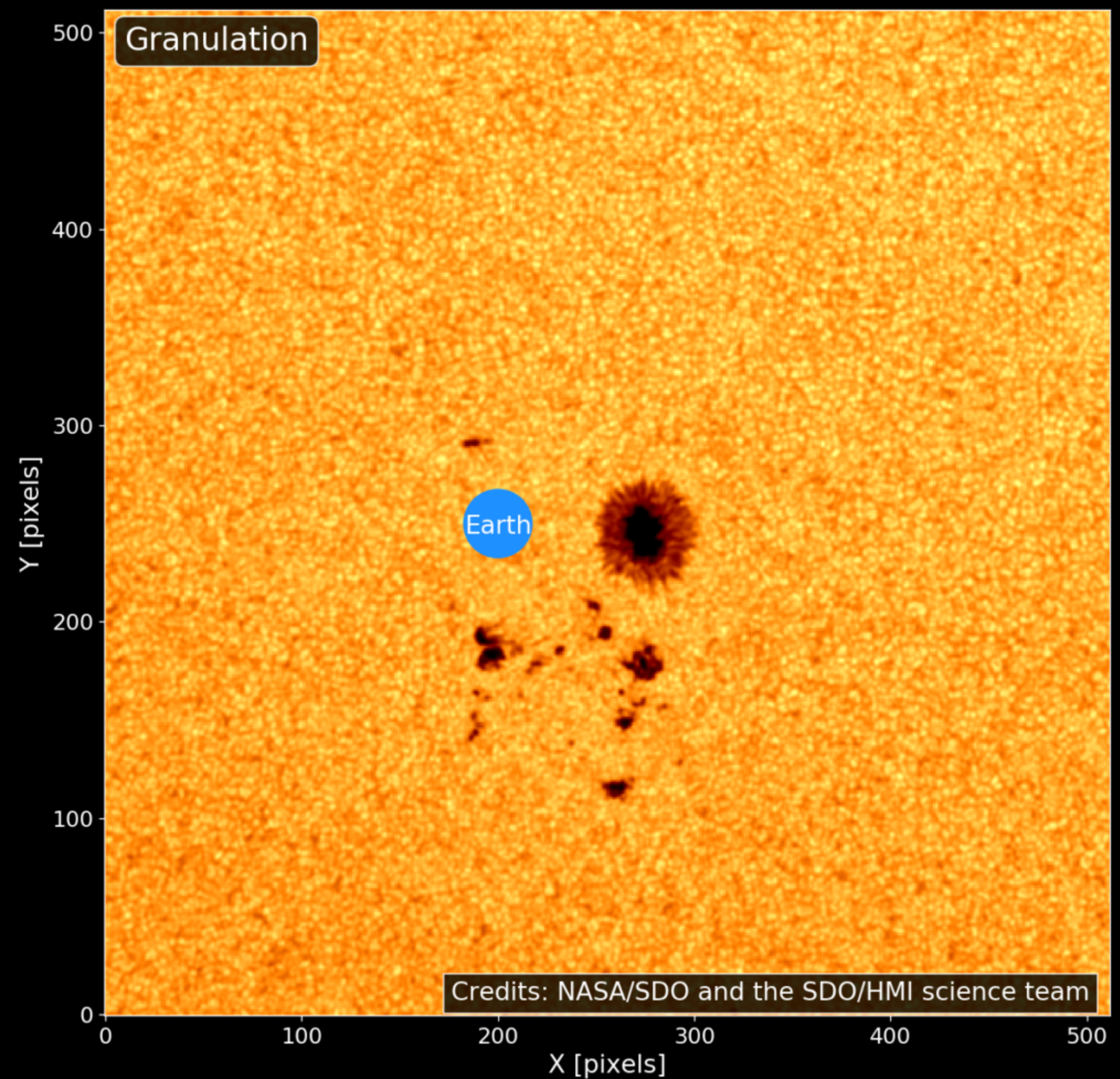
- We observe convection cells called “granulation”



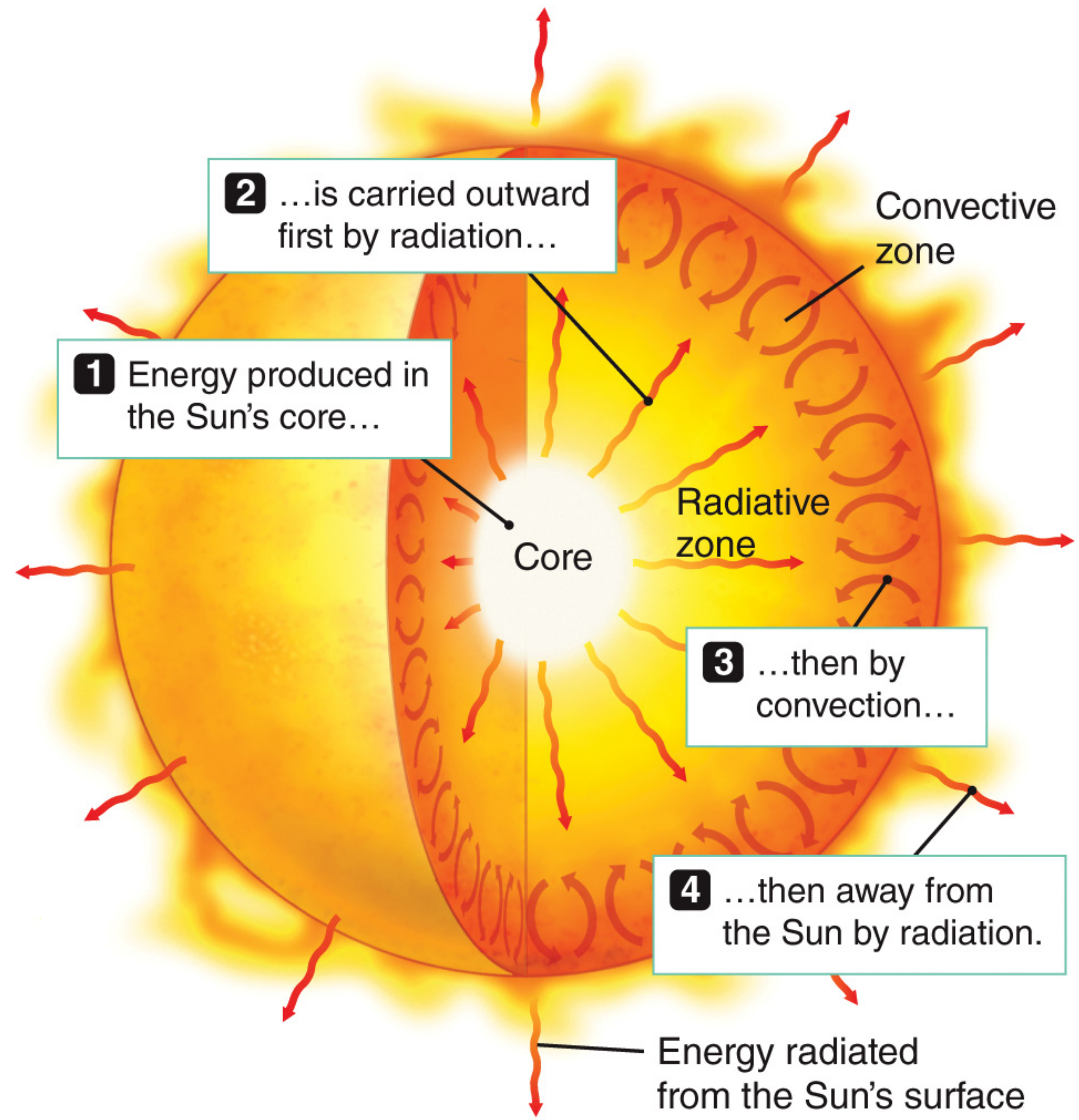
Size Comparison: Solar Granulation vs. Earth





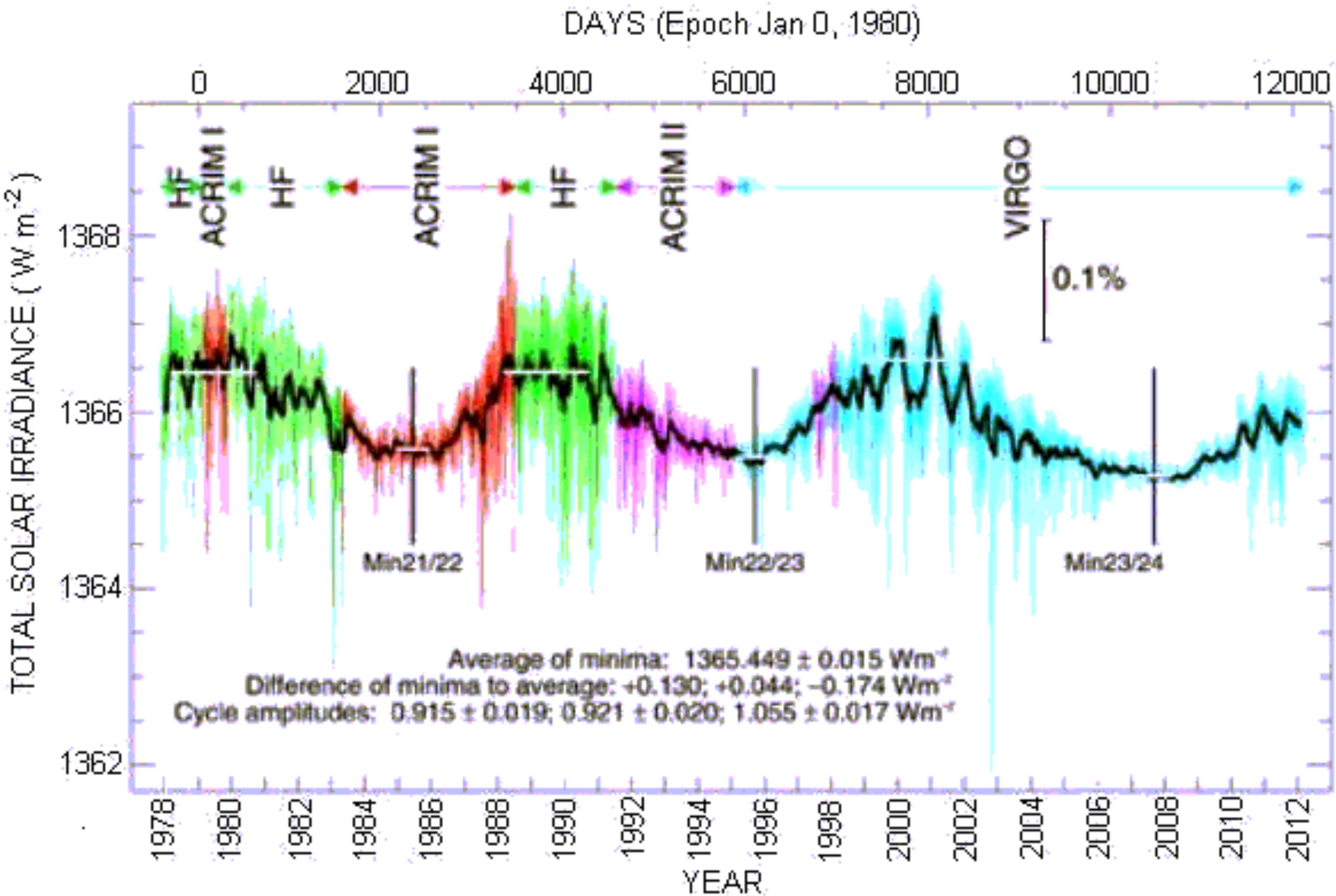
Summary: Energy Generation and Transport in the Sun

- Energy produced in the core must get out. **The rate of energy loss must equal the rate of energy gain** to maintain stability.
- The core maintains a constant fusion rate by using the weight of its envelop as a **gravitational thermostat**.
- In the inner layer, **radiation** transfers energy via photons.
- In the outer layer, **convection** carries energy by moving hot gas up and cool gas down.



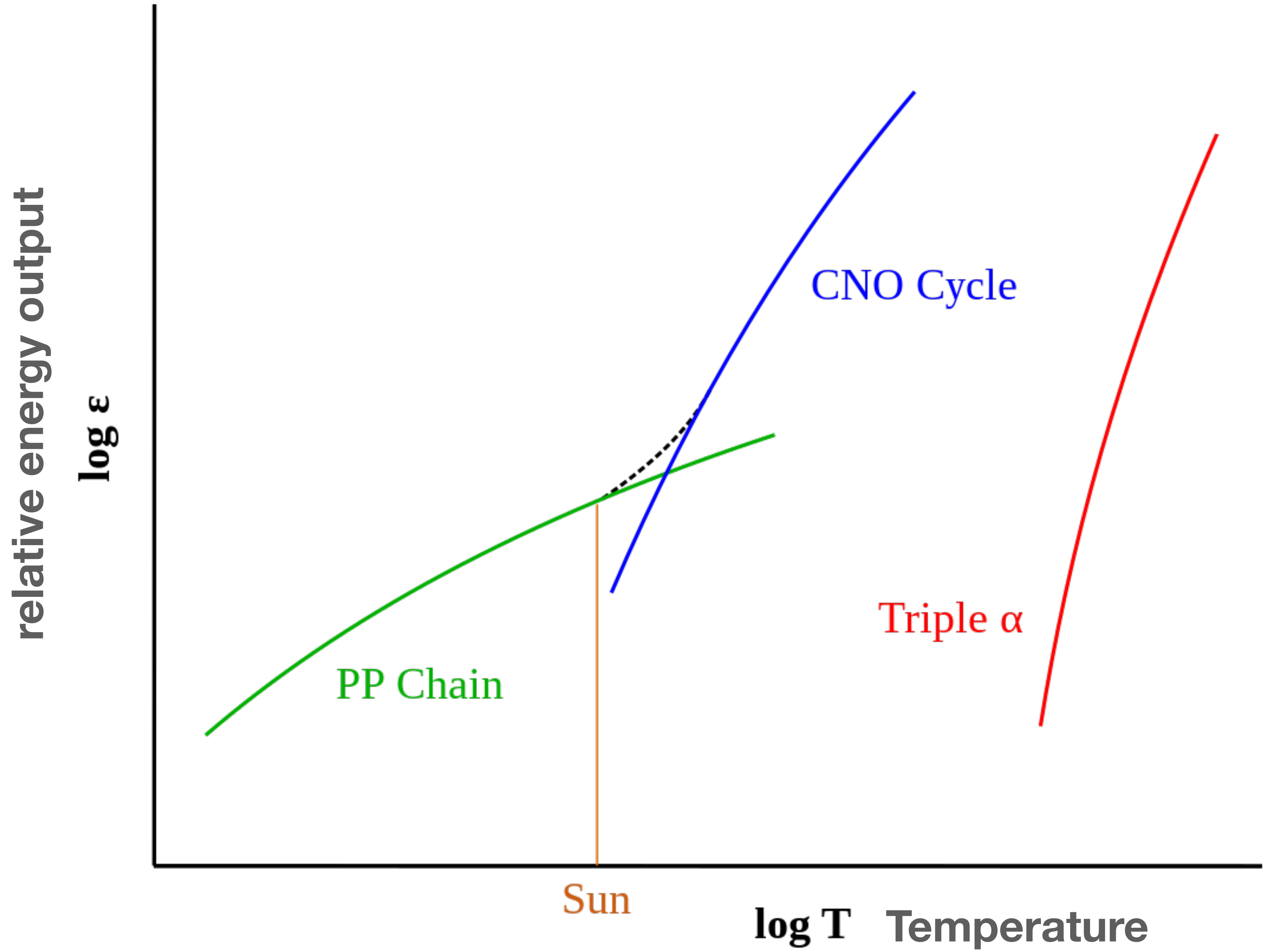
The gravitational thermostat

Solar “Constant” vs. Time (corrected for orbit ellipticity)



Hydrogen Fusion's Strong Temperature Dependency

- The rate of fusion is **sensitive to temperature:**
  - For PP chain, reaction rate ~ T^4; for CNO cycle, rate ~ T^{20}



Steady Fusion in the Cores of MS stars:

A gravitational ``thermostat''

H fusion rate increases

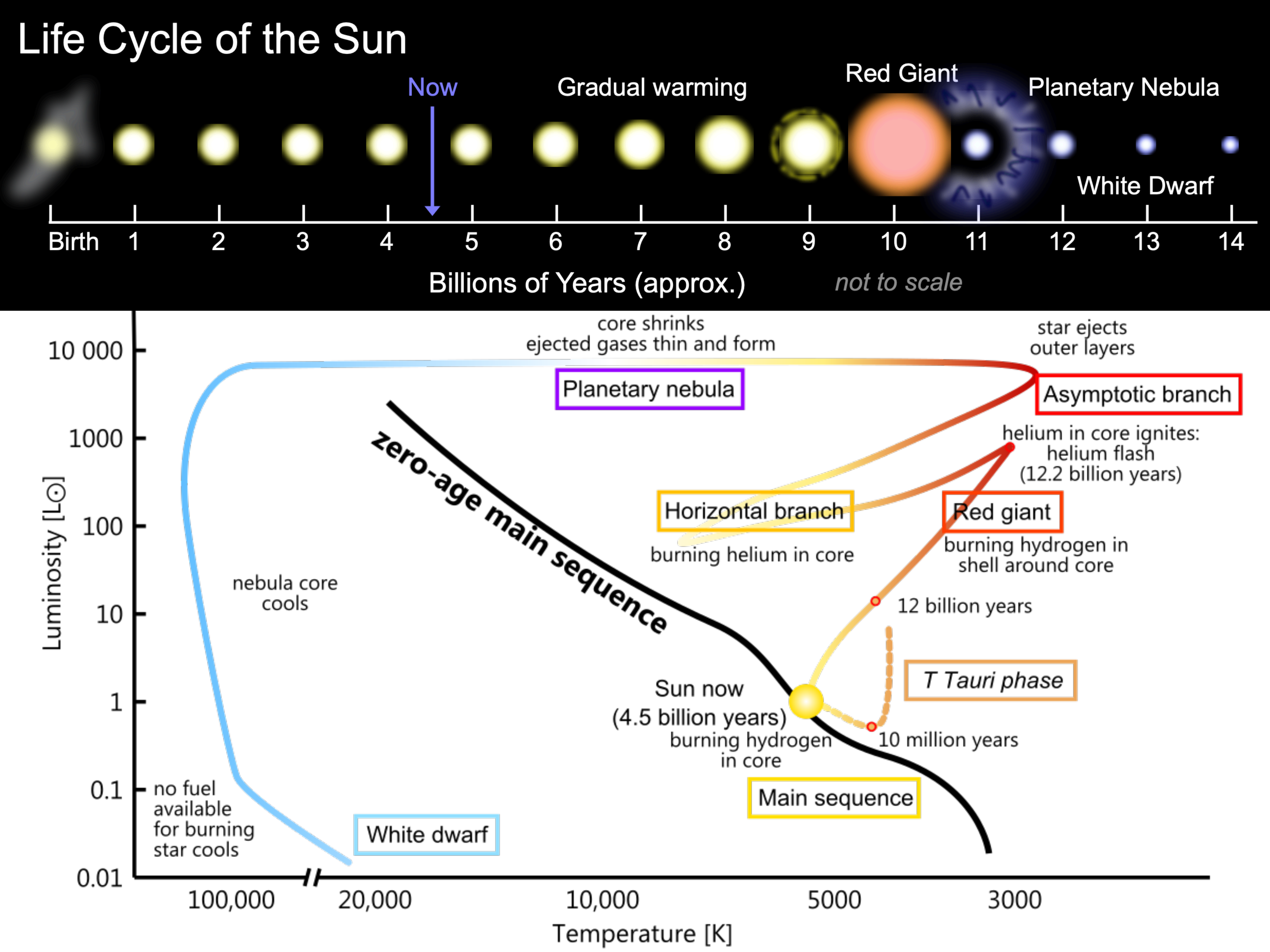
T & P increases

Core expands,
work against Gravity

T decreases

H fusion rate decreases

Steady H Burning


Life Cycle of the Sun
Now
Gradual warming
Red Giant
Planetary Nebula
White Dwarf
Birth
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Billions of Years (approx.)
not to scale
core shrinks
ejected gases thin and form
star ejects
outer layers
Planetary nebula
Asymptotic branch
helium in core ignites:
helium flash
(12.2 billion years)
zero-age main sequence
Horizontal branch
Red giant
burning helium in core
burning hydrogen in
shell around core
nebula core
cools
12 billion years
T Tauri phase
Sun now
(4.5 billion years)
burning hydrogen
in core
10 million years
Main sequence
no fuel
available
for burning
star cools
White dwarf
Luminosity [L⊙]
10 000
1000
100
10
1
0.1
0.01
100,000
20,000
10,000
5000
3000
Temperature [K]

How to test the hypothesis that nuclear fusion powers the Sun?

measure solar neutrino flux

Estimate neutrino flux from Solar EM flux

How many neutrinos pass through a m² area on Earth per second?

Solar luminosity = Solar constant $\times 4\pi(1\ \text{AU})^2$

Neutrino luminosity = Solar luminosity $/E \times n_\nu$

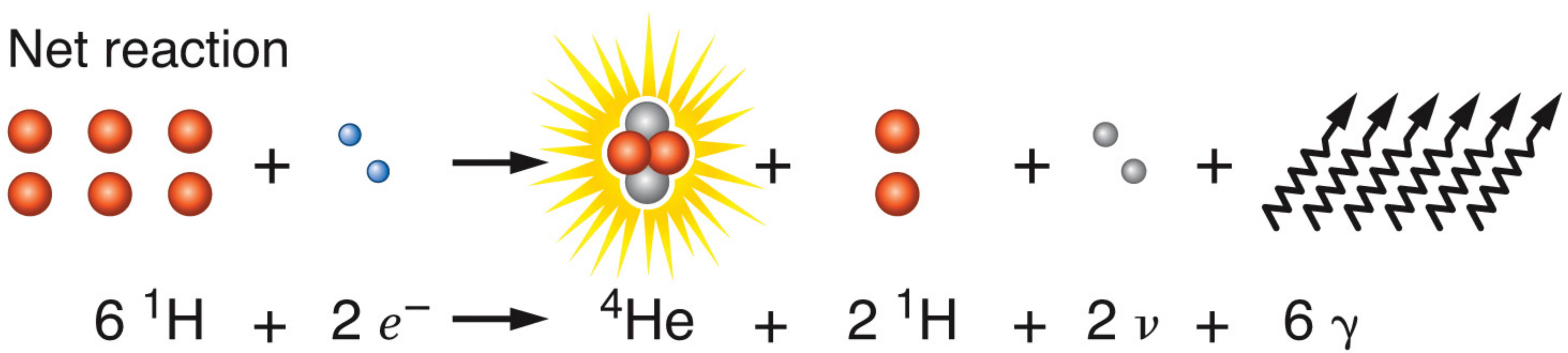
Neutrino luminosity = Neutrino flux $\times 4\pi(1\ \text{AU})^2$

$E = \Delta m \cdot c^2$ = 4×10^{-12} J/fusion

$n_\nu = 2$ neutrino/fusion

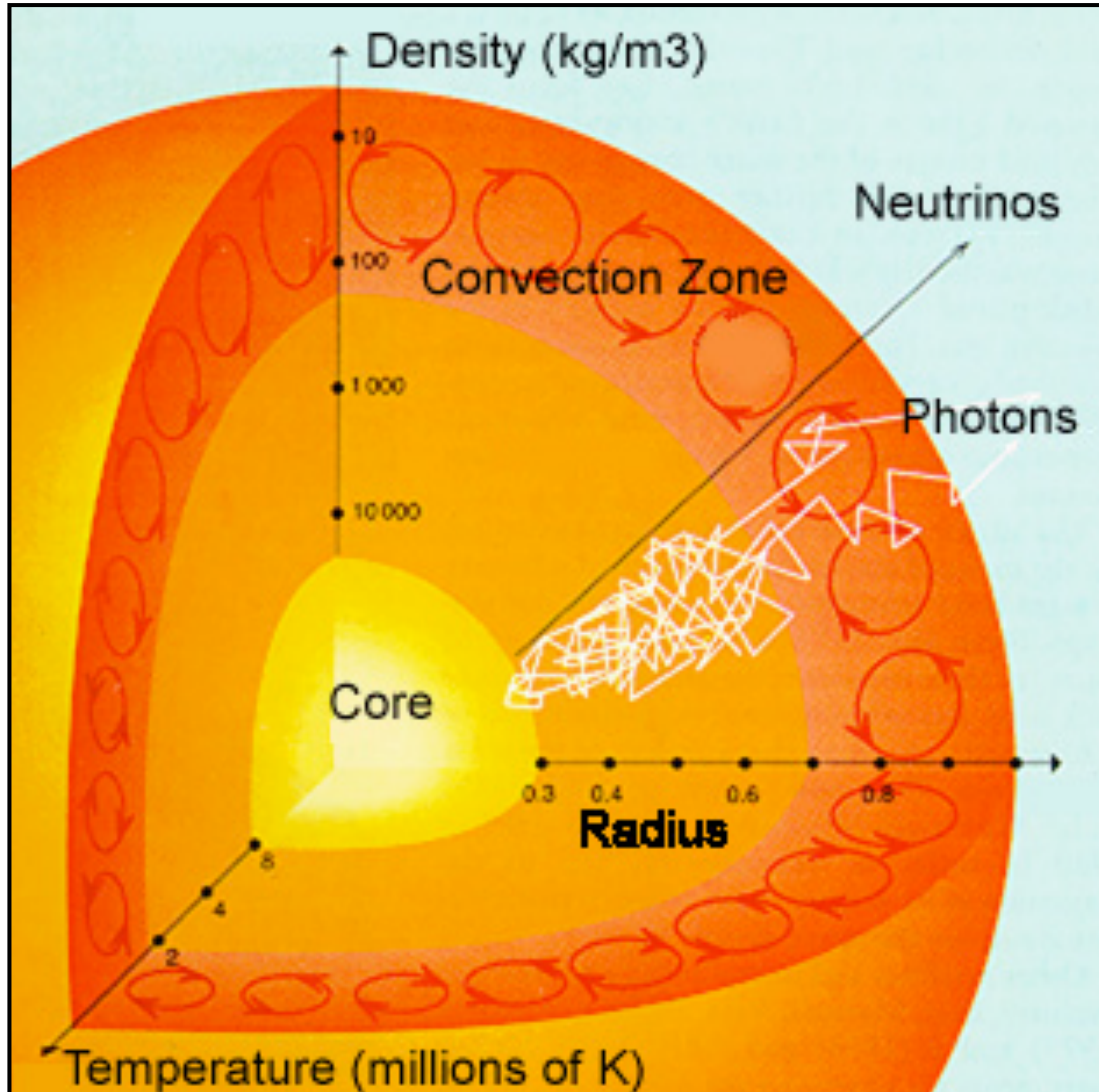
It is evident from the above equations that:

Neutrino flux = Solar constant $/E \times n_\nu$ **= 6.4×10^{14} neutrino/s/m²**



Testing the fusion model: Solar neutrino flux

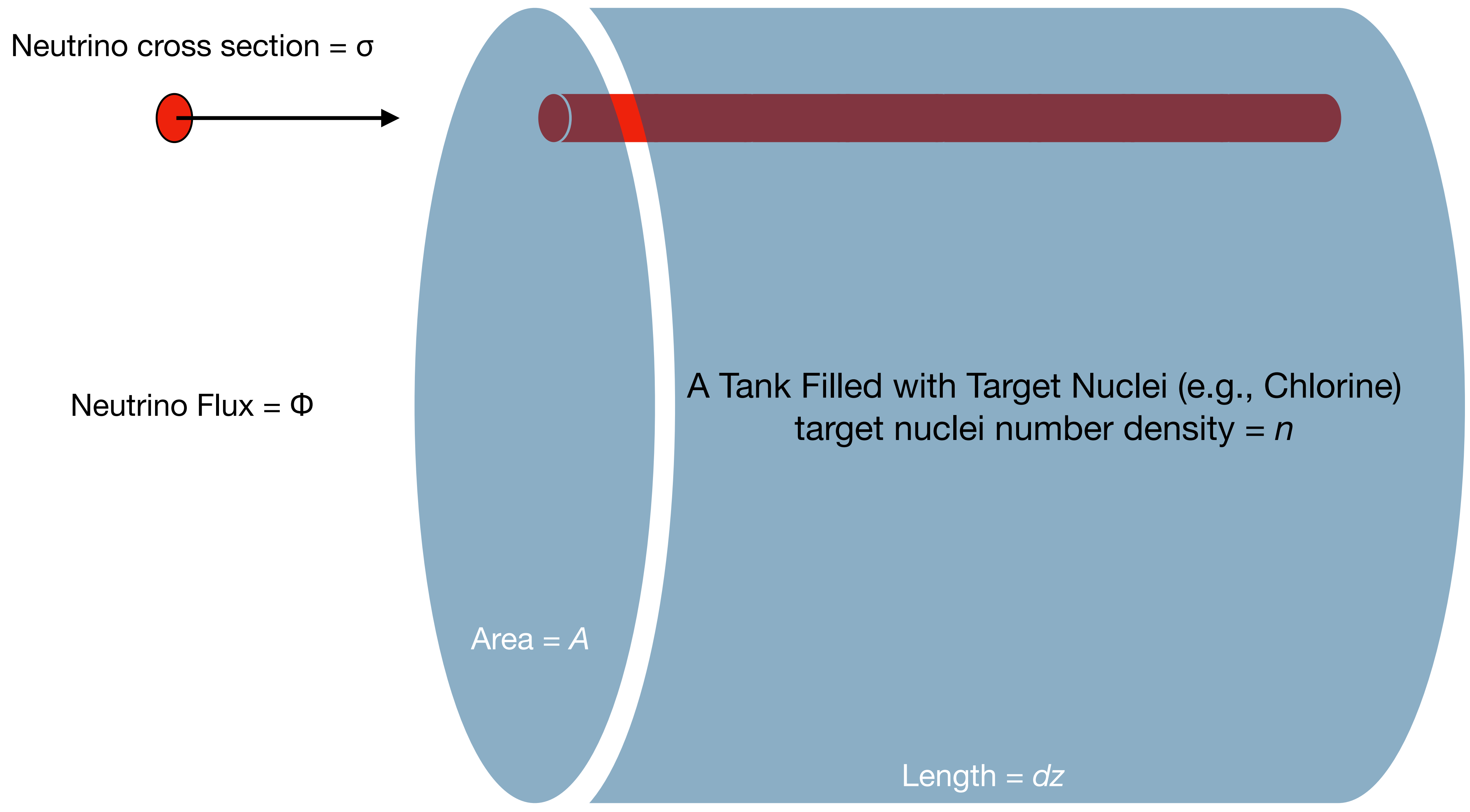
- Hydrogen fusion emits neutrinos.
- *Neutrinos*: weakly interacting particles, little mass, no charge.
- *Very* weak interactions with matter.
- Should escape the Sun freely at near speed of light.



Interaction Cross Section

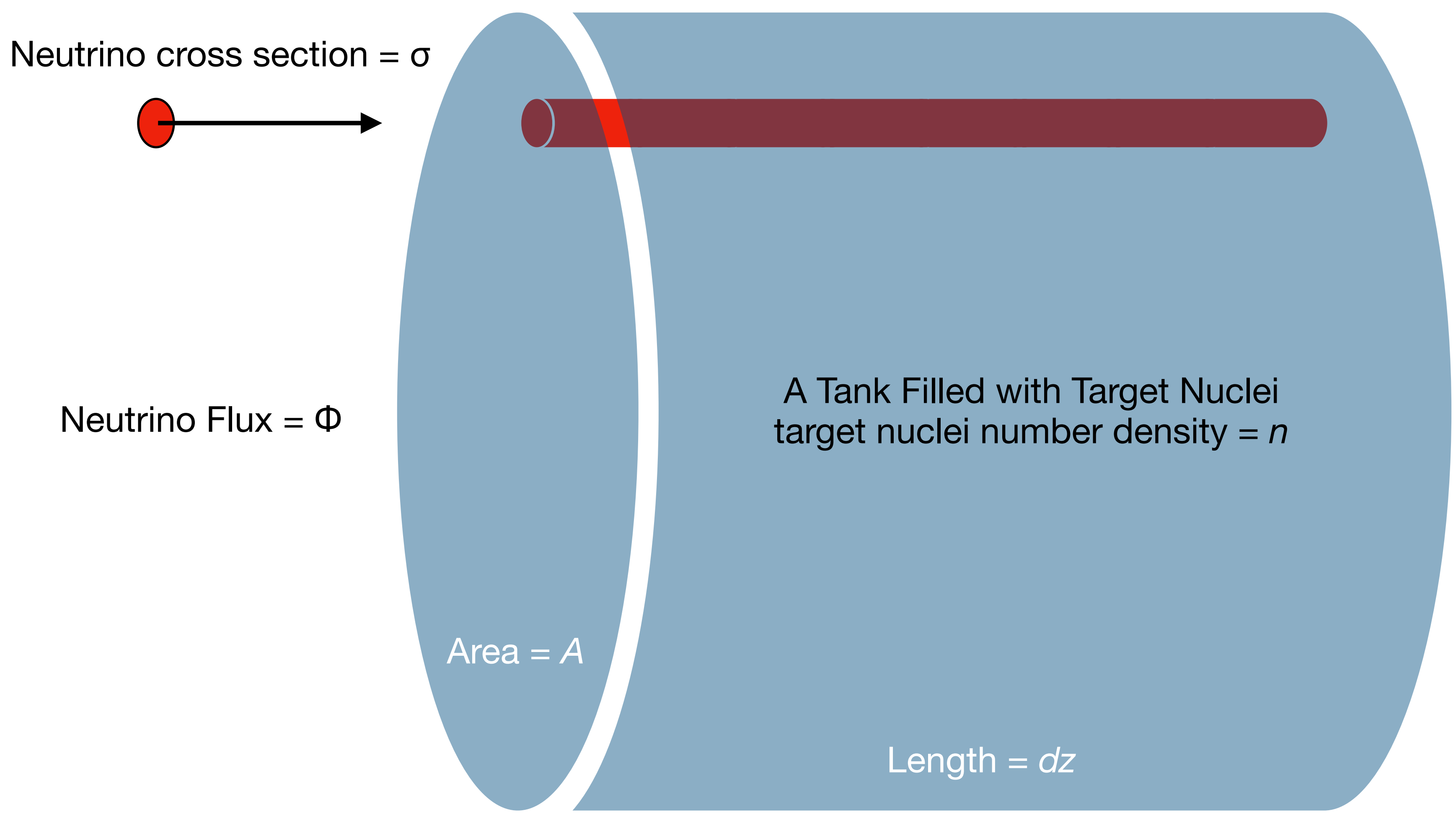
Understanding the attenuation equation

- When a neutrino travels through a cylindrical tank, how many target nuclei would have interacted with it? That would equal to the number of target nuclei in the volume carved through by the neutrino's cross section.
- How many neutrinos pass through the tank per unit time?
- What's the total number of reactions that could happen in the tank per unit time?



The Attenuation Equation

- No. of reactions per neutrino: $n_{\text{interaction}} = n\sigma dz$
- Reaction rate over the volume: $n\sigma dz \cdot \Phi \cdot A$
- No. of neutrinos removed per unit time by the volume: $-d\Phi \cdot A$
- The above two should equal, leading to the attenuation equation: $-d\Phi = \Phi n\sigma dz = \Phi(dz/l_{\text{mfp}})$



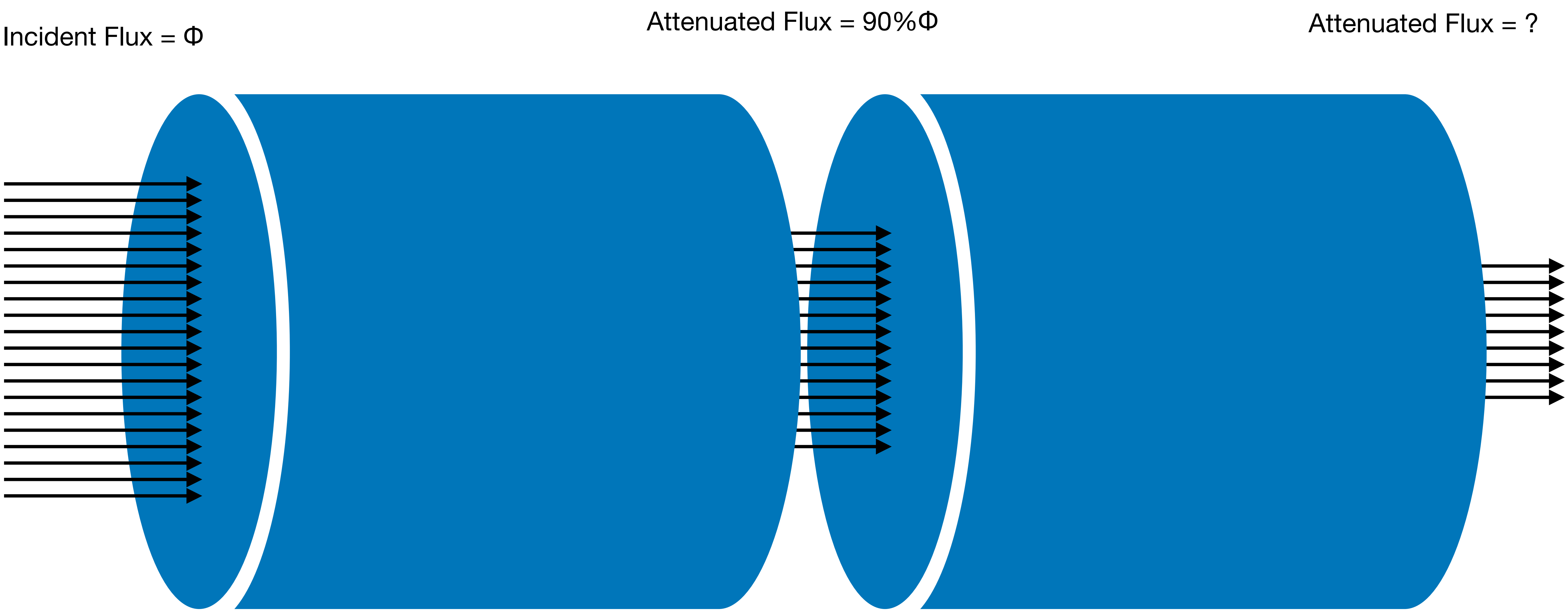
Solution of the attenuation equation

By integrating the attenuation equation: $$\frac{d\Phi}{\Phi} = -n\sigma dz$$

We obtain: $$\Phi = \Phi_0 \exp(-n\sigma z) = \Phi_0 \exp\left(-\frac{z}{l_{\rm mfp}}\right)$$

Applying the attenuation equation

- Suppose the particle flux is reduced to 90% of the incident flux by a tank that is 1 km long, what would be the particle flux if the beam passes through a tank that is 2 km long?



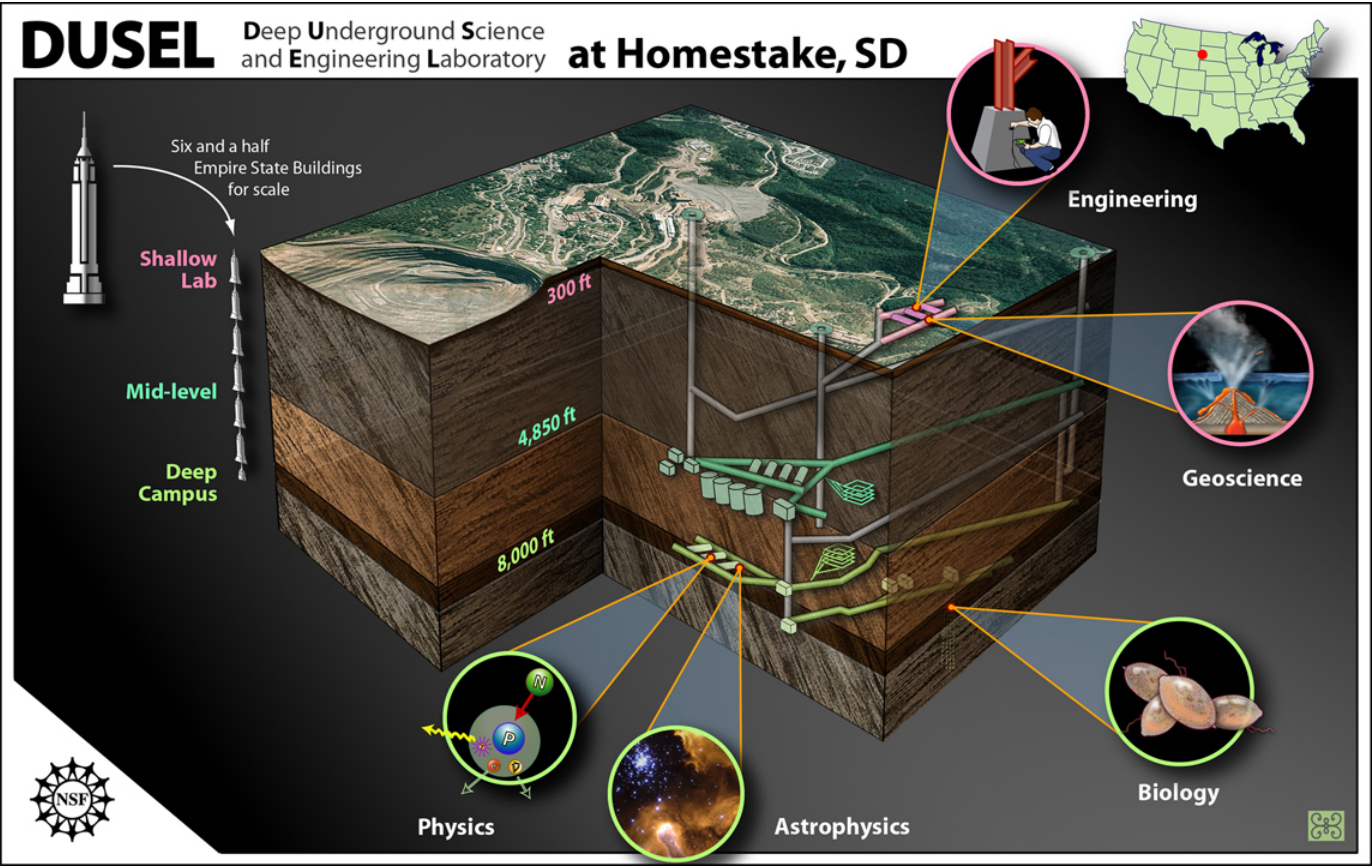
Solar neutrino detectors

First Neutrino Detection Experiments based on Chlorine (^{37}Cl), 1965-1967

- Although only a tiny fraction of neutrinos interact with matter, there is an enormous flux of solar neutrinos!
- Need large volume detectors to increase detection probability. To fill such large volumes, the detector material better be cheap!
- The Homestake experiment in 1960s used **PERC (C_2Cl_4, tetrachloroethene)**, a common dry-cleaning fluid rich in **Chlorine**

$$\nu_e + {}^{37}Cl \mapsto {}^{37}Ar + e^-$$

This is an *inverse* beta-decay reaction requires neutrino energy > 0.814 MeV



Interaction Cross Section: Neutrino-Chlorine Interaction

- The Homestake tank is filled with **615 metric tons** of **C_2Cl_4** (mean molecular weight: 164 = 12*2 + 35*4), given the mass density 1.62 g/cm^3 and proton mass (1.67e-24 g), what's the volume density of Chlorine (n in cm^{-3})?
- Given a neutrino cross section of **10^{-46} cm^2**, and the neutrino flux of **6×10^{14} s^{-1} m^{-2}**, calculate the amount of attenuated neutrino flux over a 15 m long tank (dz = 1500 cm).

$$\frac{d\Phi}{\Phi} = -n\sigma dz$$

((1.62 g) / (164 * proton mass)) * 4 =

$2.36229088 \times 10^{22}$

n of molecules = mass density / mu m_H

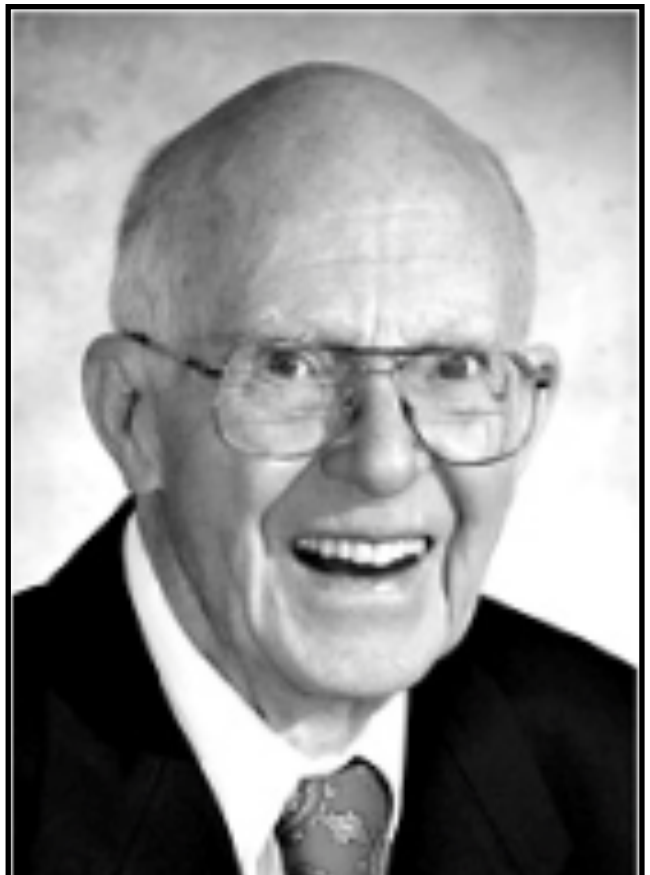
n of Cl = 4x n of molecules = 2.4e22 cm-3

d Phi/Phi = n sigma dz = 2.3e22 x 1e-46 x 1500 = 3.45e-21

=> d Phi = 2e-6 neutrinos/s/m^2

Two Nobel Prizes in Physics awarded to Solar Neutrino Experiments

- 2002 Nobel Prize in Physics: Davis & Koshiba for the detection of solar neutrinos (confirming the fusion model)
- 2015 Nobel Prize in Physics: Kajita & McDonald for the discovery of neutrino oscillation (solving the missing neutrino problem)

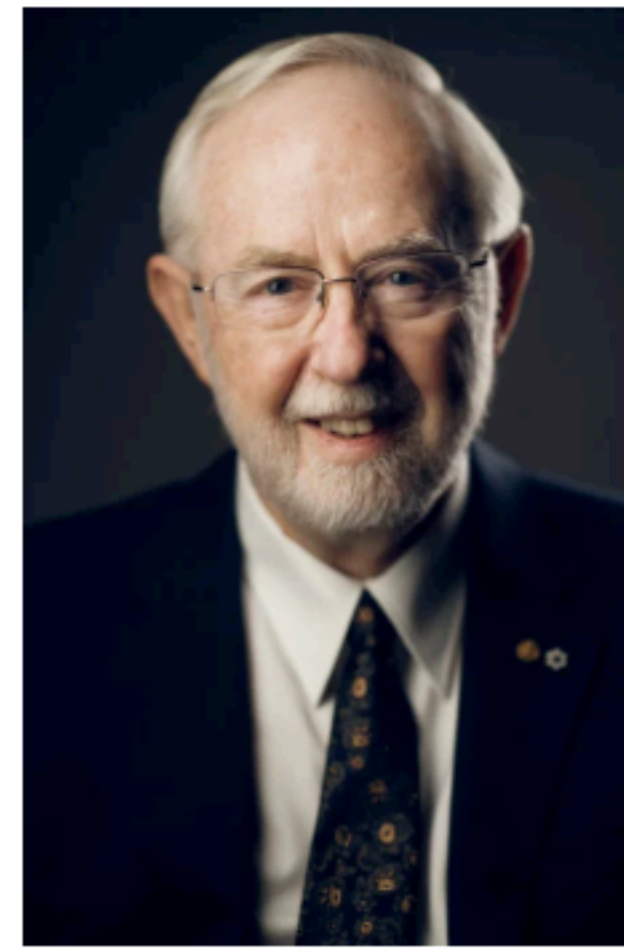


Raymond Davis Jr.

Masatoshi Koshiba

© Nobel Media AB. Photo: A. Mahmoud

Takaaki Kajita

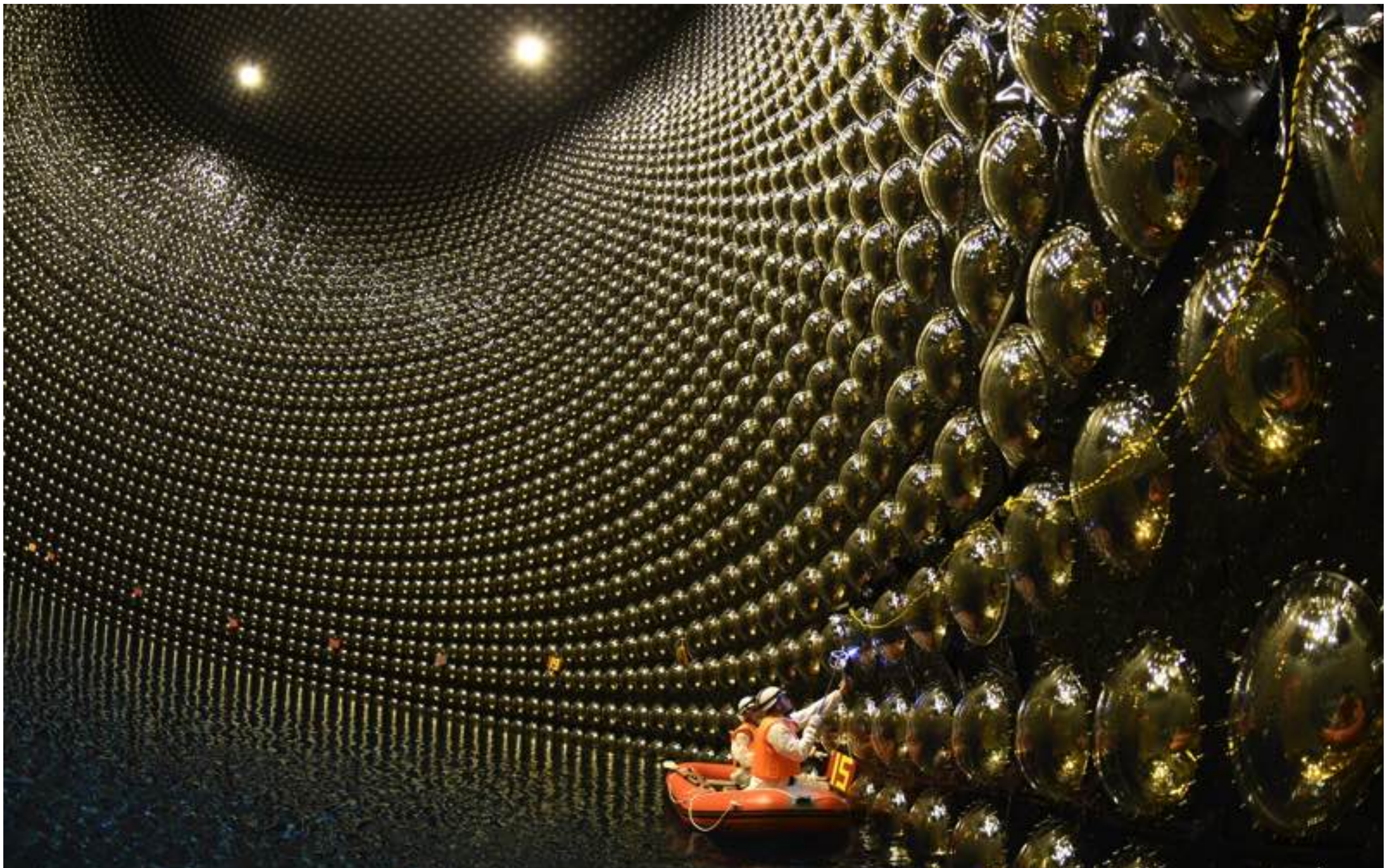


© Nobel Media AB. Photo: A. Mahmoud

Arthur B. McDonald

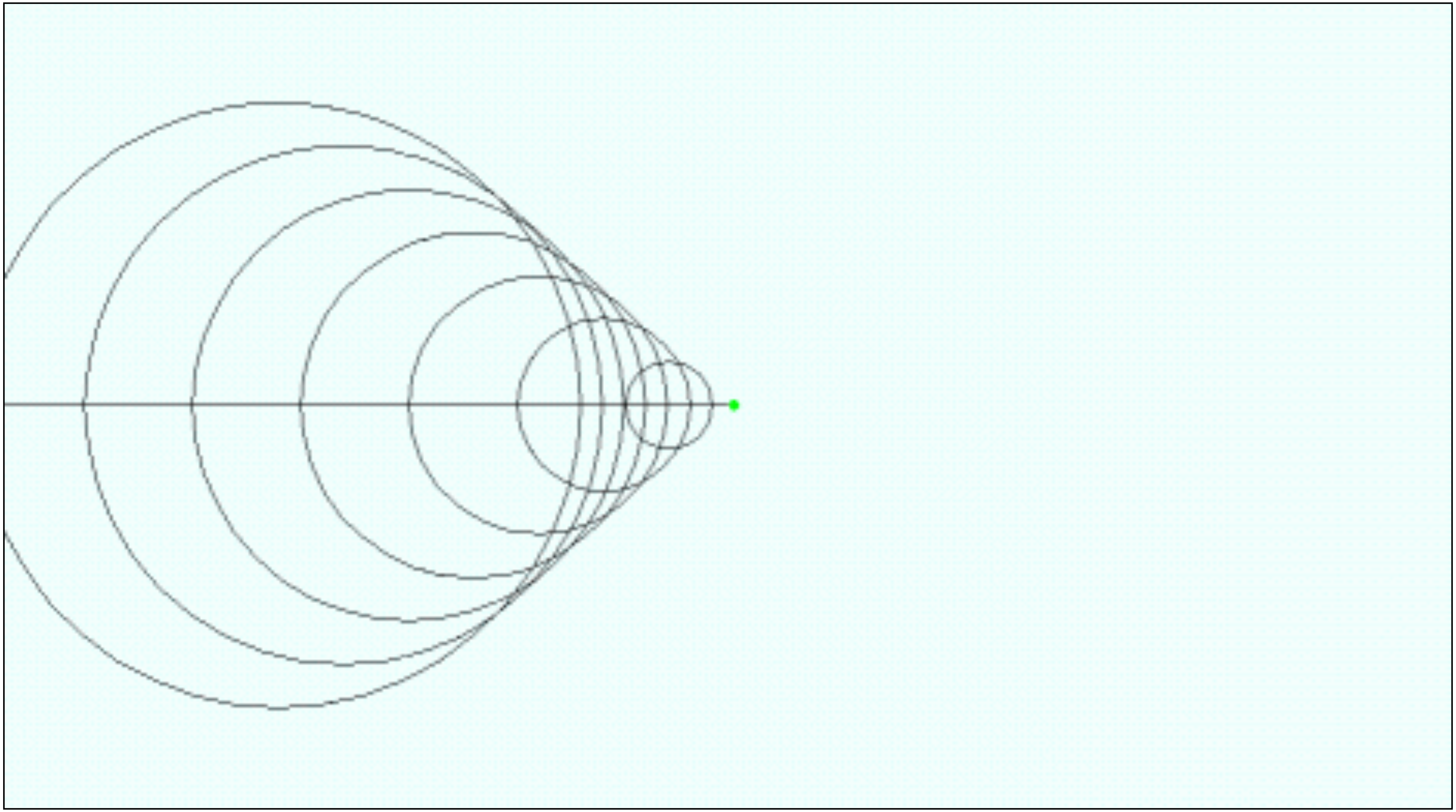
Cherenkov Neutrino Detectors: Super-Kamiokande

The detector is a cylindrical tank holding **50,000 tons of pure water**. The inside of the wall is mounted with 13,000 **photomultipliers** that can detect **Cherenkov radiation**

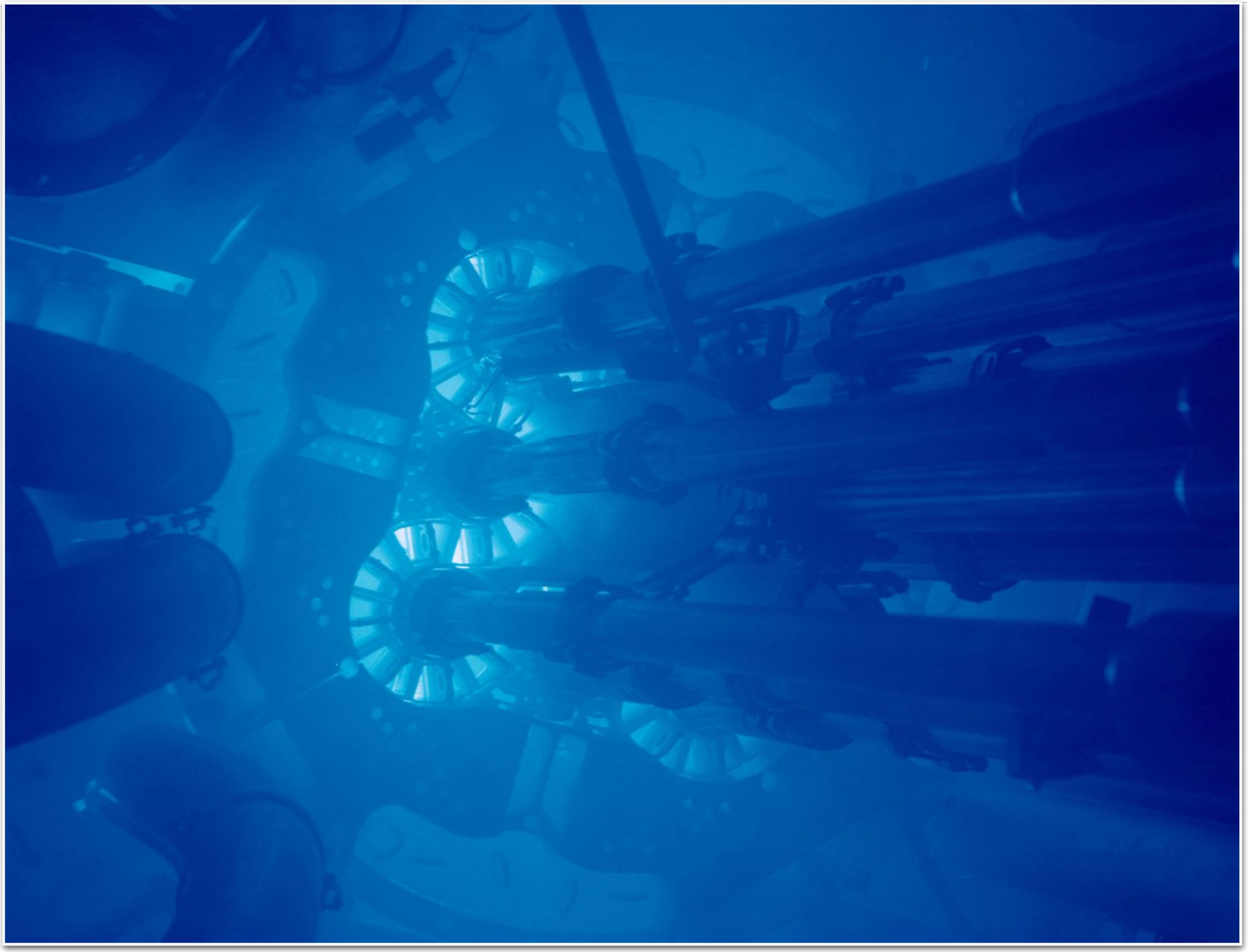


Cherenkov Neutrino Detectors: the Principle

- A neutrino interaction with water produce a charged particle (electron or position) that travels faster than the speed of light in water, creating Cherenkov radiation, similar to a sonic boom



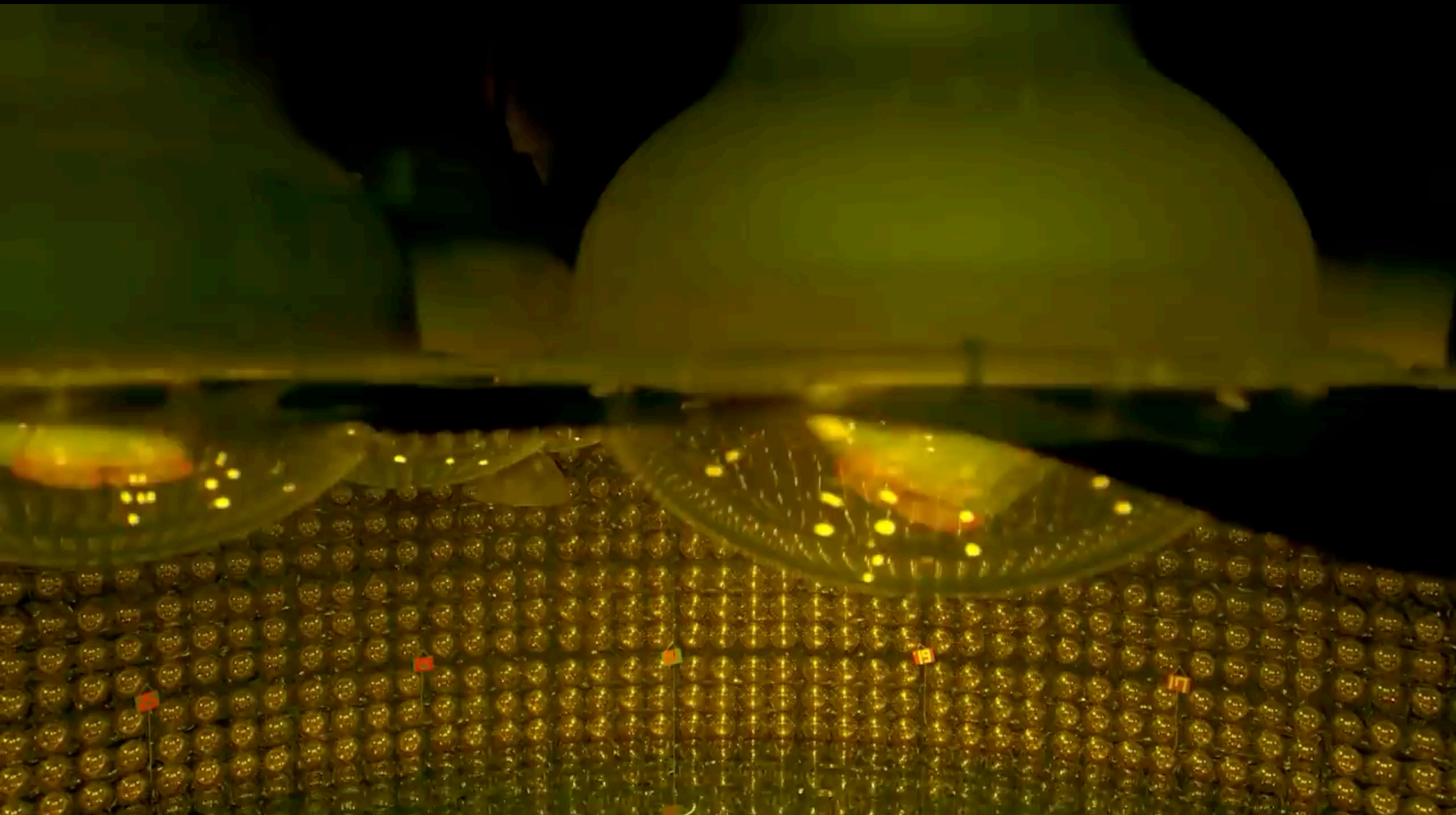
Cherenkov radiation: blue glow of water in a nuclear reactor



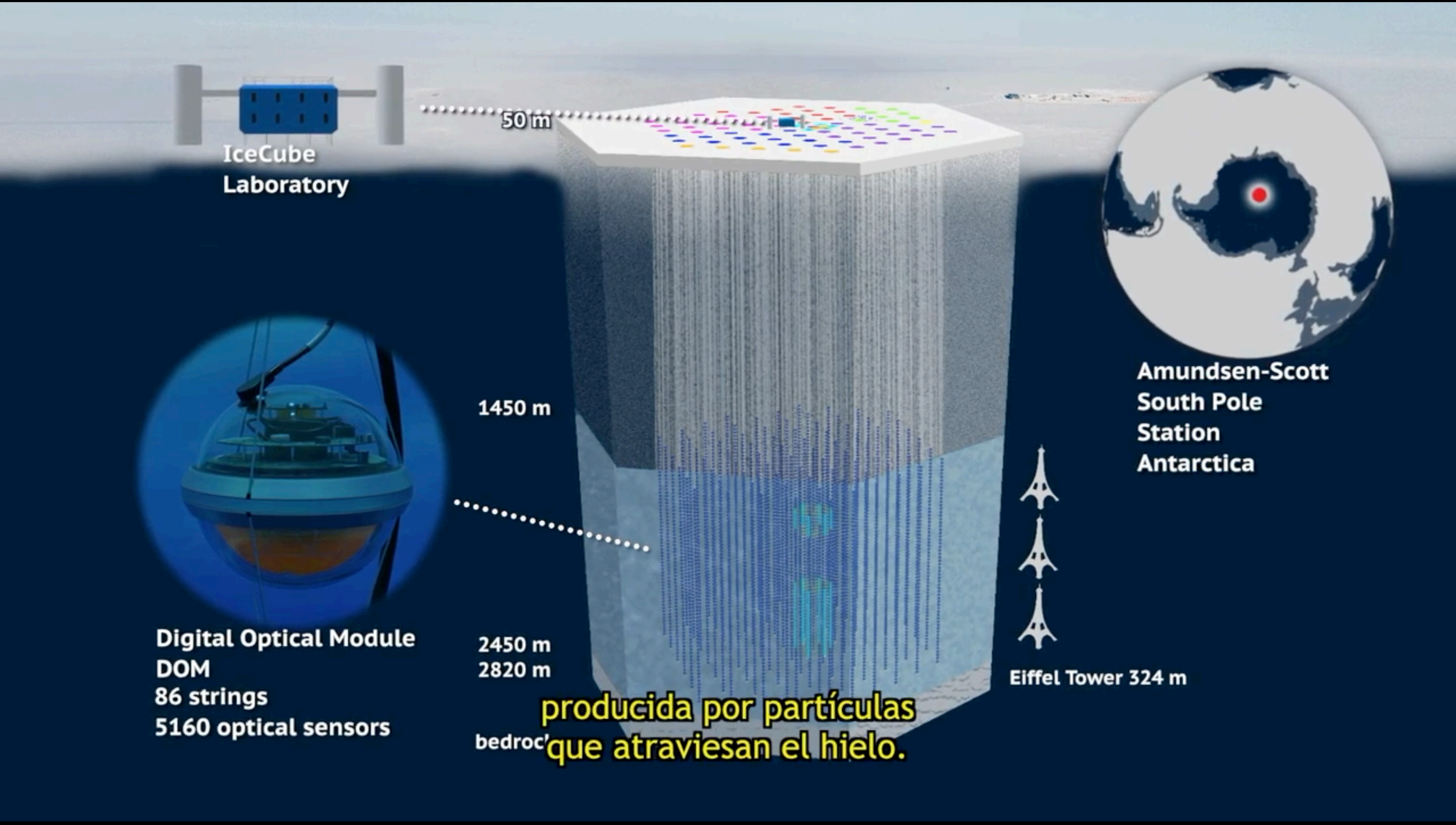
Sonic boom from a volcanic explosion

Sonic boom from a supersonic fighter jet

Cherenkov Neutrino Detectors: Super-Kamiokande



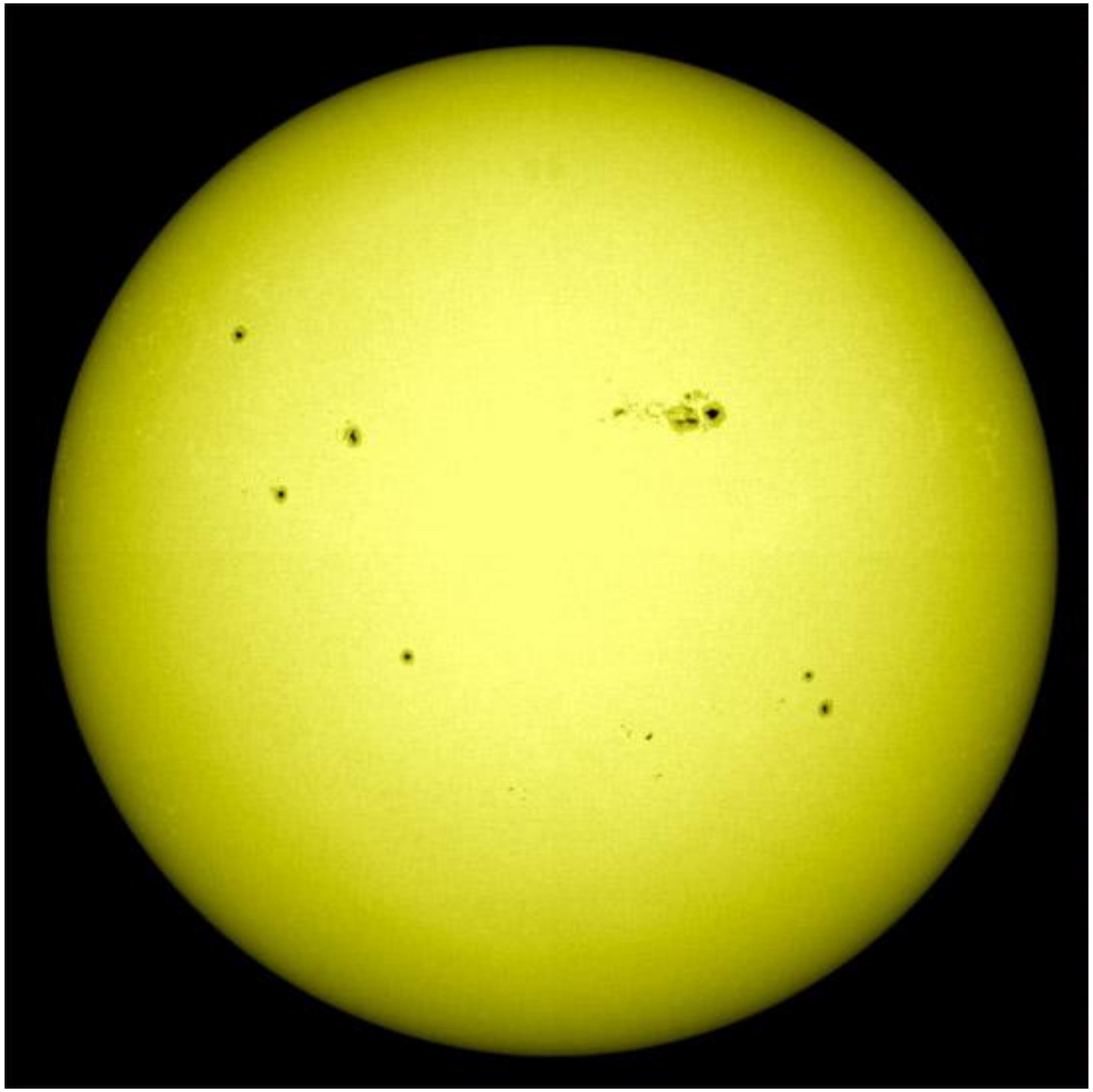
What about neutrinos from sources other than the Sun? We not only need to detect but also to trace the direction of the neutrinos



The Atmospheres of the Sun

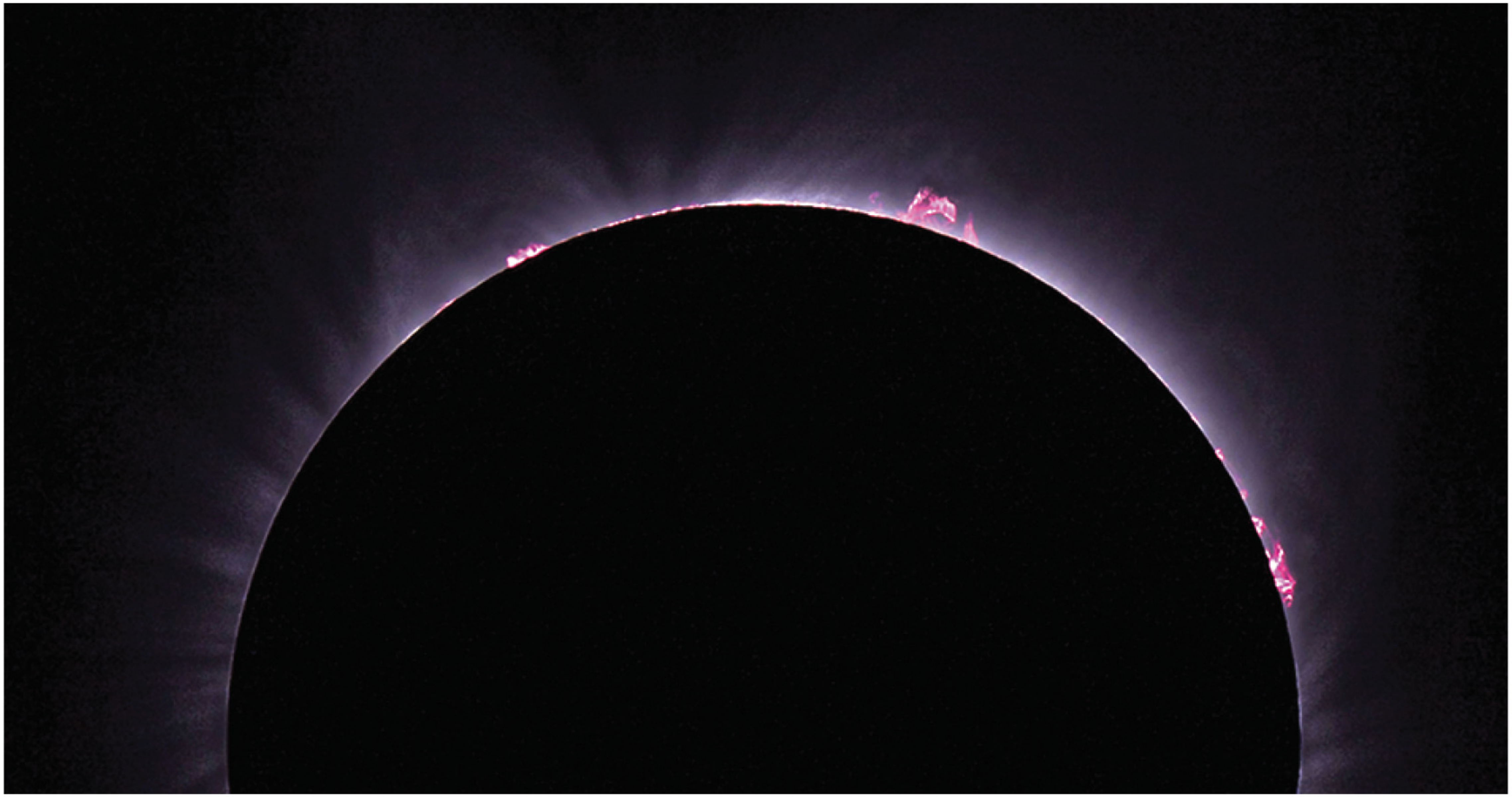
Solar Atmosphere: Photosphere

- **Photosphere**: apparent surface in **white light**
- **Effective temperature**: 5780 K
- **Thickness**: ~500 km
- Temperature decreases outward in the photosphere.
- Atmosphere density drops rapidly with increasing altitude



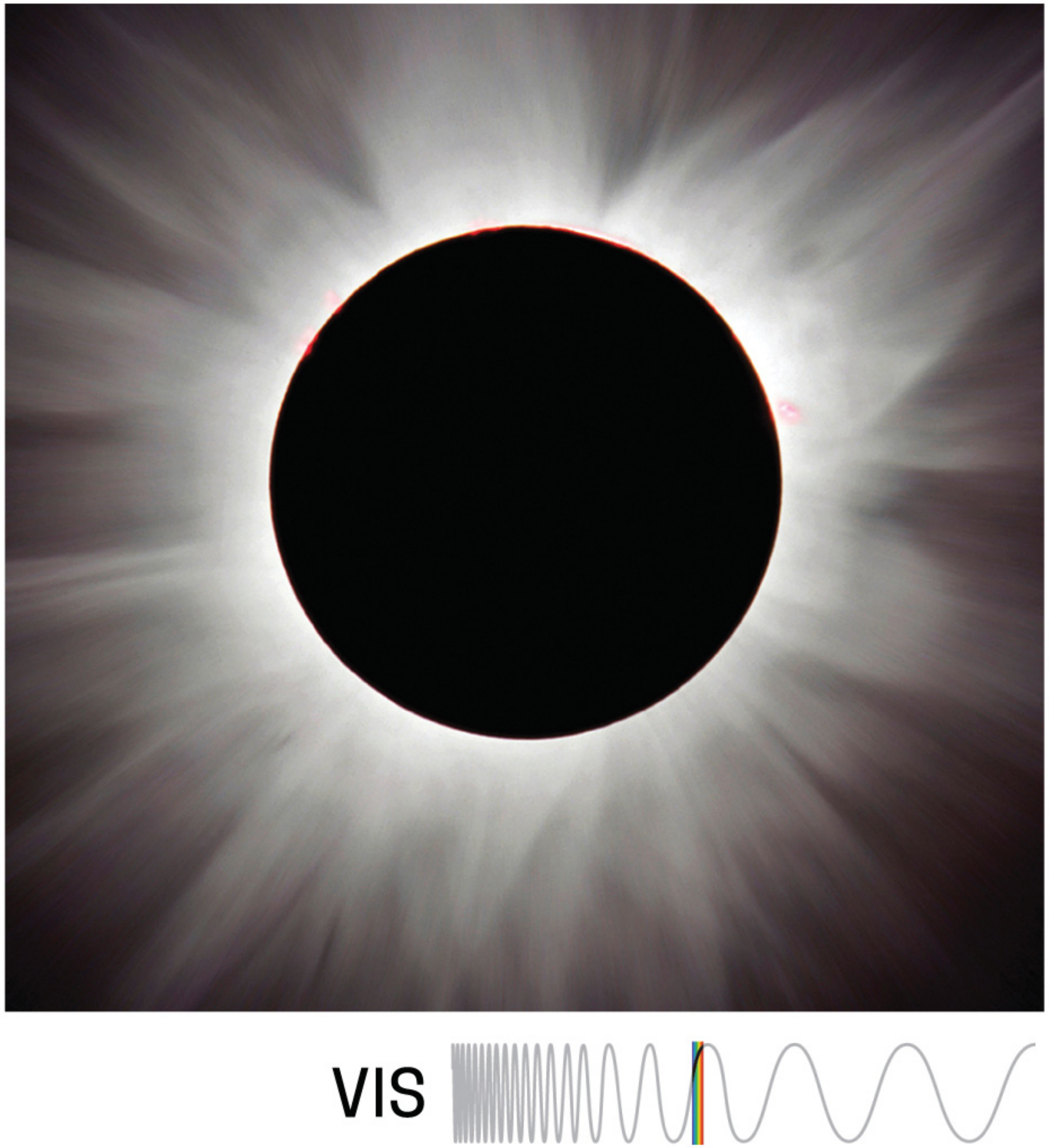
Solar Atmosphere: Chromosphere

- **Chromosphere**: above the photosphere. **Seen through Hα filter**
- Higher temperature than the photosphere (~8000 K)
- It gives off a **reddish emission-line spectrum from hydrogen**.
- The red color is what gives the chromosphere its name, because "**chromosphere**" means the "**place where the color comes from**."



Solar Atmosphere: Corona

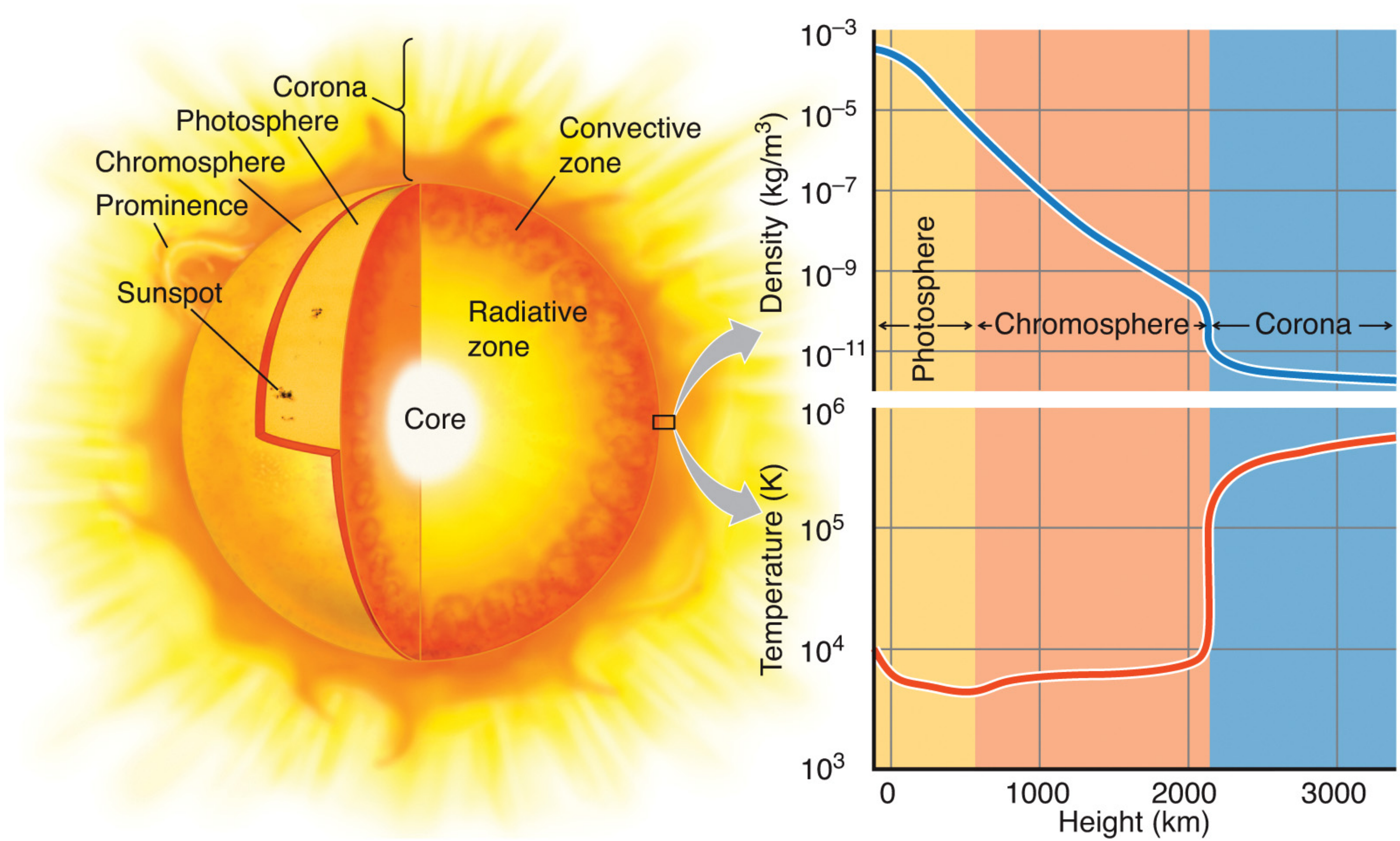
- **Corona**: above the chromosphere
- **Seen in white light during total eclipse**
- Temperature inversion: Very hot — $T = 1$ to 2 million K (emits X-rays as well as visible light).
- It can extend for several solar radii (8 million km above surface).
- The emission is so diffuse that it is not visible on Earth unless there is a solar eclipse.



Westend61 GmbH/Alamy Stock Photo

Solar Atmosphere: Temperature & Density vs. Height

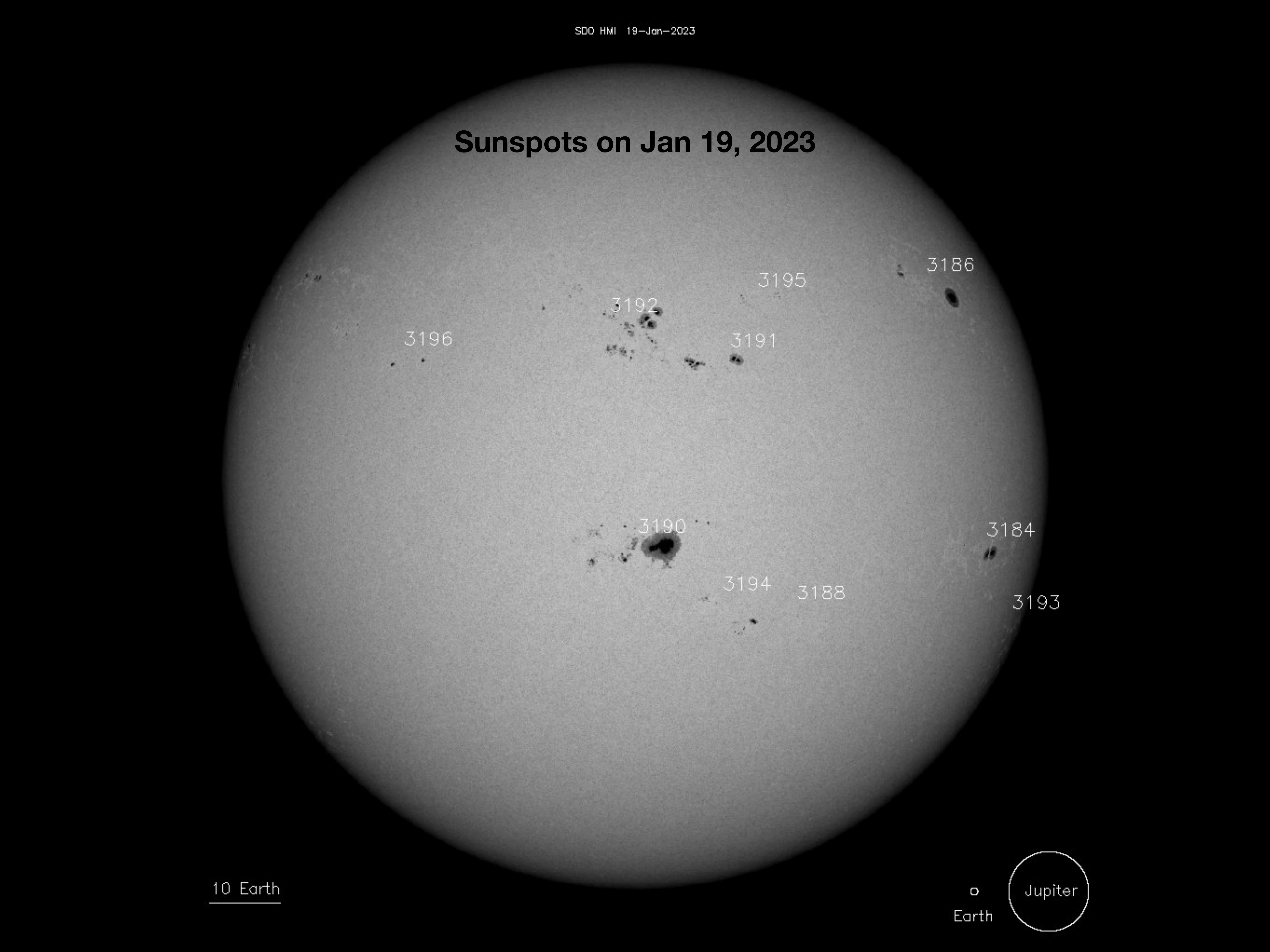
- The density consistently decreases with increasing distance from the Sun.
- The temperature decreases in the photosphere, but rises in the chromosphere and corona, potentially due to magnetic field energy.



Solar Atmosphere Activities

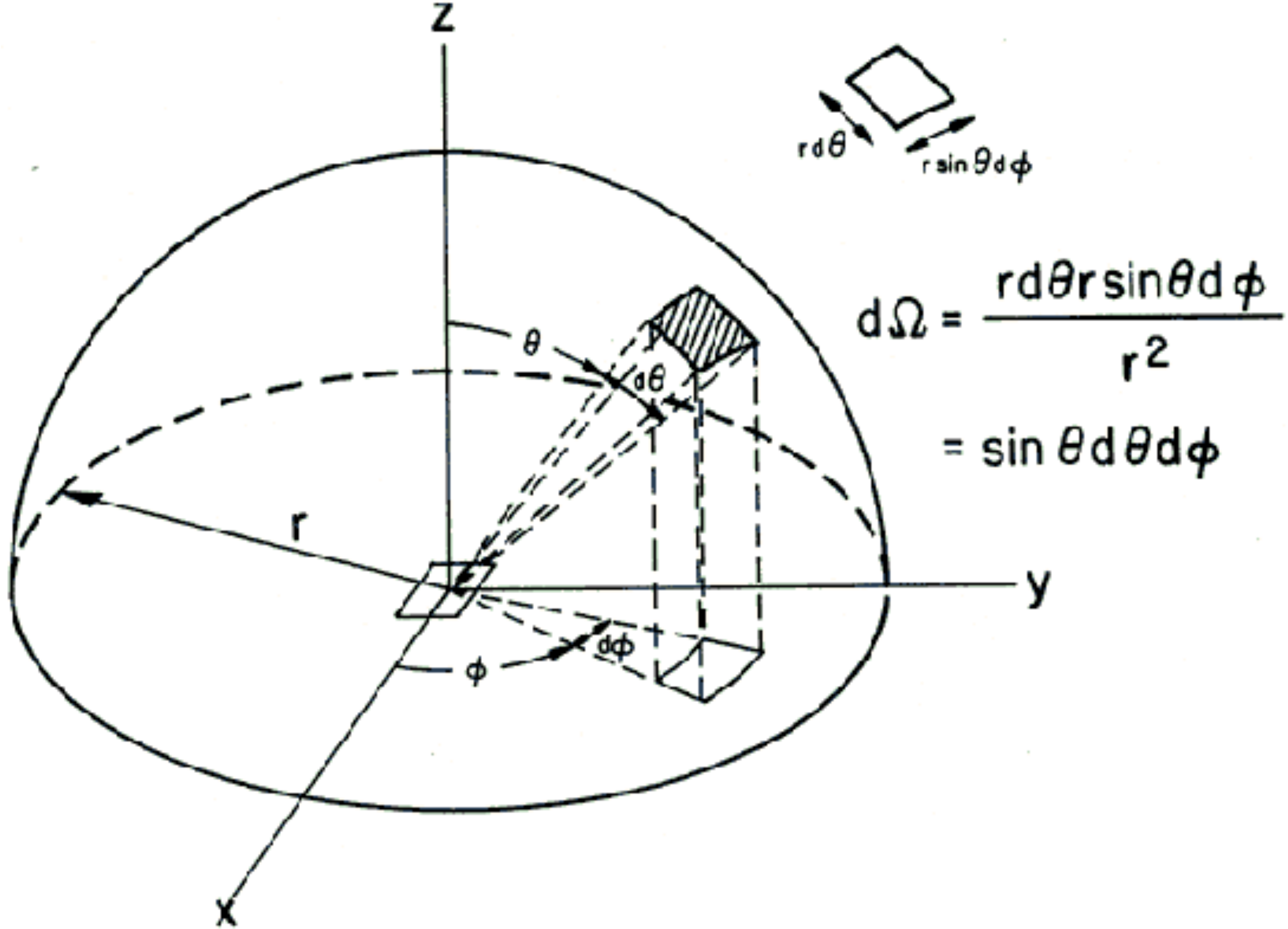
sunspots

rotation period ~25 days


SDO HMI 19-Jan-2023
Sunspots on Jan 19, 2023
3186
3195
3192
3196
3191
3190
3184
3194
3188
3193
10 Earth
Earth
Jupiter

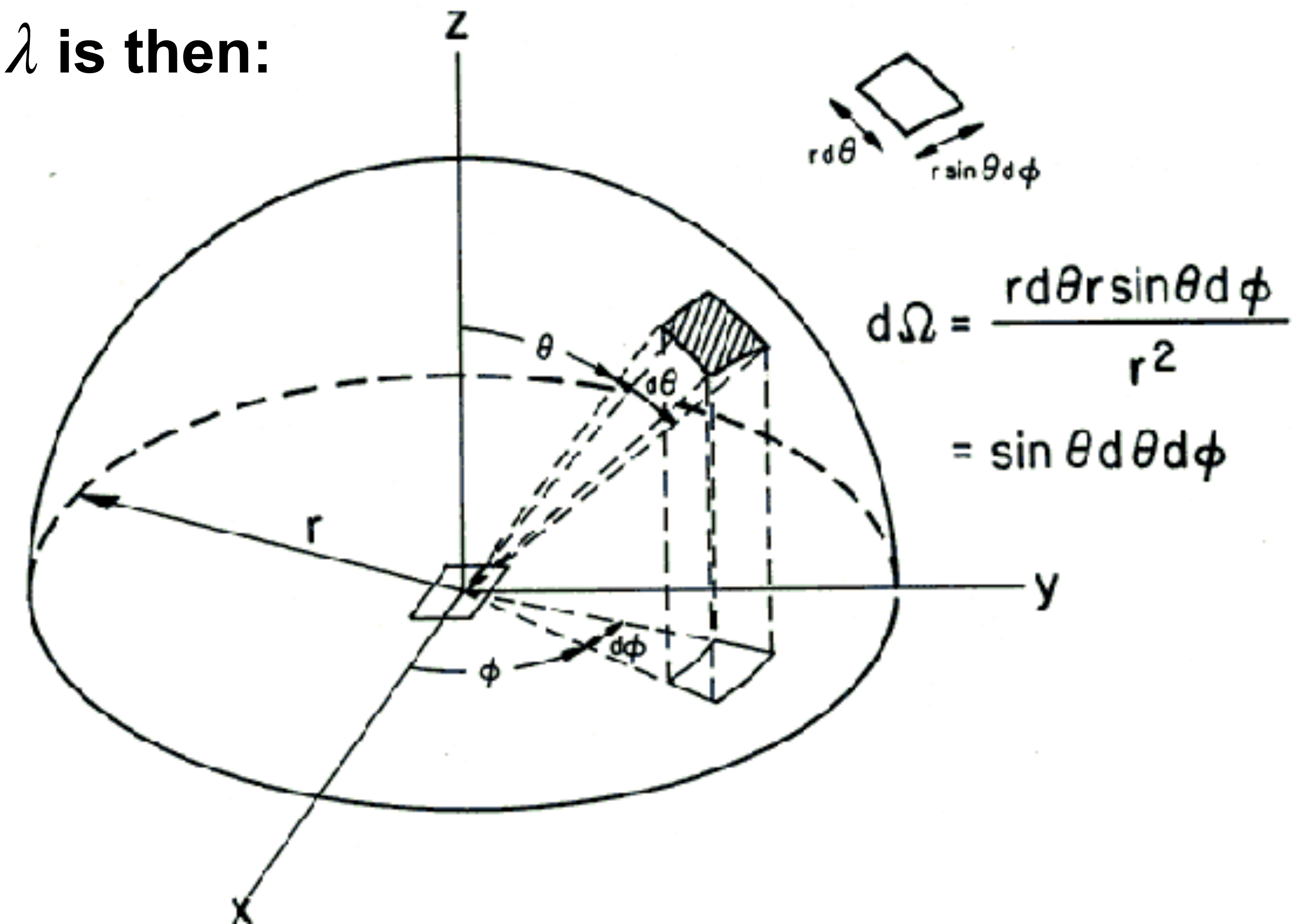
Geometric setup of surface flux

- **Differential solid angle in polar coordinates (see diagram below):** $d\Omega = \sin\theta d\theta d\phi$
- **Projected area towards direction** θ**:** $A\cos\theta$



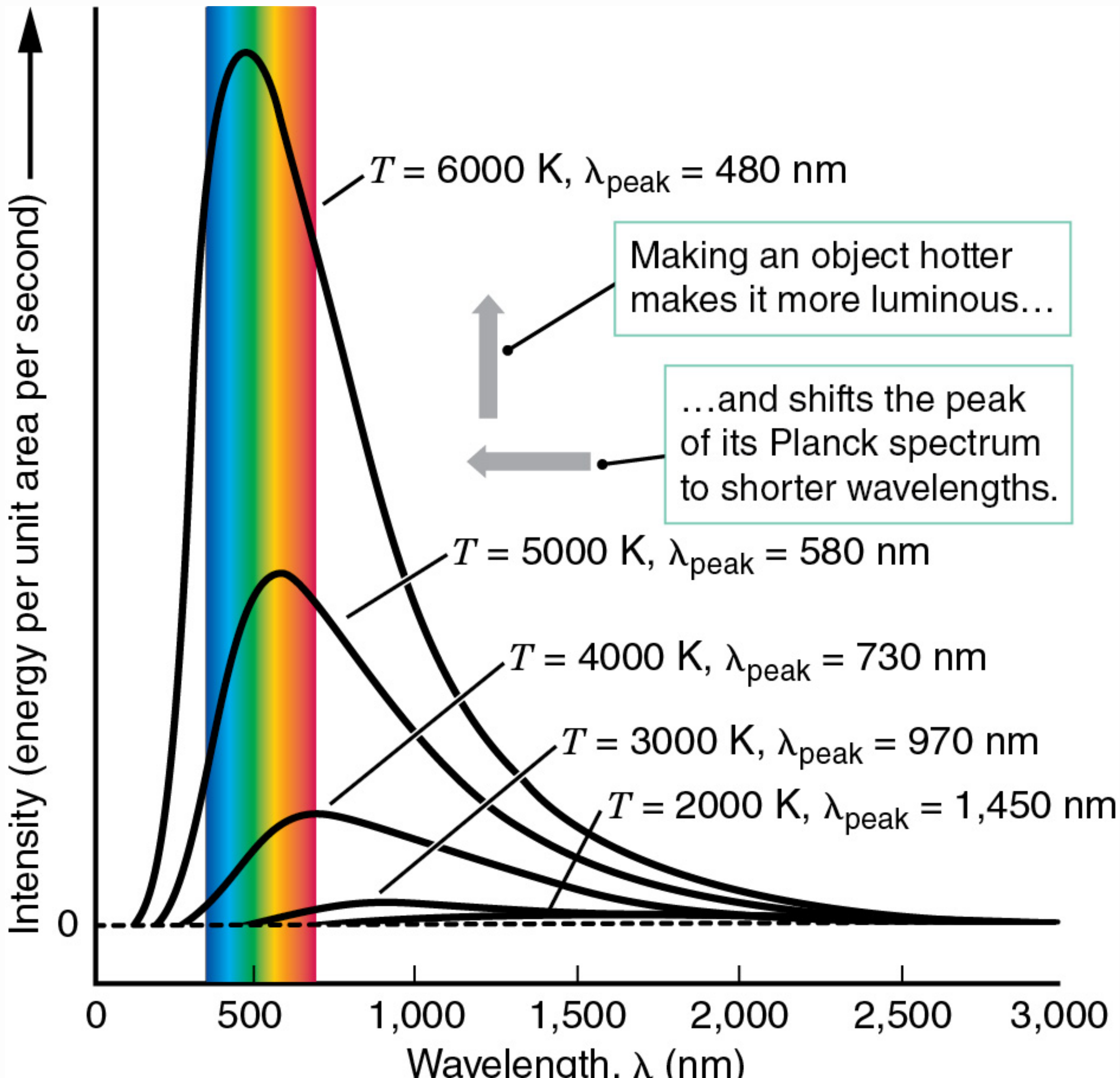
Bolometric surface flux (F_S) vs. surface flux density ($F_{S,\lambda}$)

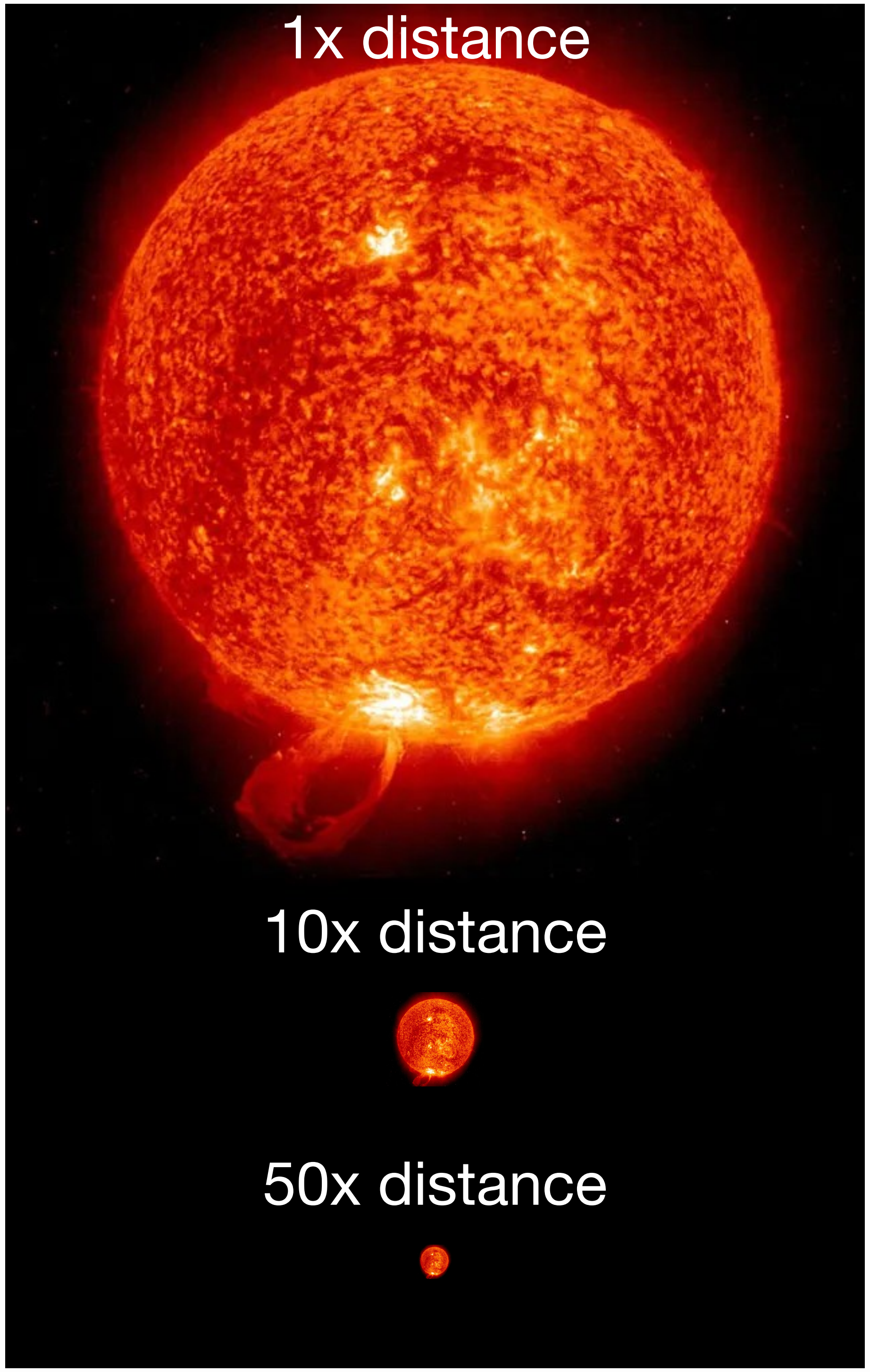
- **Power radiated into $d\Omega$ from A between $\lambda, \lambda + d\lambda$:**
  $dP = B_\lambda(T) A \cos\theta d\Omega d\lambda$
  given that $B_\lambda(T)$ is the power (dP) emitted per unit projected surface area ($A\cos\theta$) per unit solid angle ($d\Omega$) per unit wavelength ($d\lambda$)
- **Bolometric surface flux emitted to the upper half hemisphere is:**
  $$F_S = \frac{P}{A} = \int_{\lambda=0}^{\infty}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} B_\lambda(T)\ \cos\theta\ \sin\theta\ d\theta\ d\phi\ d\lambda = \pi\int_{\lambda=0}^{\infty} B_\lambda(T) d\lambda = \sigma_{\rm SB} T^4$$
- **The surface flux density at λ is then:**
  $F_{S,\lambda} = \pi B_\lambda(T)$



What $B_\lambda(T)$ is to a distant observer?

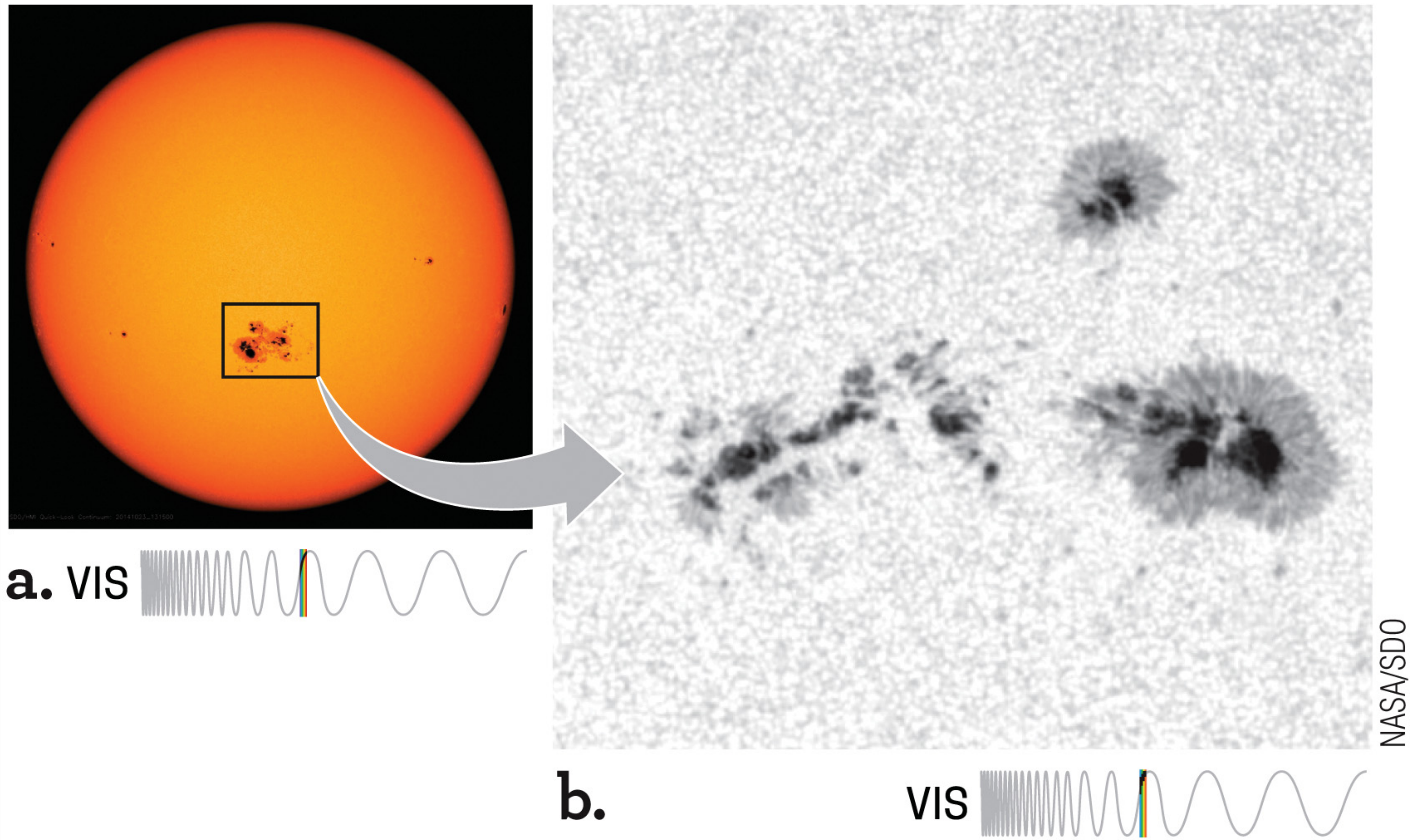
- **Observed flux density** can be derived from **luminosity density** using the **inverse distance square law**: $F_\lambda = L_\lambda/4\pi d^2 = (4\pi R^2 \cdot \pi B_\lambda(T))/(4\pi d^2) = B_\lambda(T) \cdot \pi R^2/d^2$
- Since $\pi R^2/d^2$ is the **angular area** of the source, hence **Planck function** $B_\lambda(T)$ gives the **surface brightness** of the source *(at λ), which is* ***distance invariant.***



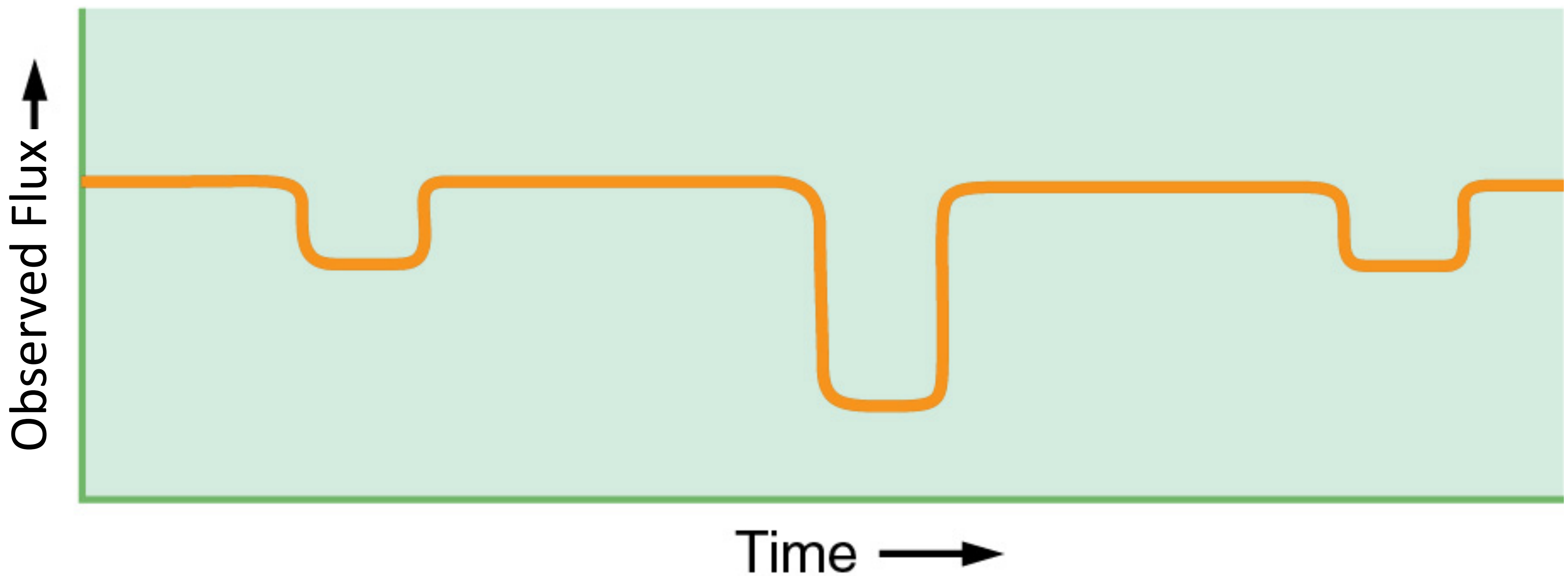


Use Planck Function to Explain Sunspots

- **Planck function** $B_\lambda(T)$ gives the **observed surface brightness** of the source *at λ. It is **a distance-invariant quantity***
- **Sunspots** appear darker against the rest of the solar disk because they have **lower temperature**, thus lower surface brightness



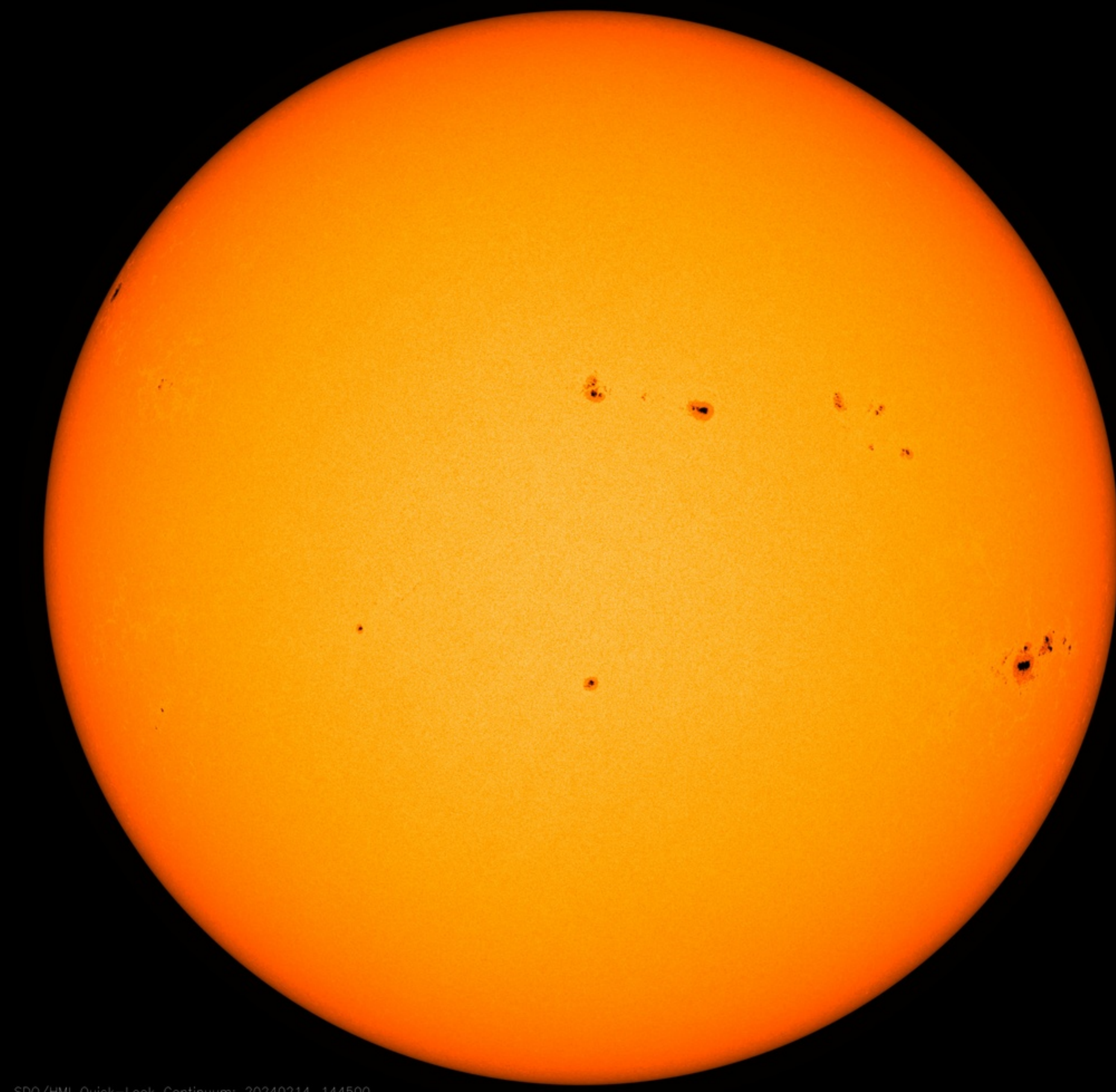
Eclipsing Binary Stars - Understanding the Light Curve



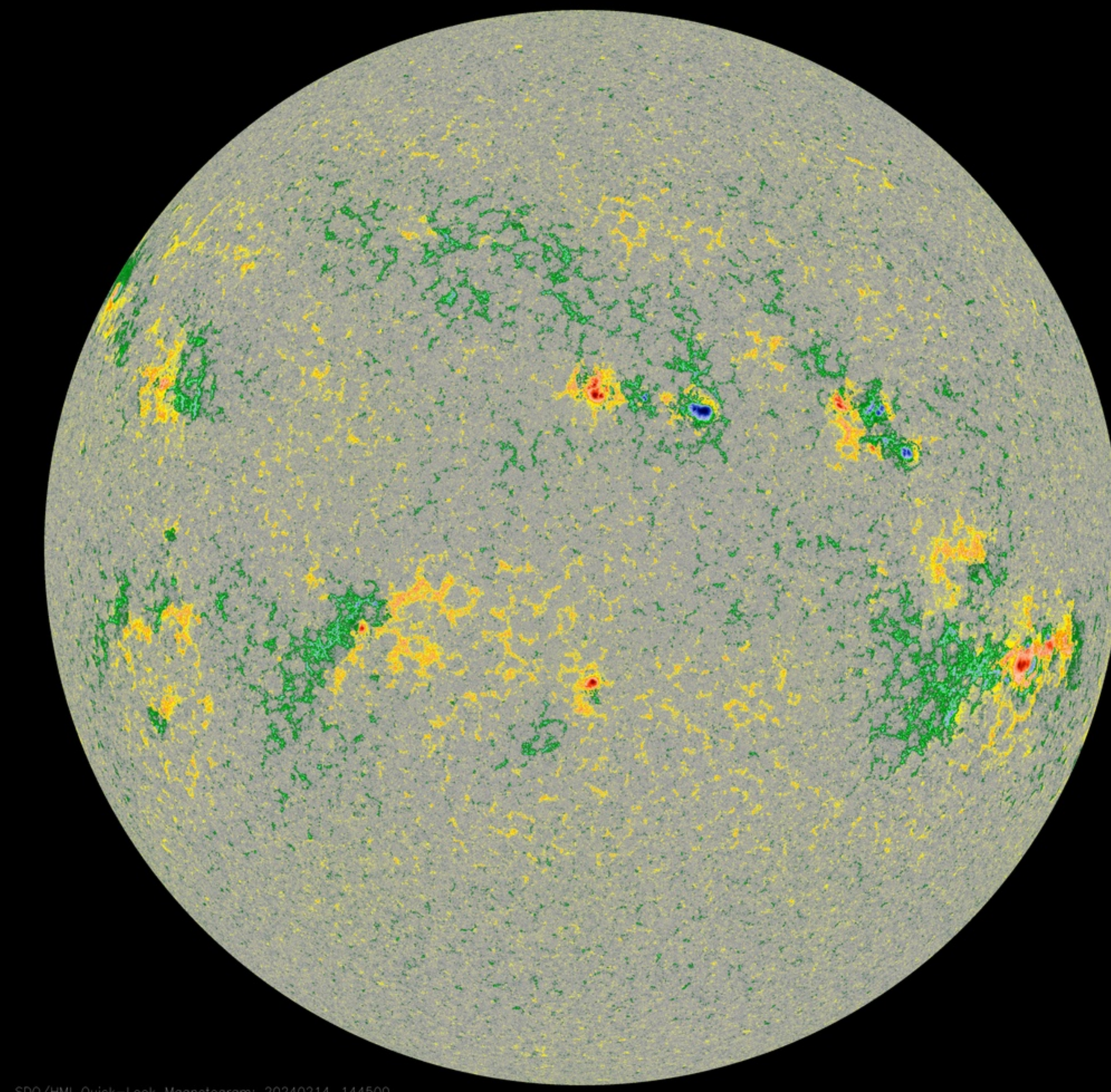
- In an **eclipsing binary** system, the total light coming from the star system decreases when *either* star passes in front of the other.
- When the hotter object is eclipsed, the eclipse is deeper. This is because the **surface brightness (observed flux per unit solid angle: F_λ/Ω)** of the hotter object is greater: $F_\lambda/\Omega = B_\lambda(T)$

https://sdo.gsfc.nasa.gov/data/ Solar Images Now!

Intensity



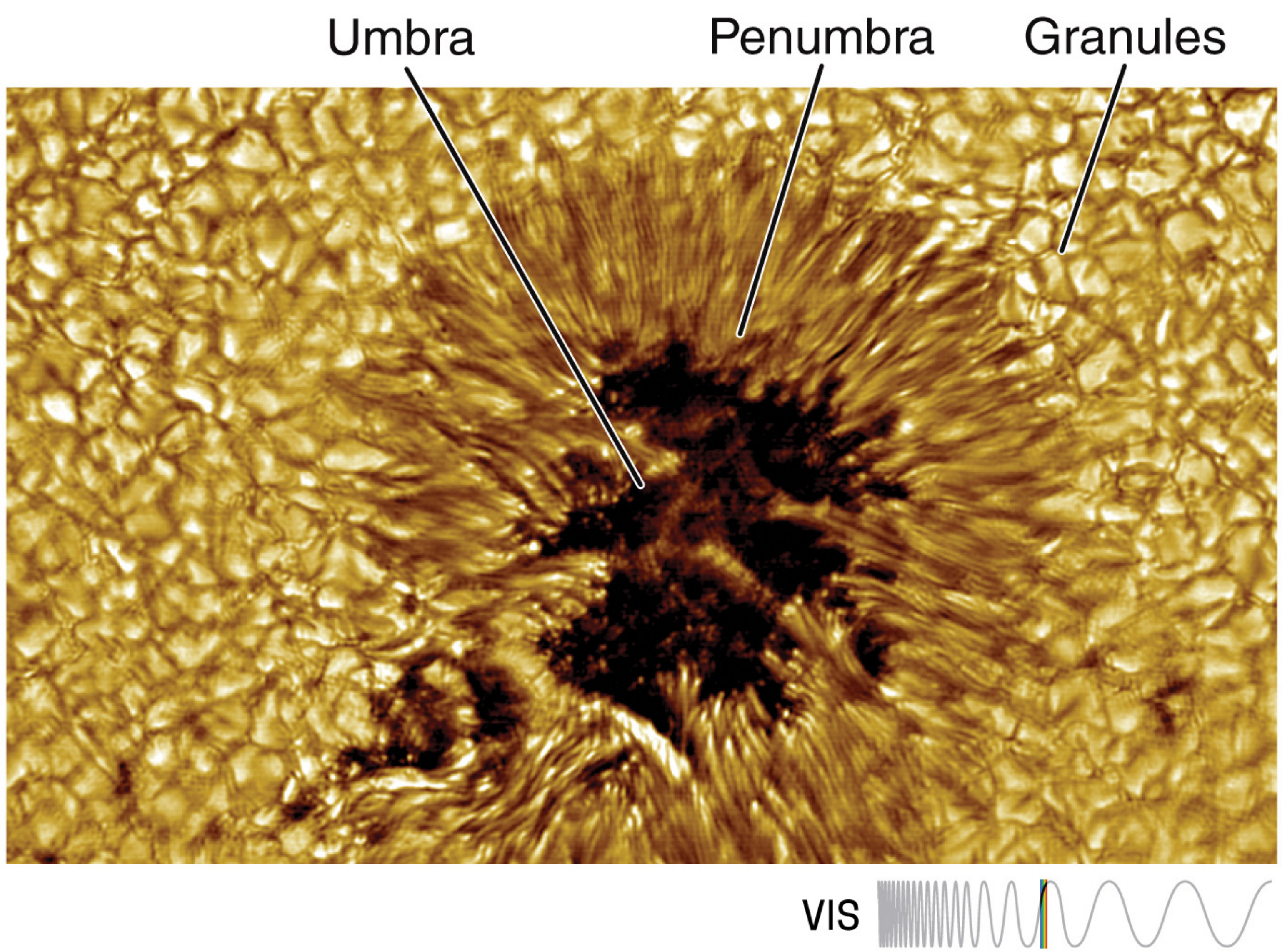
Magnetogram



- Because of differential rotation, magnetic field gets tangled. The areas where the magnetic field gets knotted up are **sunspots**.

Anatomy of a Sunspot

- **Sunspots**: cooler areas in the photosphere
- Sunspot structure: dark inner **umbra** with surrounding **penumbra**
- Sunspots are caused by tangled magnetic fields that trap gas at the surface, prohibiting them from sinking and warming (**impeding convection**).
- Sunspots occur in pairs connected by a **magnetic loop**.
- Sunspots last approximately **2–11 days**.



Solar Atmosphere Activities

prominences, flares, CMEs

Solar Prominences & Filaments

hot plasma extending from the photosphere into the corona

Prominences are called **filaments** when viewed against the solar disk, instead of viewed on the edge of the disk

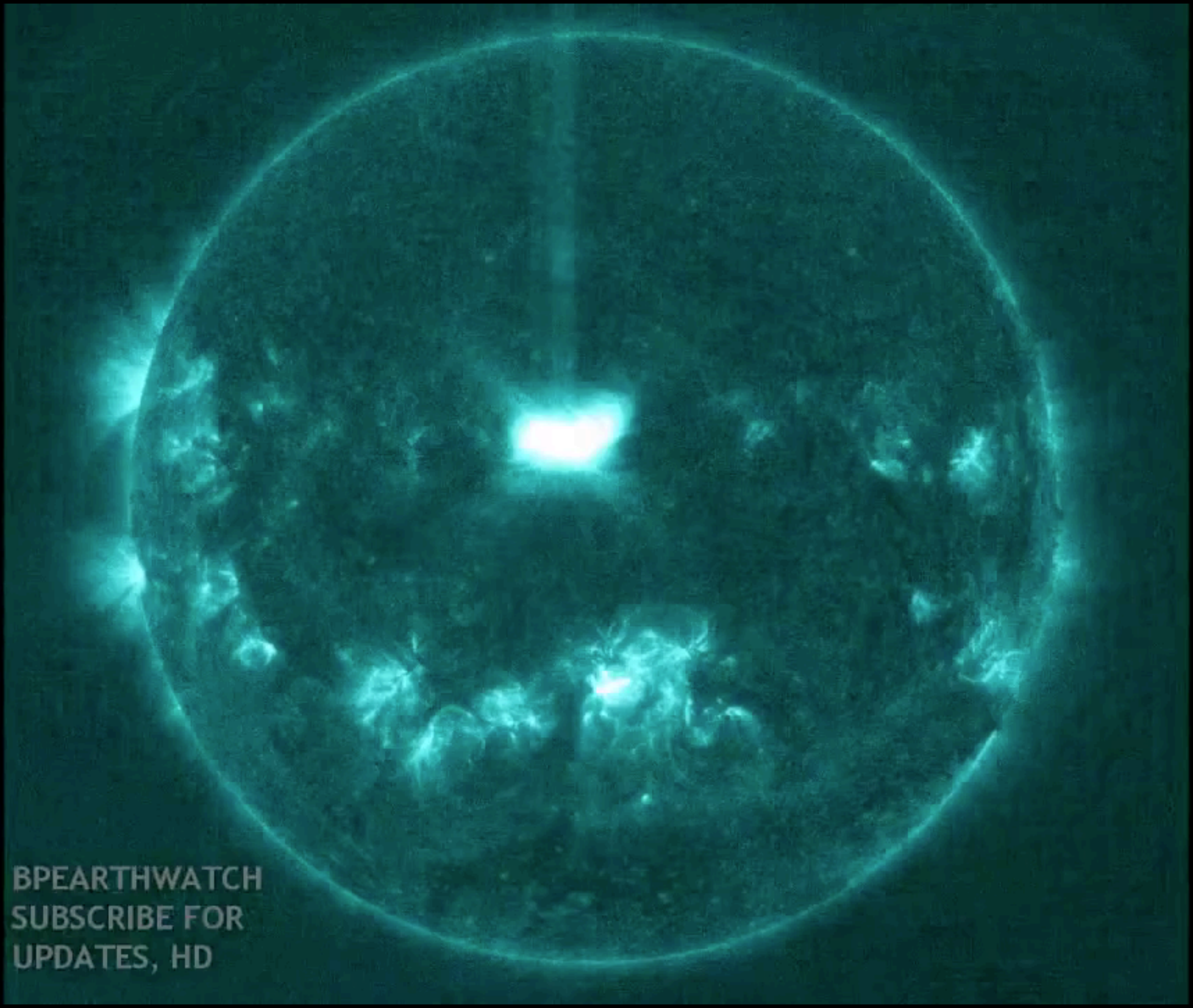
appear constrained by magnetic fields and anchored on sunspots

Some stable ones can persist over a month

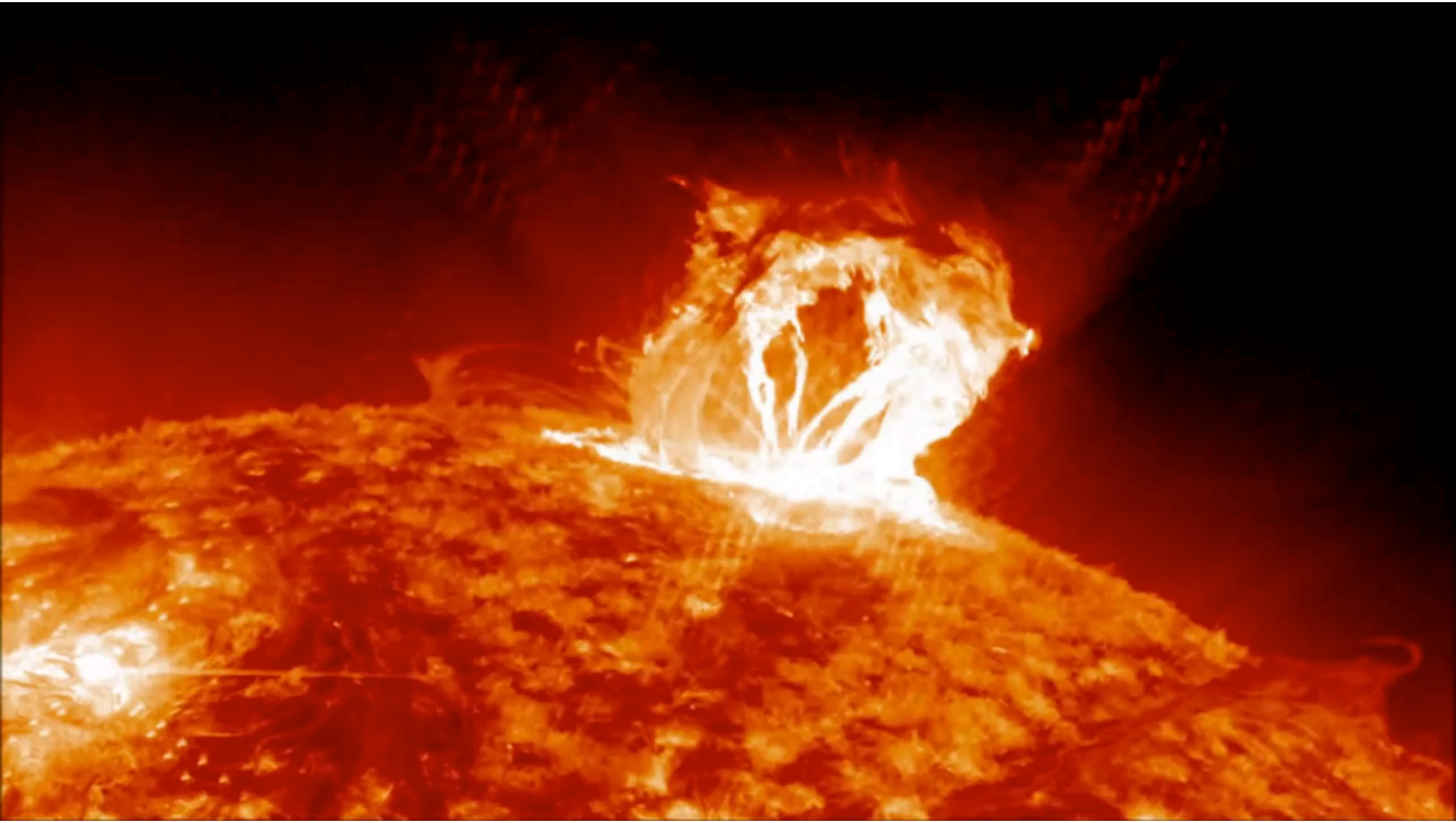
1 Solar Radius = 110 Earth Radii = 10 Jupiter Radii

1 AU = 215 Solar Radii = 108 Solar Diameter

- **Solar flares**: violent eruptions powered by the sudden release of magnetic energy. Last only a few minutes to a few hours

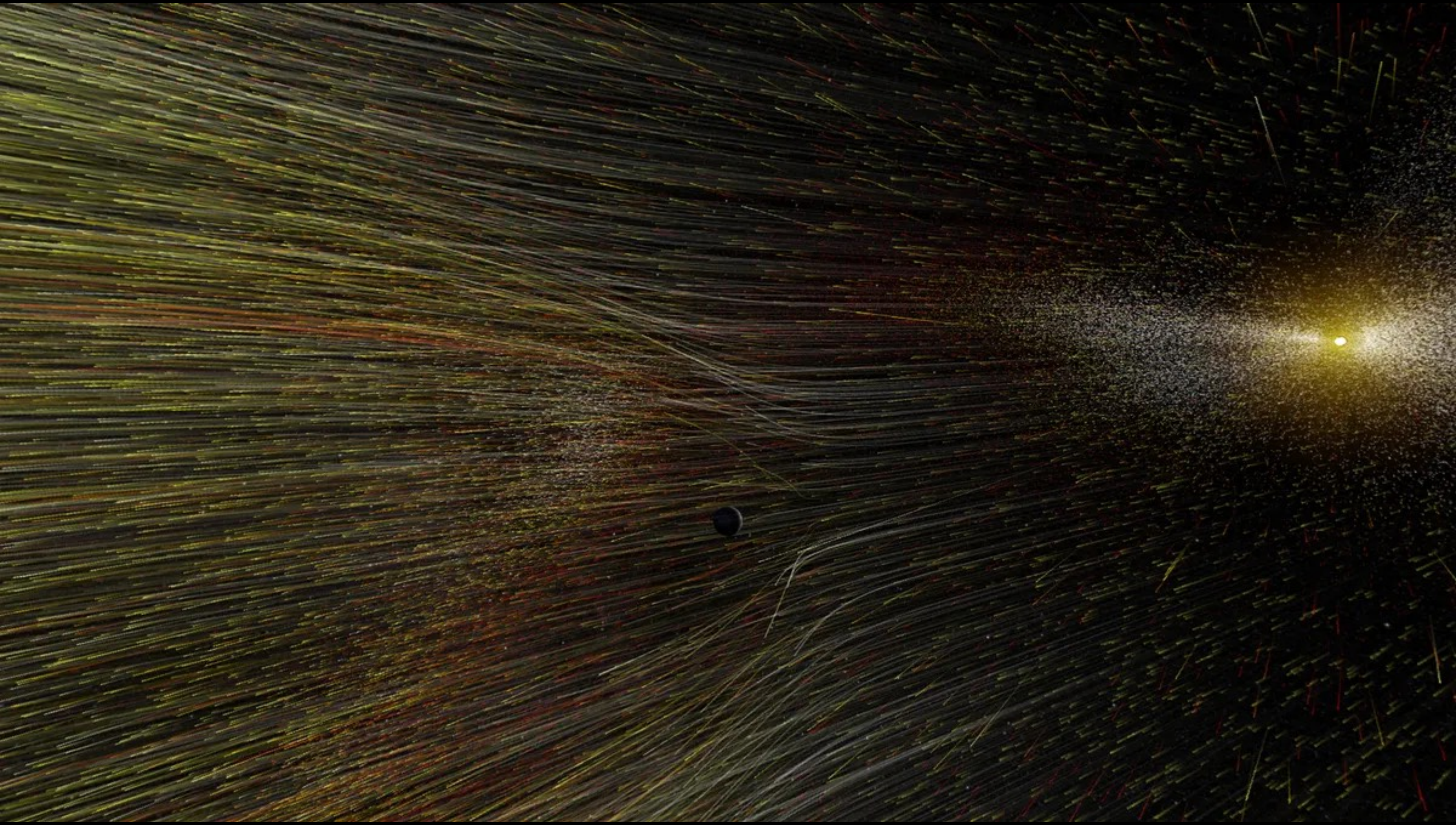


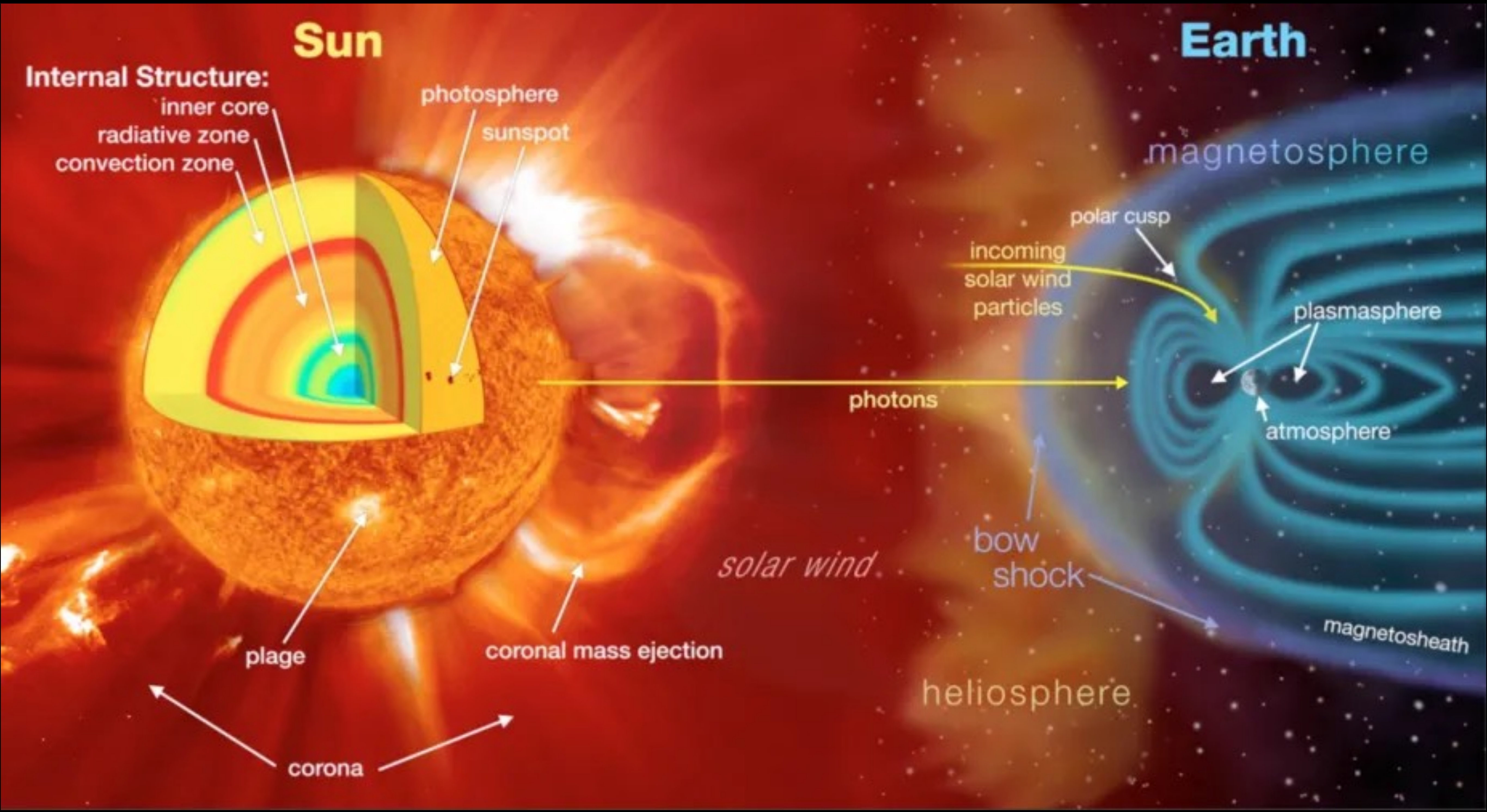
- Solar flares are often followed by **coronal mass ejections (CME)**
- Hot plasma ejected at speeds up to 1,500 km/s
- Powerful bursts of energetic particles

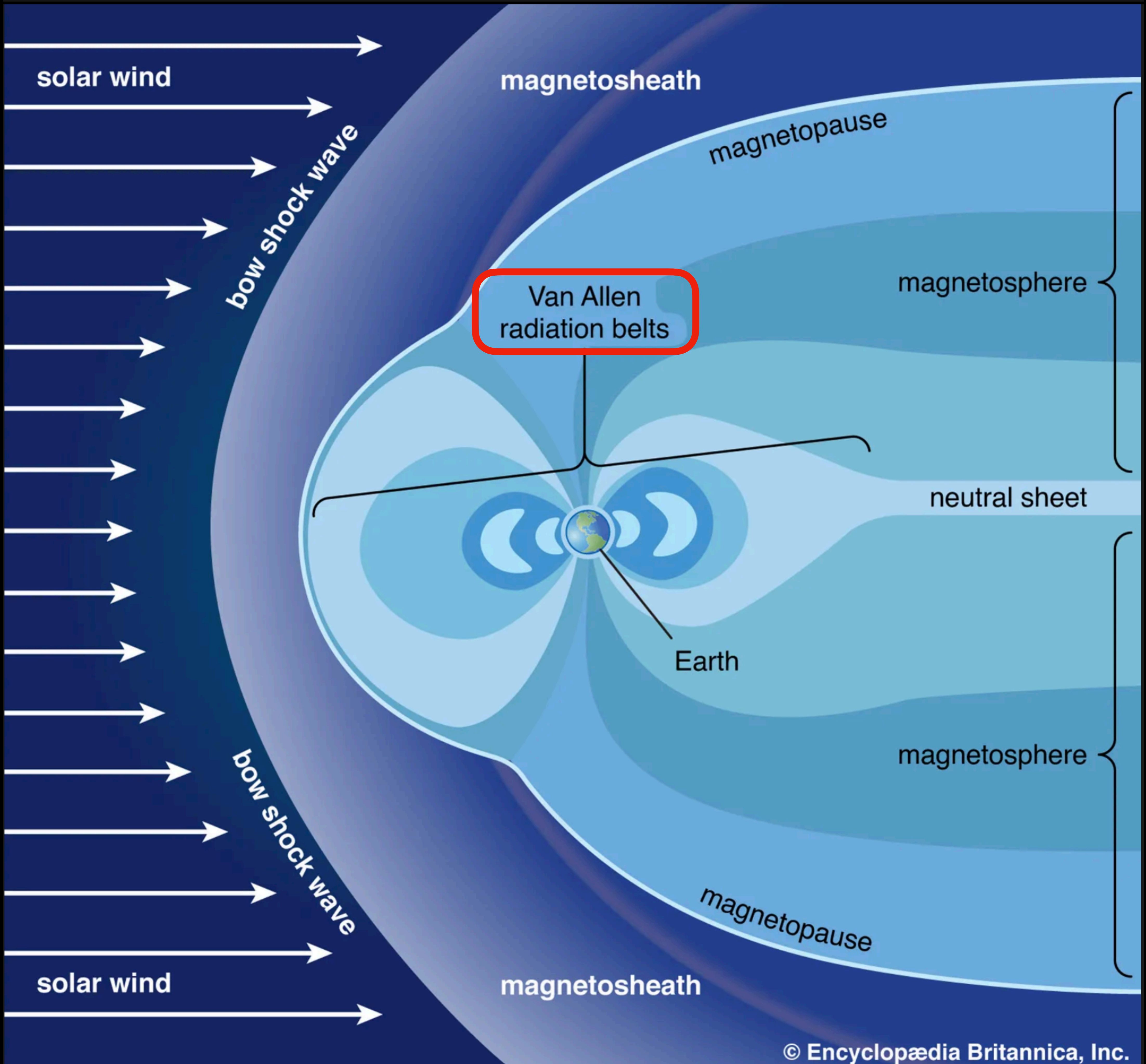


Solar wind and the Heliosphere

Artist’s impression of solar wind




Sun
Internal Structure:
inner core
radiative zone
convection zone
photosphere
sunspot
plage
coronal mass ejection
corona
solar wind
photons
heliosphere
bow
shock
Earth
magnetosphere
polar cusp
incoming
solar wind
particles
plasmasphere
atmosphere
magnetosheath


solar wind
magnetosheath
bow shock wave
magnetopause
Van Allen
radiation belts
magnetosphere
neutral sheet
Earth
magnetosphere
bow shock wave
magnetopause
solar wind
magnetosheath
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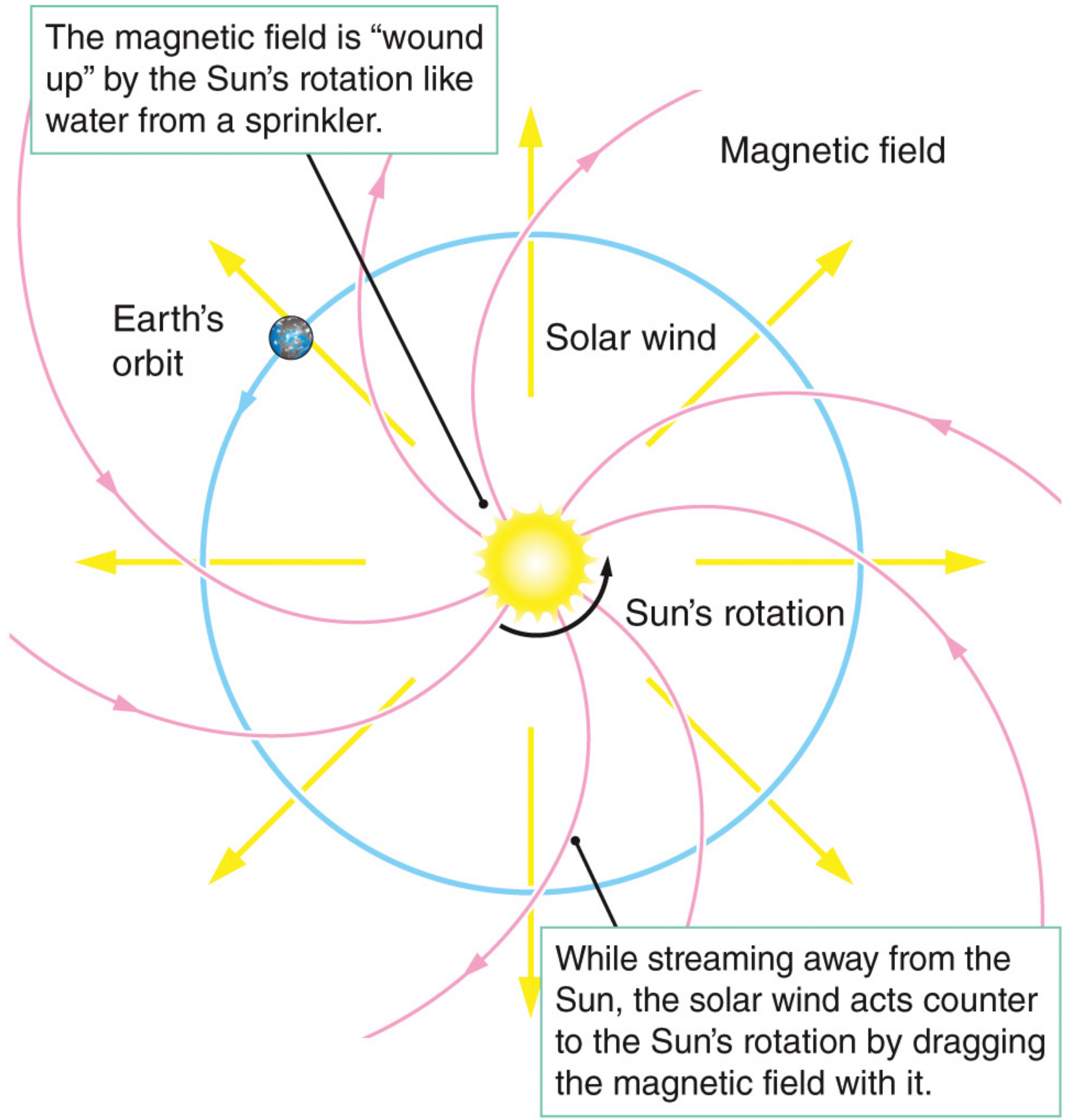
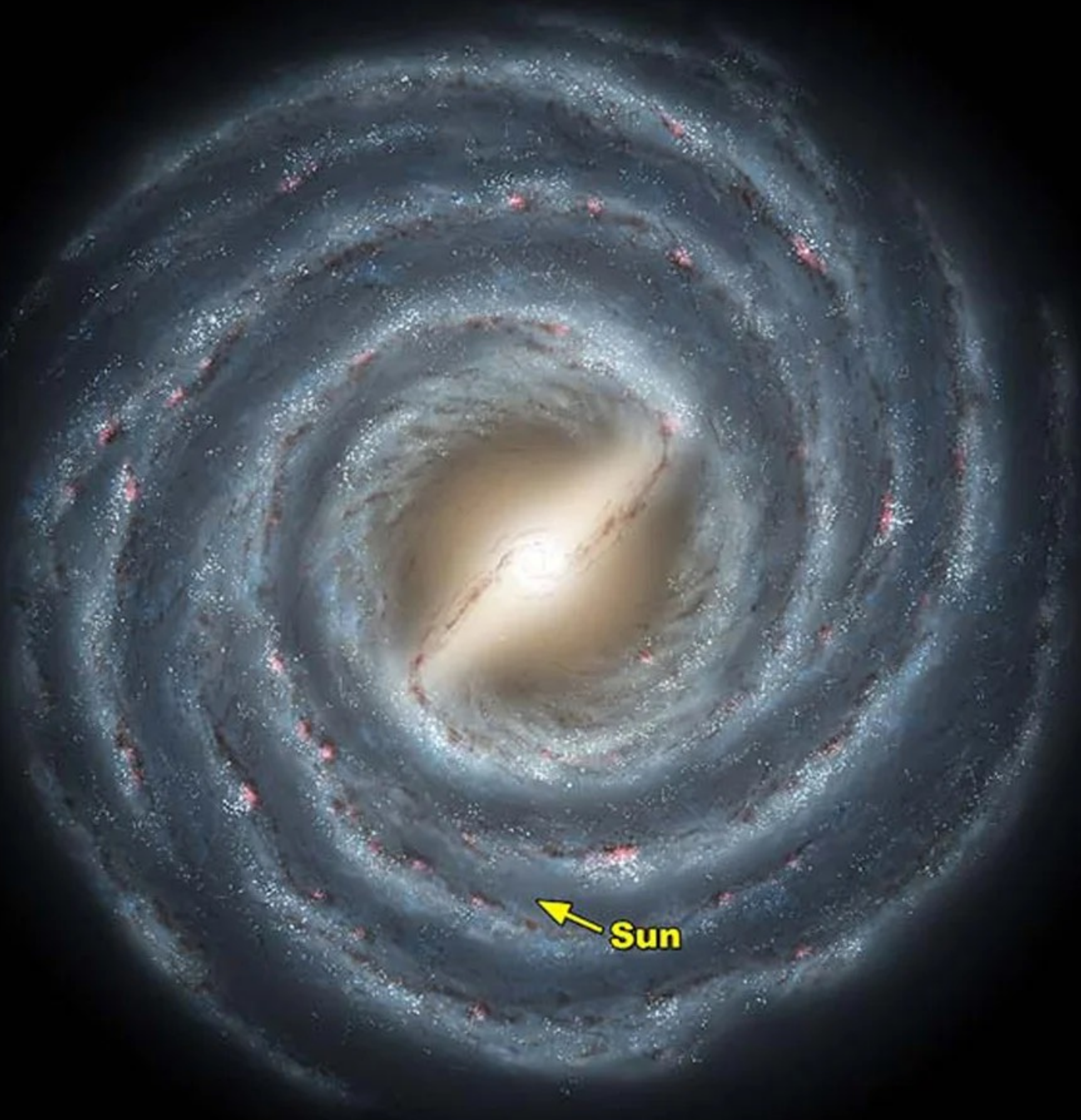
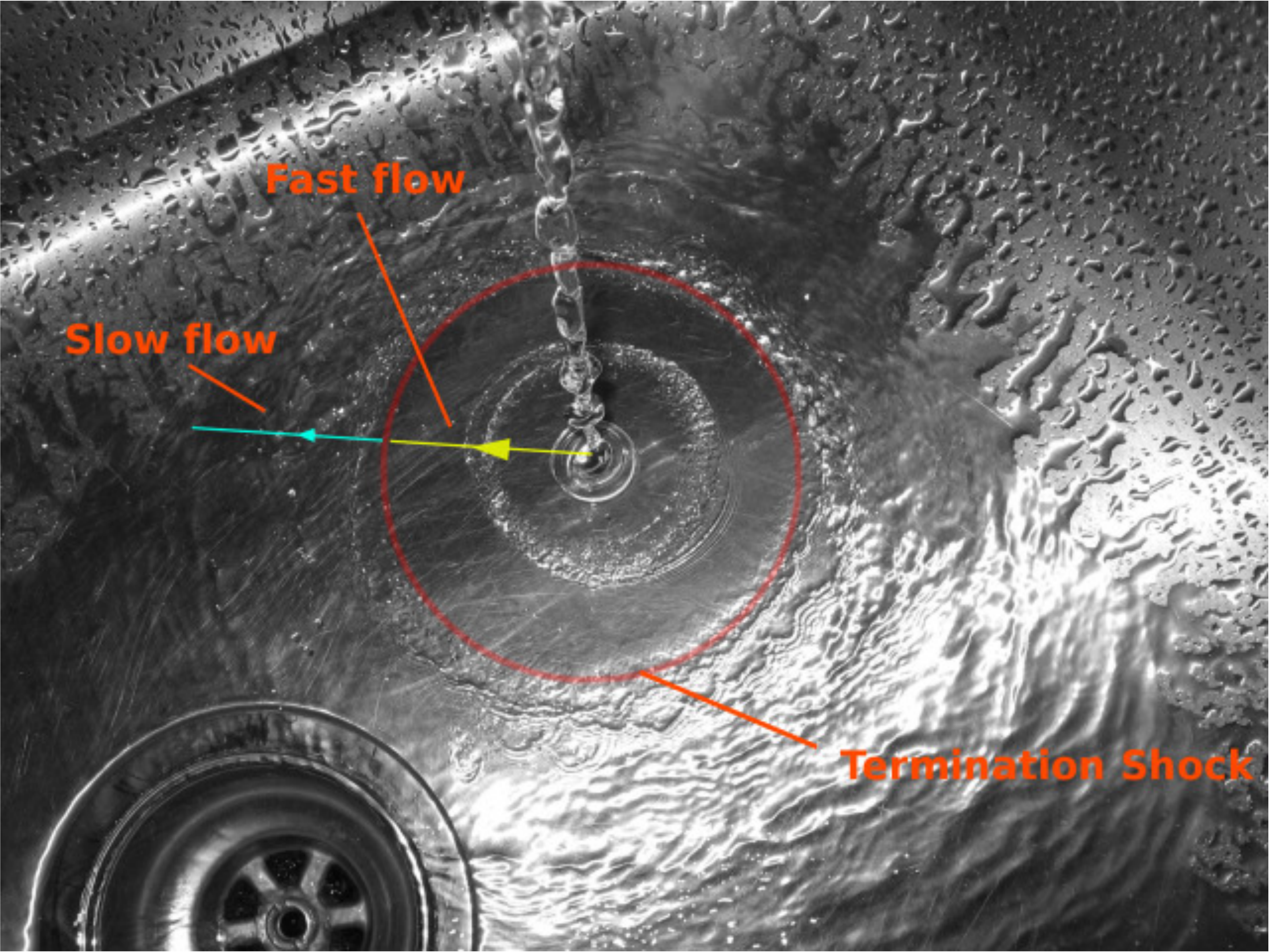
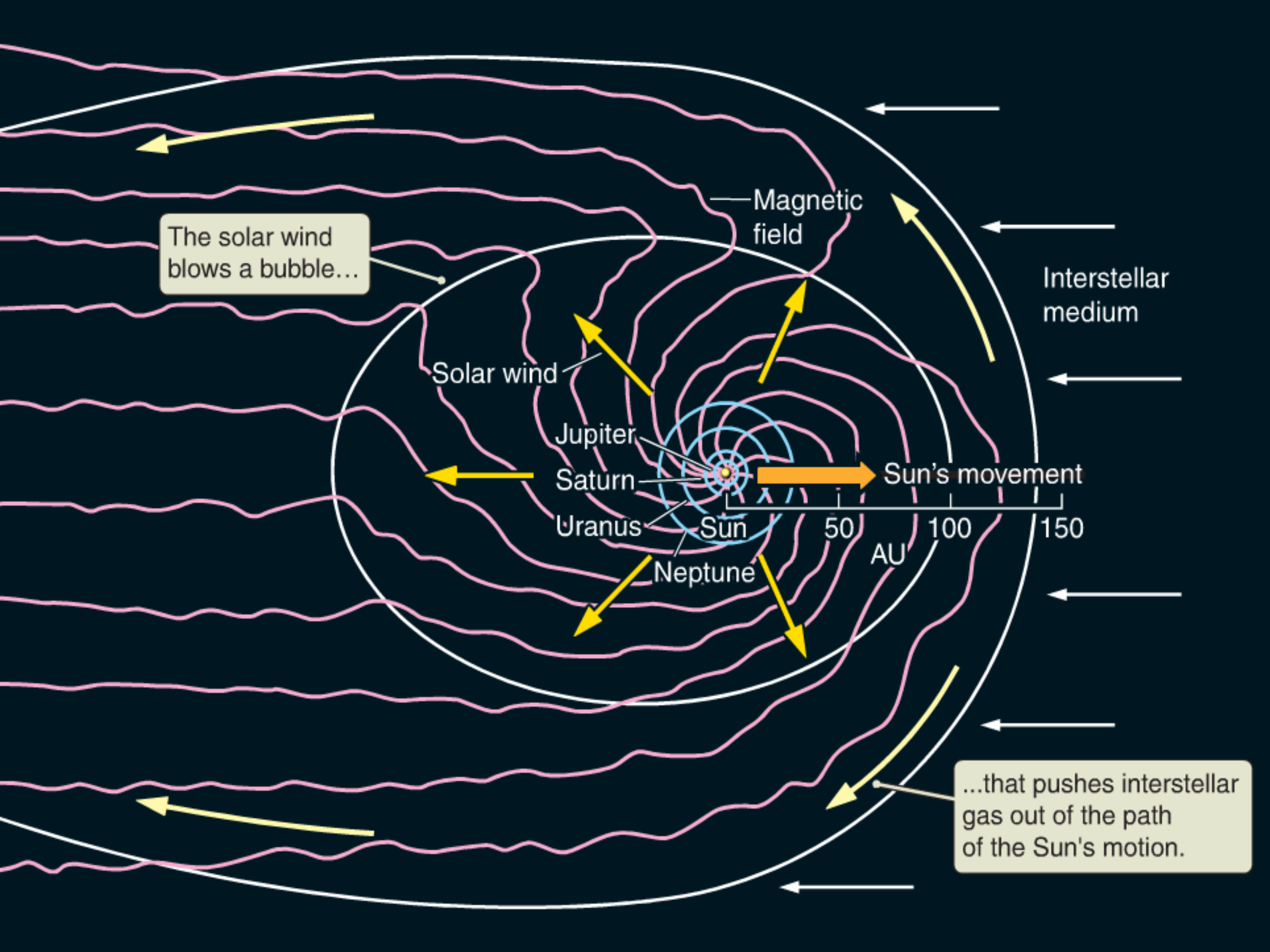
Solar Wind: the constant mass loss of the Sun

- The solar wind blows the **tails of comets** away from the Sun and powers **auroral** displays on planets.
- The solar wind interacts with the **interstellar medium**, pushing it out of the way, forming the boundary of the Solar system.
- The Sun loses its mass at a rate of **2×10^{-14} M_{sun}/yr,** from **solar wind**. This is **4e16 kg/yr** (1 Msun = 2e30 kg)
- **In 10 Gyrs, it only loses 0.02%, even less than the mass loss due to nuclear fusion 0.07% (assuming 10% core mass)**

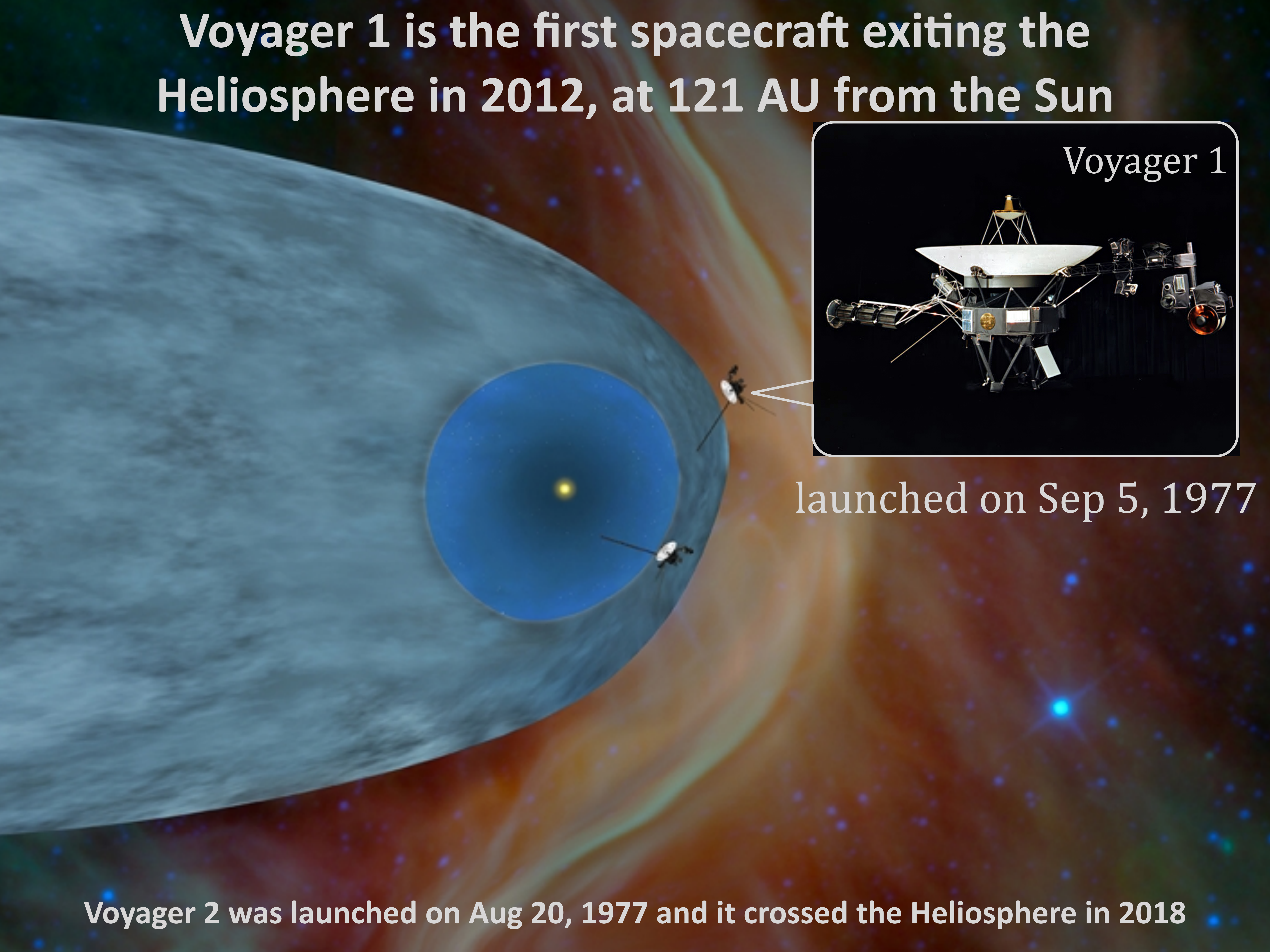



Sun

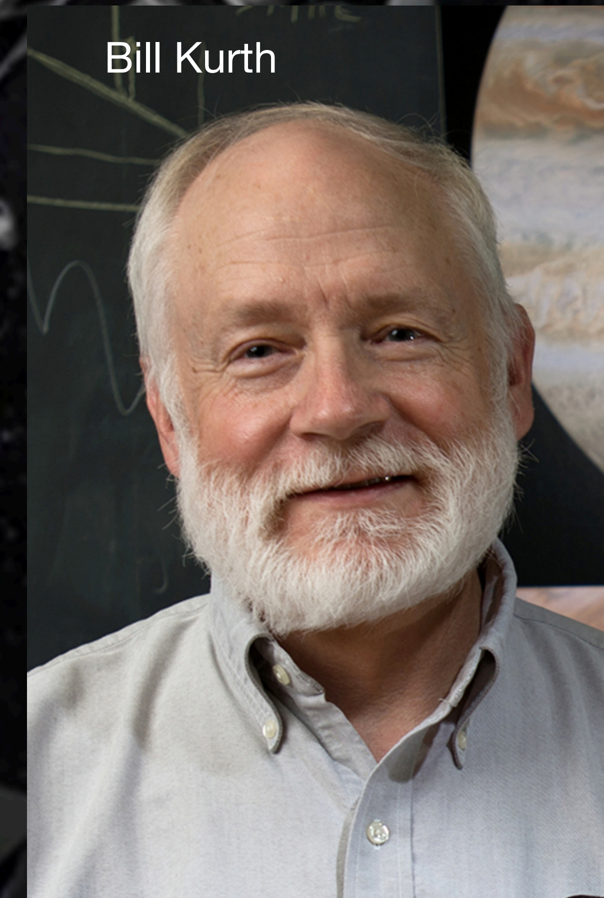

Fast flow
Slow flow
Termination Shock



Magnetic field
The solar wind blows a bubble…
Interstellar medium
Solar wind
Jupiter
Saturn
Uranus
Sun
Neptune
Sun's movement
50
100
150
AU
...that pushes interstellar gas out of the path of the Sun's motion.


Voyager 1 is the first spacecraft exiting the Heliosphere in 2012, at 121 AU from the Sun
Voyager 1
launched on Sep 5, 1977
Voyager 2 was launched on Aug 20, 1977 and it crossed the Heliosphere in 2018

IOWA



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Voyager 1 spacecraft reaches interstellar space

An artist's concept shows the Voyager spacecraft traveling through space against a field of stars. Image courtesy of NASA/JPL-Caltech.

University of Iowa-led study confirms historic achievement in space exploration

Thursday, September 12, 2013

Written by: Gary Galluzzo

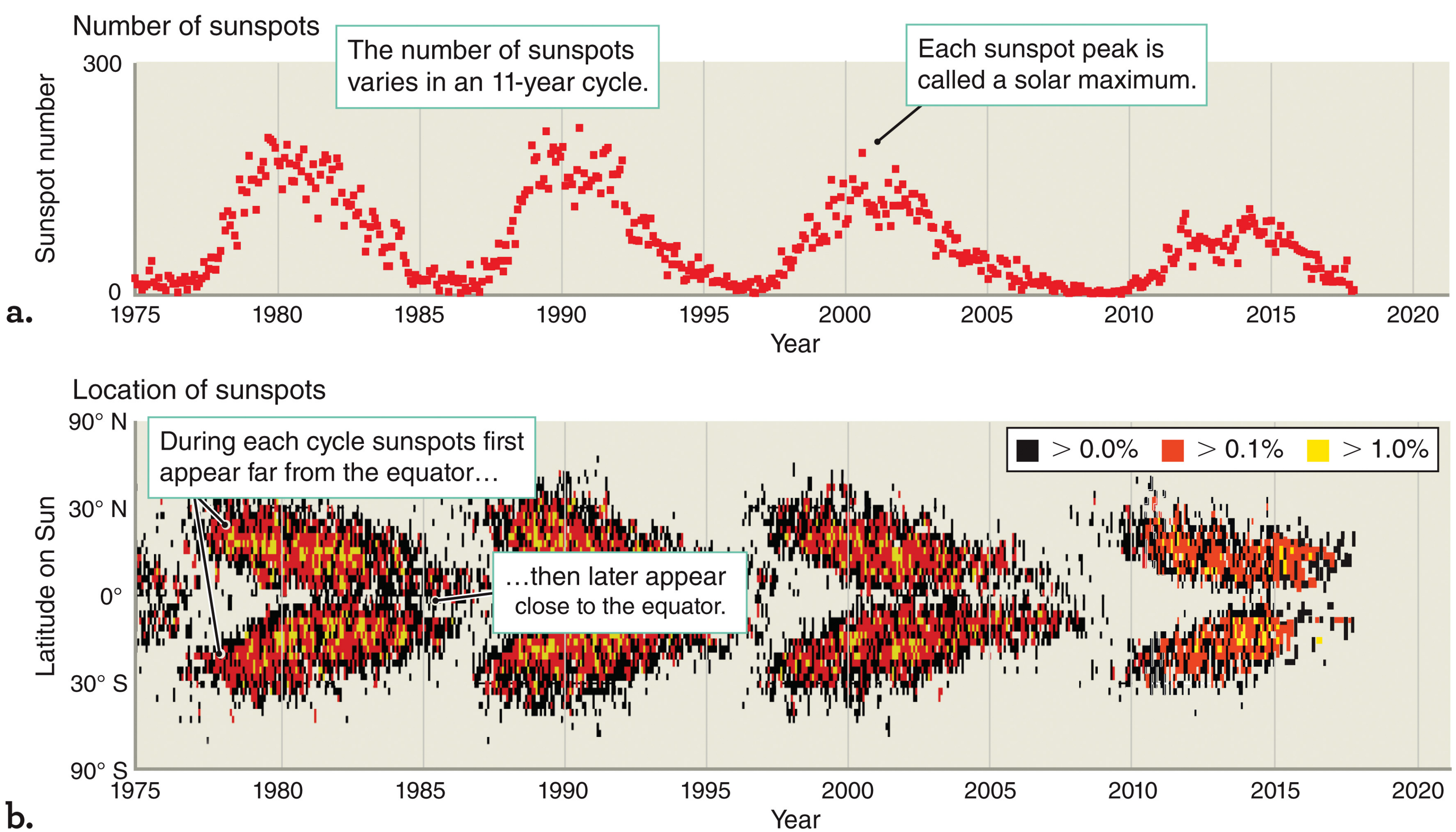
University of Iowa space physicist Don Gurnett says there is solid evidence that NASA's Voyager 1 spacecraft has become the first manmade object to reach interstellar space, more than 11 billion miles distant and 36 years after it was launched.

The finding is reported in a paper published in the Sept. 12 online issue of the journal Science.

Sunspot Cycles

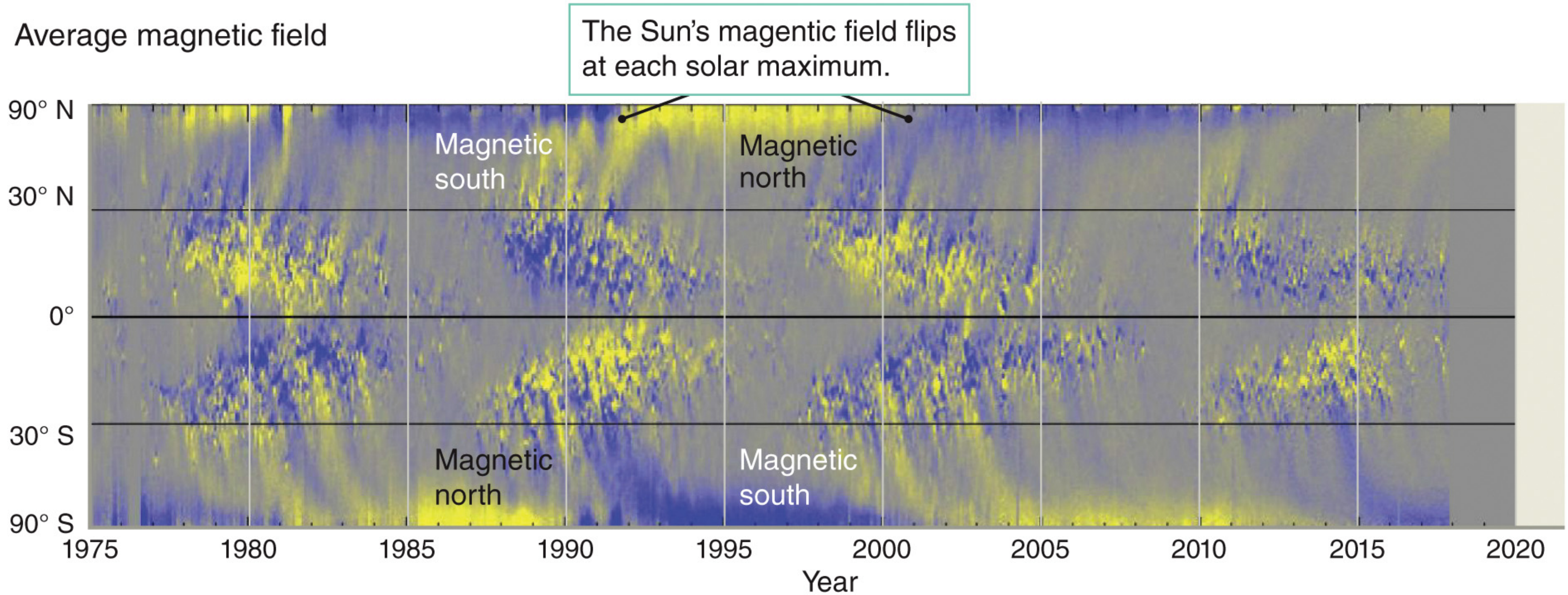
The Sunspot Cycle

- The Sun shows an approximate 11-year **sunspot cycle** (part of a 22-year magnetic cycle).
- **Solar maxima**: most sunspots and activity. **On Earth, the intensity of sunlight increases by 0.1% during solar maxima when compared to minima.**



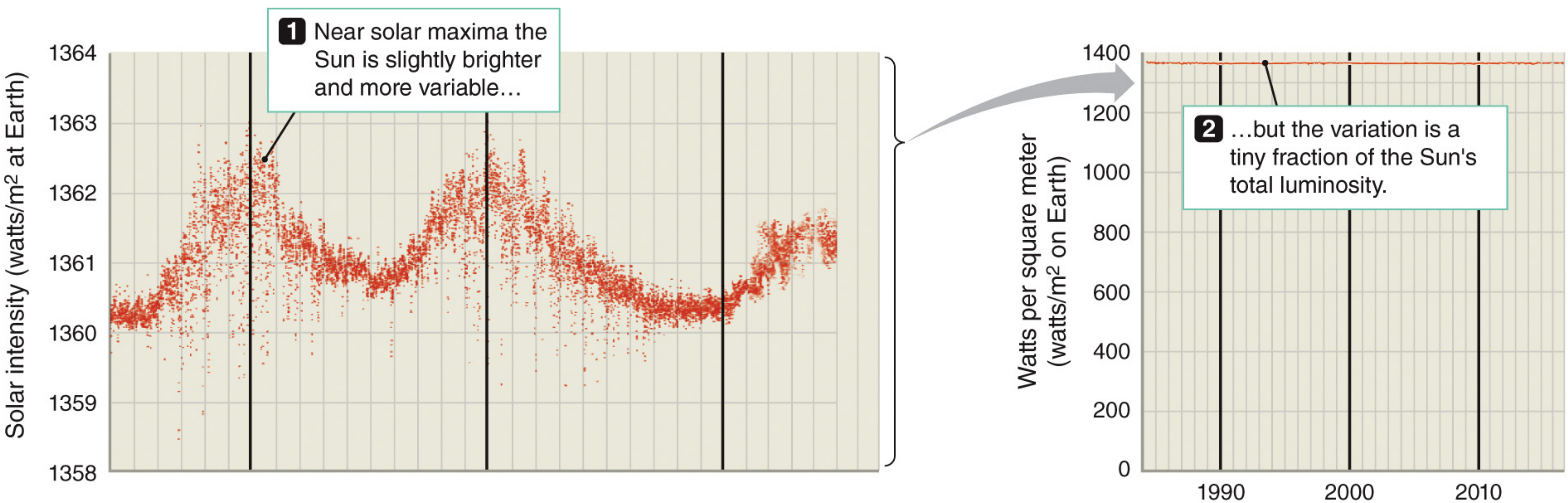
Sunspot Cycles Caused by Global Magnetic Field Flips

- The Sun's magnetic field flips every 11 years, during the maximum of the sunspot cycle.
- Sunspots come in pairs. During one cycle, the south magnetic pole sunspot will lead, but during the next cycle, the north magnetic pole sunspot will lead.



The Sunspot Cycle

- **Solar maxima**: most sunspots and activity. On Earth, the intensity of sunlight **increases by 0.1%** during solar maxima compared to solar minima.

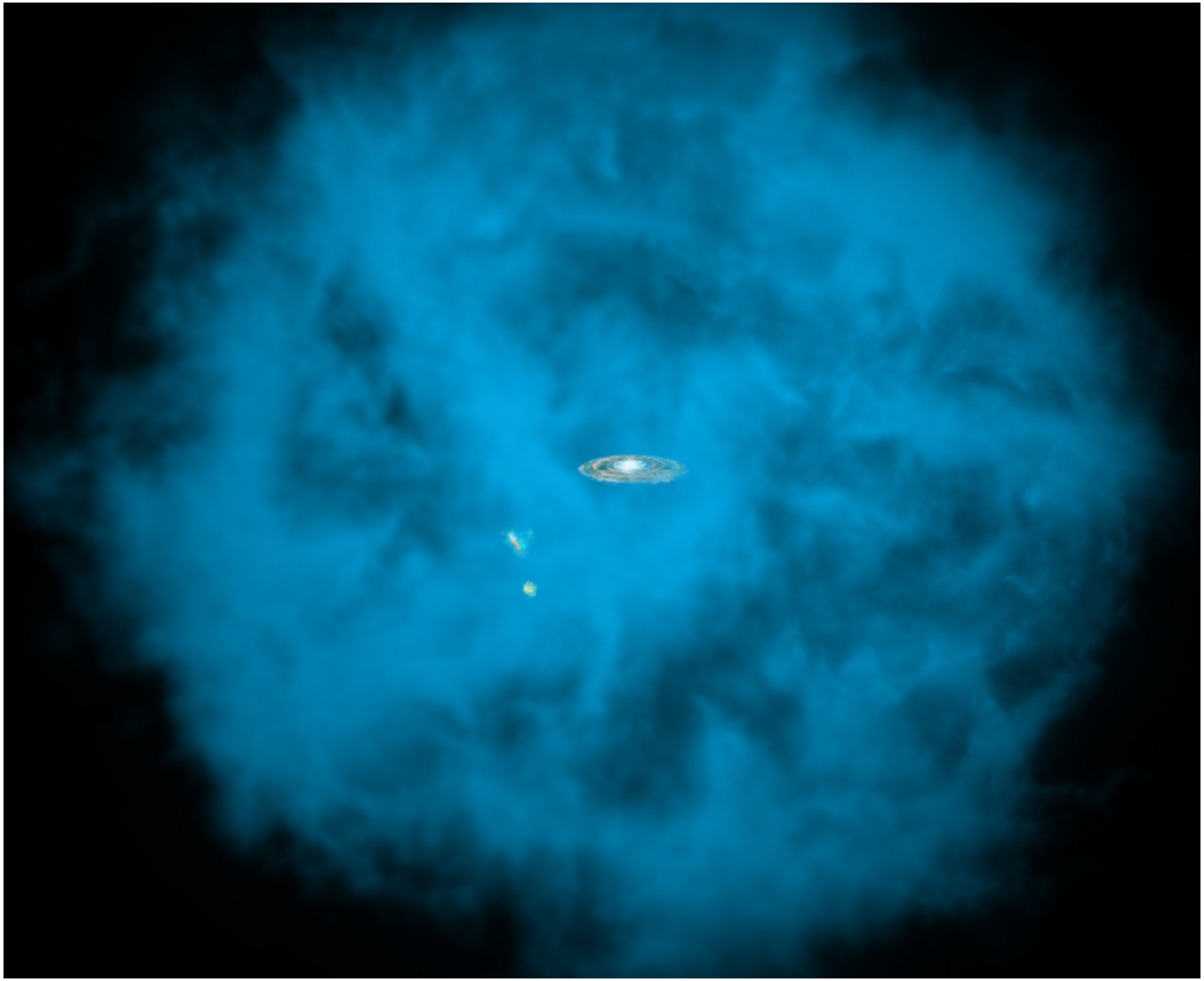


Why the Sun show sharp edges?

the scale height of the photosphere

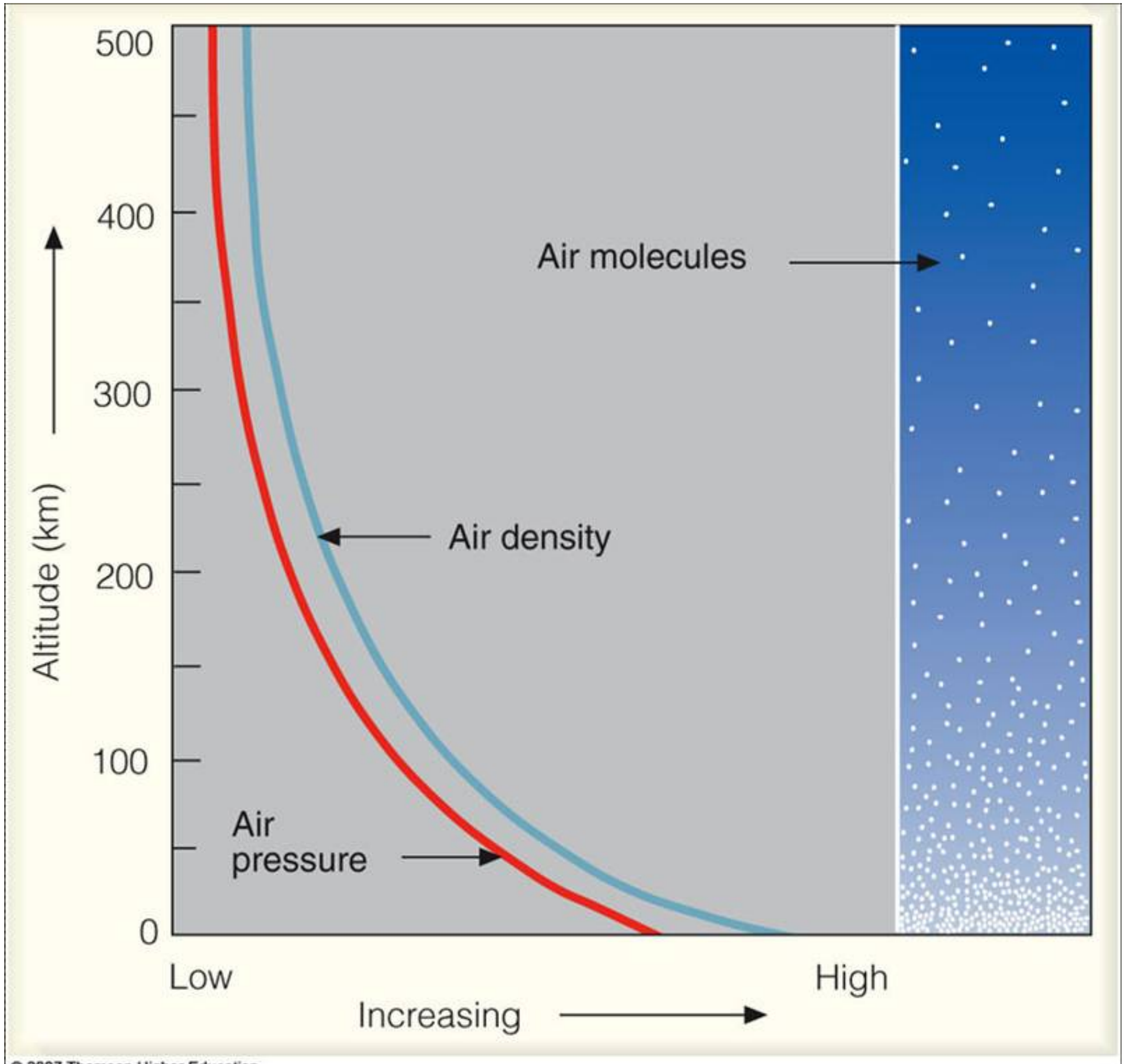
Why the Sun appears to have a sharp edge?

- Wouldn’t a spherical gas cloud look like fuzzy on the edges? Like pictured below?



Recall the scale height of an isothermal atmosphere

$n(h) = n_0 \exp\left(-\frac{h}{h_S}\right)$ where $h_S = \frac{kT}{\mu m_H g}$ is the **scale height**.



exp(x) = e^x, *where e* is Euler's number 2.71828

Recall also the Virial Theorem applied to a spherical cloud

Now we can put both equations together and then write down the virial theorem for a uniform spherical gas cloud:

$$K = \frac{3}{2}\frac{M_{\text{gas}}}{\mu m_H}kT \qquad\qquad U = -\frac{3}{5}\frac{GM^2}{R}$$

Virial theorem applies IF the cloud is stable:

$$2K = -U \Rightarrow \frac{3MkT}{\mu m_H} = \frac{3GM^2}{5R}$$

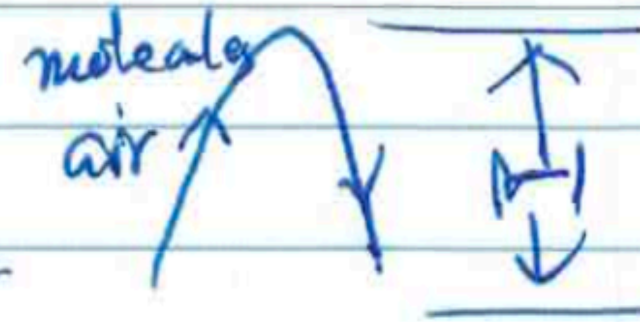
Although 2K and -U both increase as R increases, they don't increase at the same rate ($K \sim R^3$, $U \sim R^5$). So beyond some point, the virial theorem is violated as 2K < -U.

Scale height of the solar photosphere << Solar radius

Scale height for solar atmosphere.

$$\bar{m} g H = k T_s$$

$\bar{m} g H$ ← potential energy; kT_s ← kinetic energy



H is small → the sun looks like a sharp edge

$$\frac{H}{R} = \frac{k T_s}{\bar{m} g R}$$

Virial theorem : $k T_c \sim \frac{GM}{R} \bar{m} \sim \bar{m} g \cdot R$

$$\Rightarrow \frac{H}{R} = \frac{k T_s}{\bar{m} g R} = \frac{k T_s}{k T_c} = \frac{T_s}{T_c} \sim 10^{-4}$$

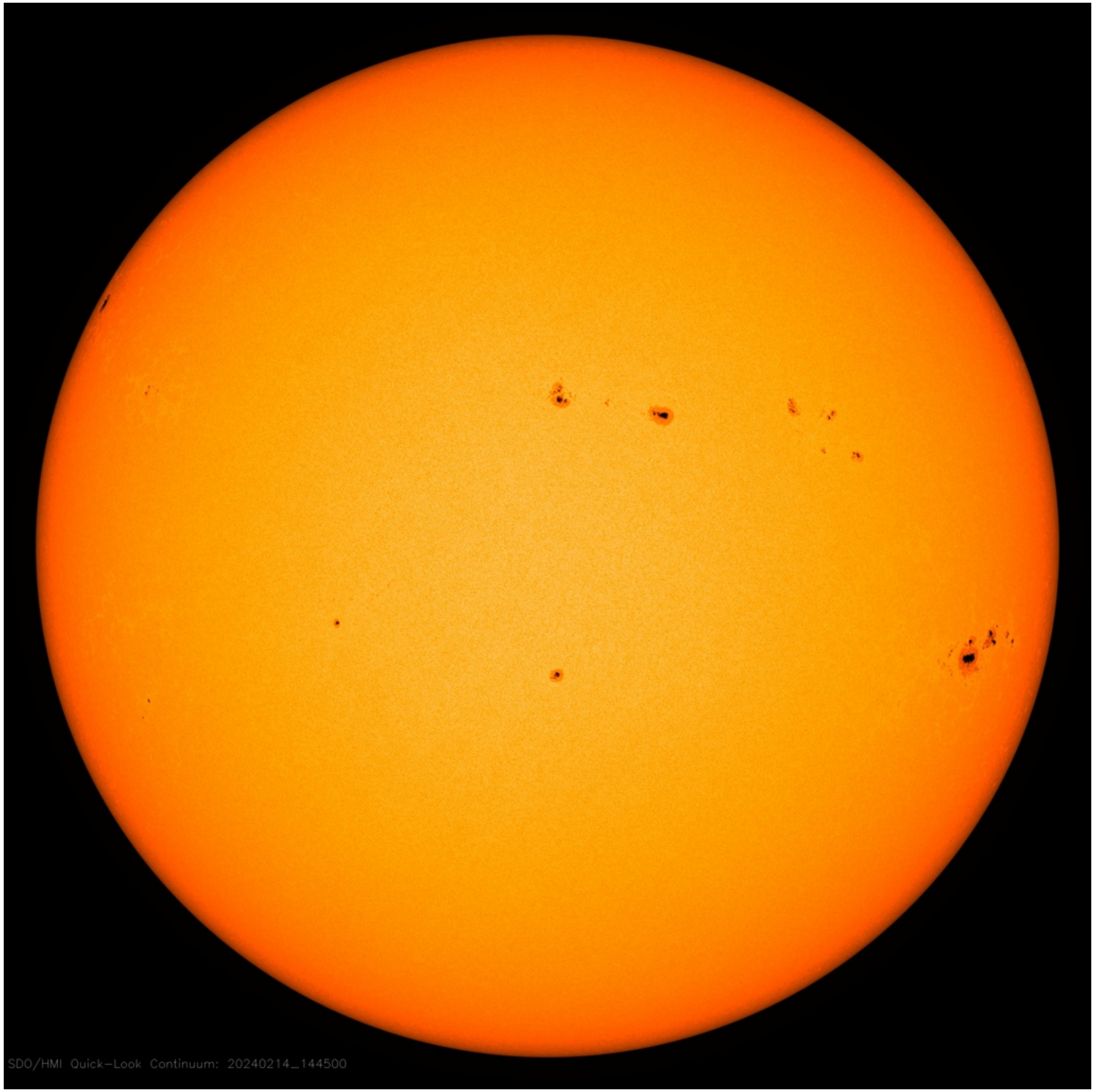
For Earth. $\frac{H}{R} \sim 10^{-3}$. The Sun appears even sharper.

Why the Sun’s limb appears darker?

the last scattering surface

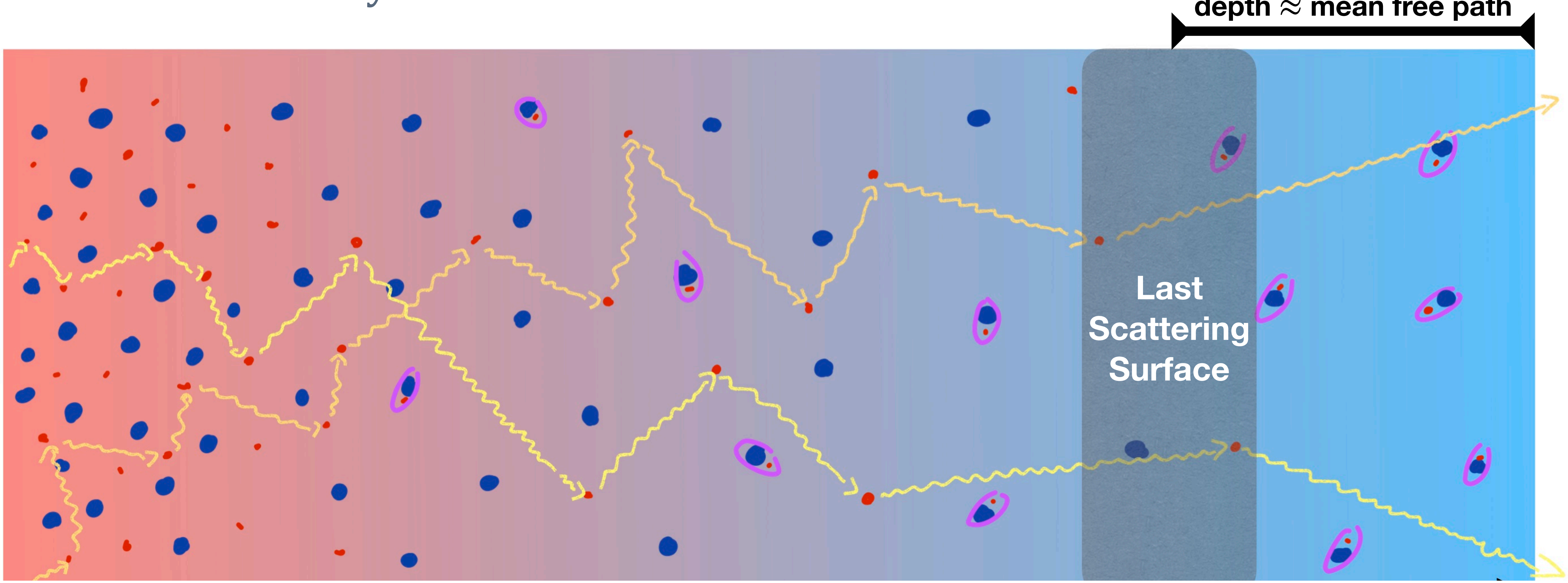
Limb Darkening of Photosphere

Limb Darkening: The Sun appears darker near its edge. **Why?**



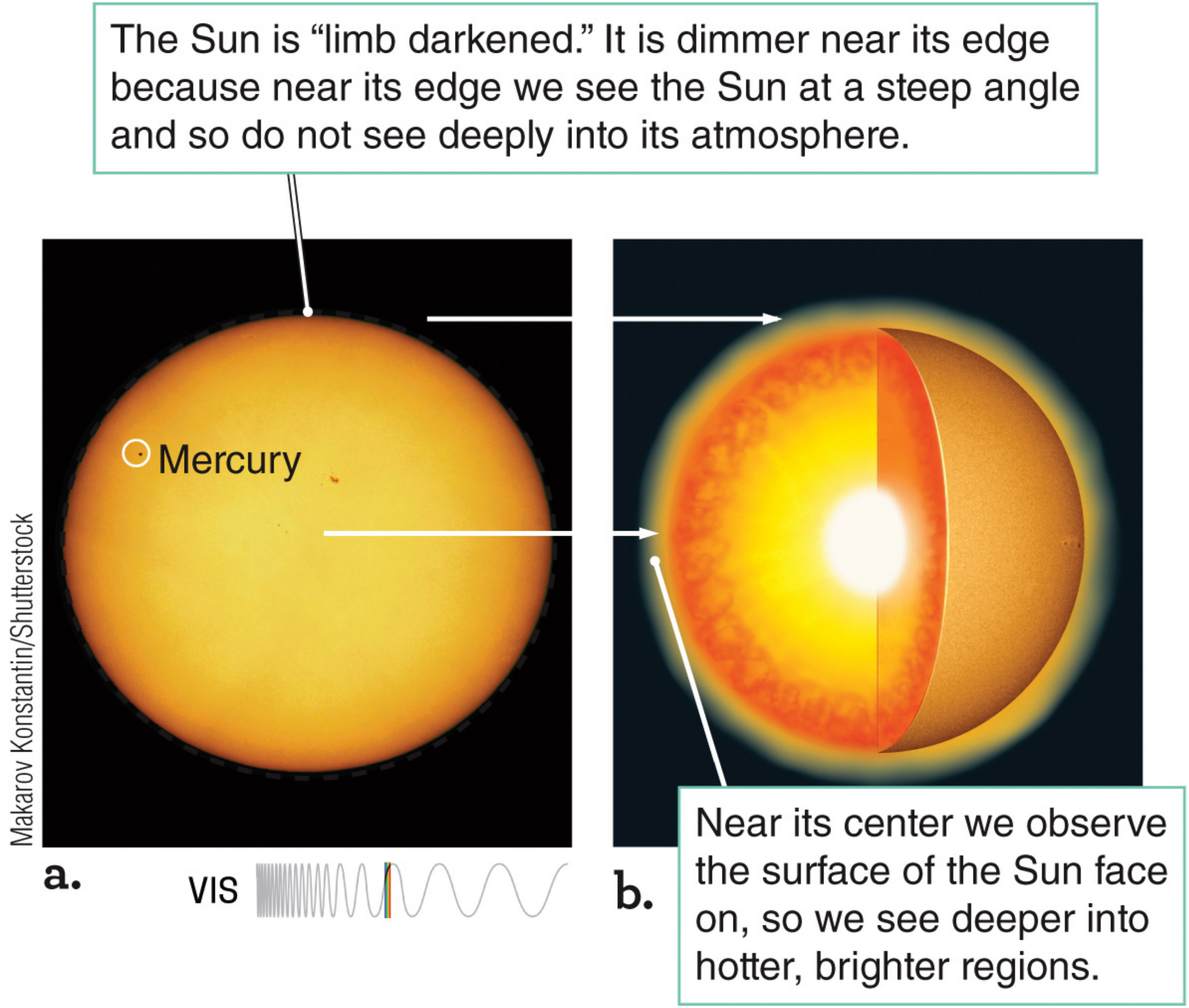
Solar Atmosphere: Last Scattering Surface

- The Sun has no solid surface, but the apparent surface of the Sun is the surface at which light can directly escape into space.
- Let's call this surface the **last scattering surface** (a concept also used in cosmology). Note that its depth depends on **(1) the angle we look into the Sun and (2) the wavelength of the photons**
- The layers above this point are known as the **atmosphere**, which can be directly observed.



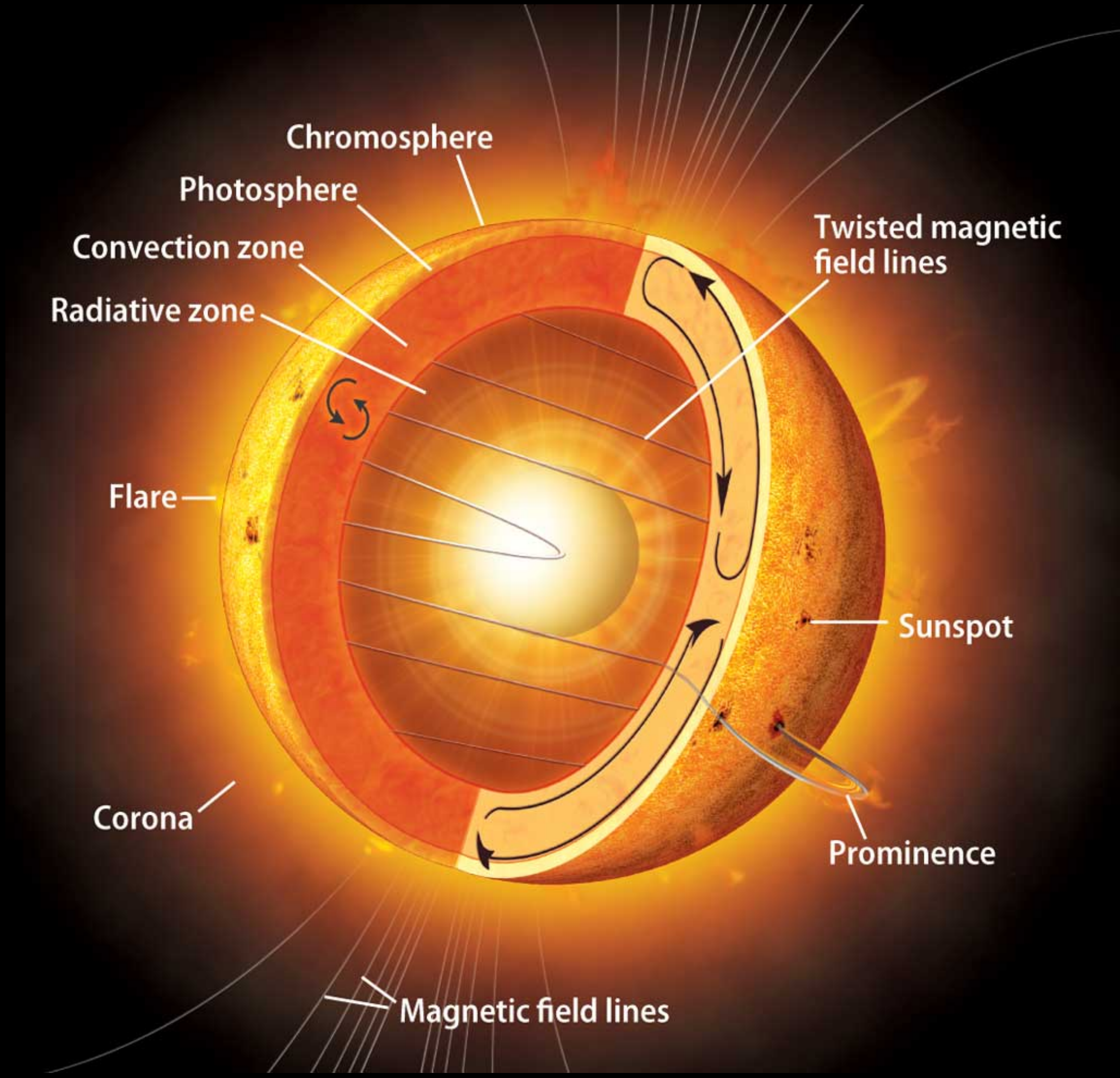
Solar Atmosphere: Limb Darkening of Photosphere

The Sun appears **darker near its edge** because our sightline penetrates less depth at a steeper angle, the **last scattering surface** is at a lower temperature.



Chap 2: Our Star - The Sun: Key Concepts

- The sheer mass of the Sun and hydrostatic equilibrium creates the necessary conditions for fusion: dense and hot gas
- Fusion can maintain Solar luminosity over billions of years
- How energy is transported out?
- Fusion model can be tested by neutrino detectors
- Interaction cross section and mean free path
- Atmospheres of the Sun
- Solar activities and cycles

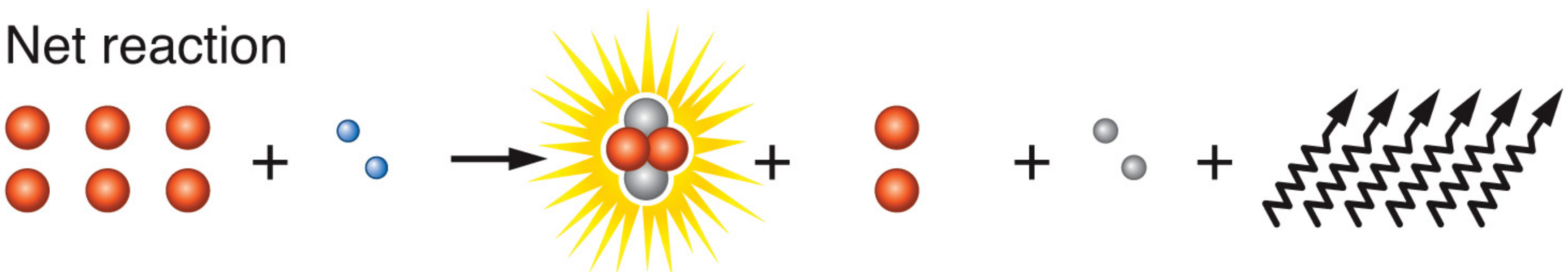


Chap 2: Our Star - The Sun: Key Equations

$$\frac{dP(r)}{dr} = -\rho(r)\ g(r)$$

$$h_S = \frac{kT}{\mu m_H g}$$

Net reaction



$$6\ {}^{1}\mathrm{H} + 2\,e^{-} \longrightarrow {}^{4}\mathrm{He} + 2\ {}^{1}\mathrm{H} + 2\,\nu + 6\,\gamma$$

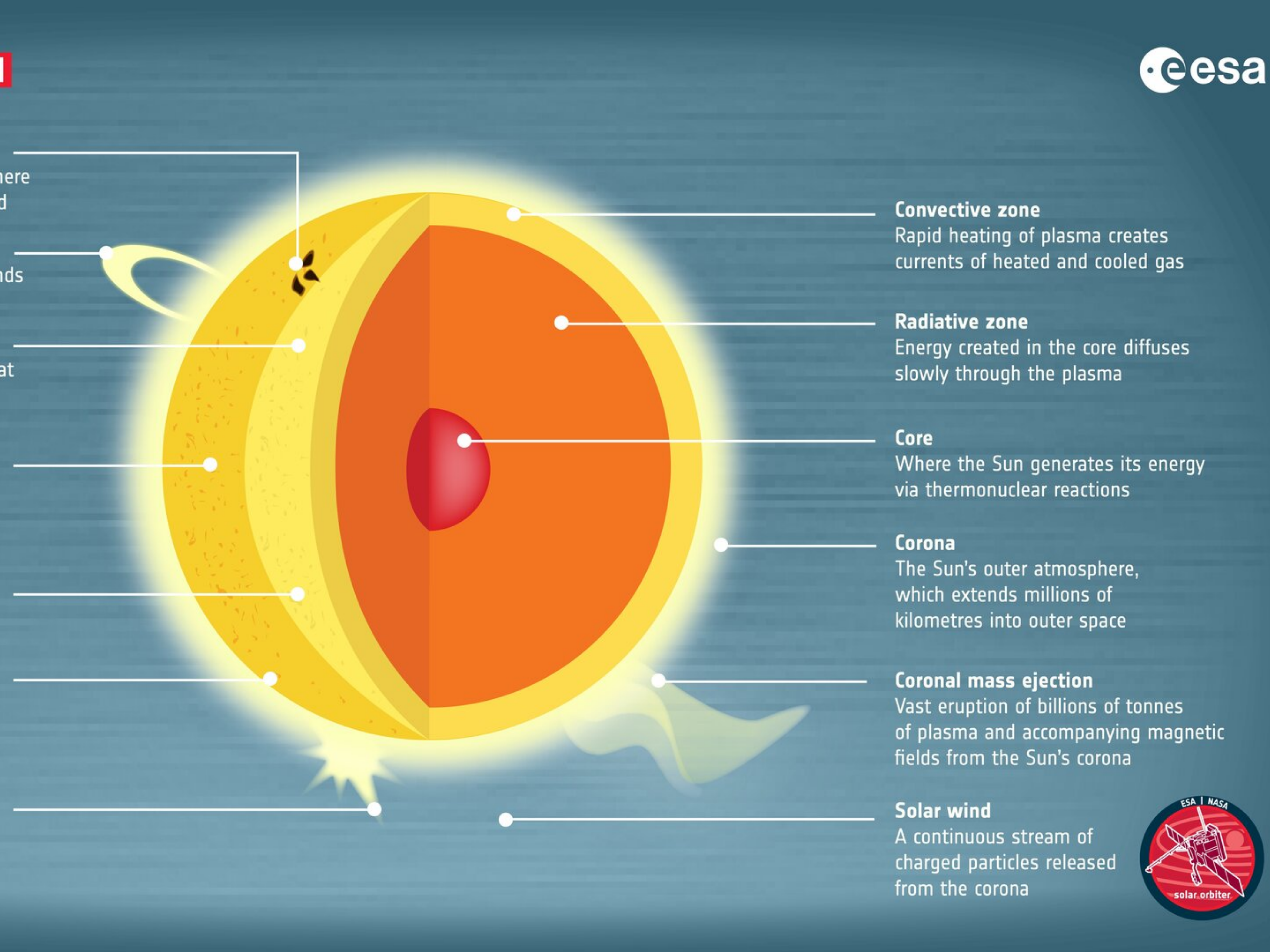
$$E = \Delta m c^2$$

$$l_{\mathrm{mfp}} = \frac{1}{n\sigma} = \frac{\mu m_H}{\rho\sigma} = \frac{1}{\rho\kappa}$$

$$d = l\sqrt{N}$$

$$\frac{d\Phi}{\Phi} = -n\sigma dz$$

$$\Phi = \Phi_0 \exp\left(-\frac{z}{l_{\mathrm{mfp}}}\right)$$



Convective zone
Rapid heating of plasma creates currents of heated and cooled gas
Radiative zone
Energy created in the core diffuses slowly through the plasma
Core
Where the Sun generates its energy via thermonuclear reactions
Corona
The Sun's outer atmosphere, which extends millions of kilometres into outer space
Coronal mass ejection
Vast eruption of billions of tonnes of plasma and accompanying magnetic fields from the Sun's corona
Solar wind
A continuous stream of charged particles released from the corona
ESA | NASA
solar orbiter

ANATOMY OF THE SUN

Sunspots
Darker, cooler areas on the photosphere with concentrations of magnetic field

Prominence
Large structure, often many thousands of kilometres in extent

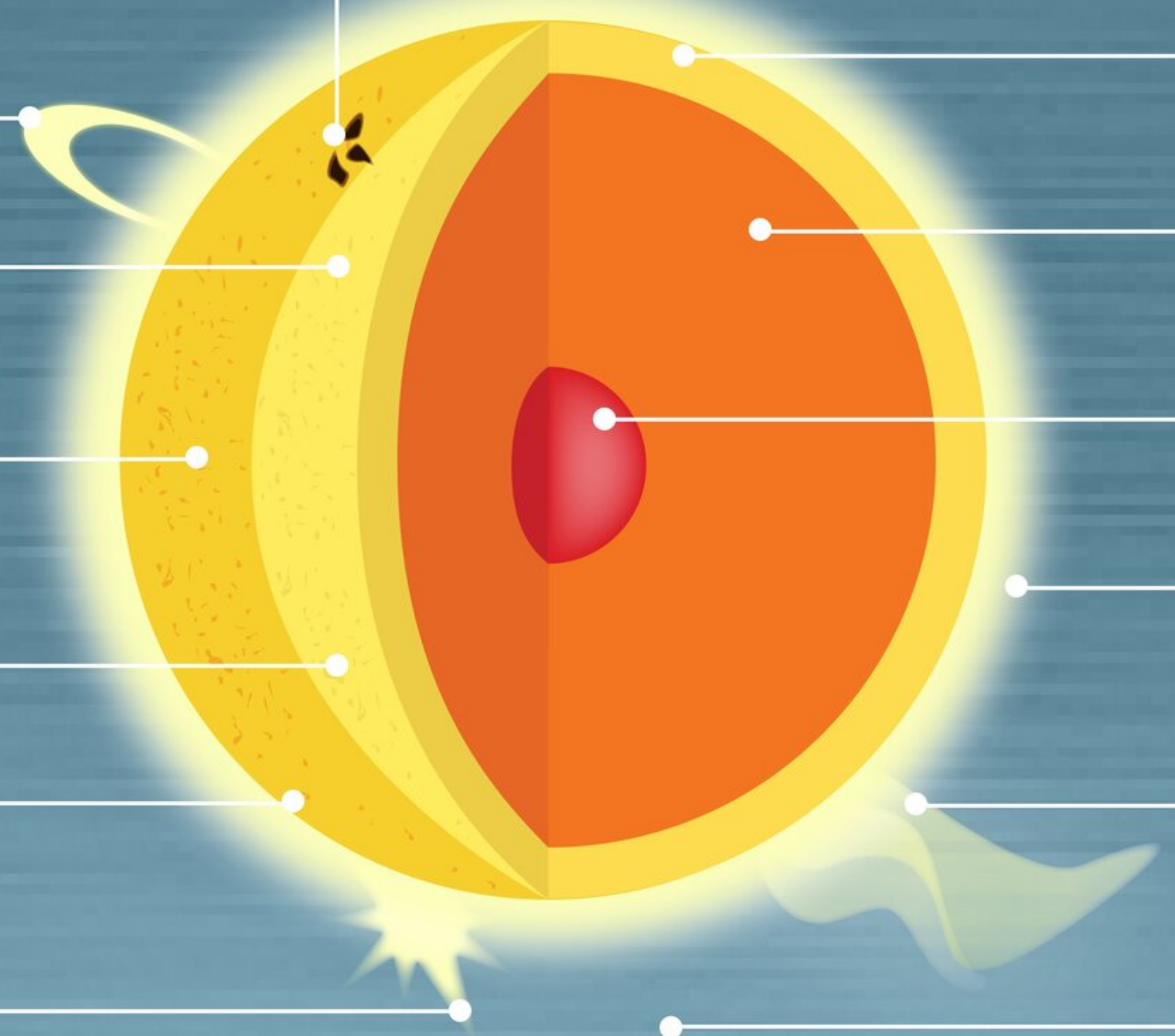
Granulation
Small, short-lived grainy features that cover the Sun, caused by thermal currents rising from below

Chromosphere
Layer above the photosphere, where the density of plasma drops dramatically

Photosphere
The visible 'surface' of the Sun

Transition region
Thin, irregular layer that separates the relatively cool chromosphere from the much hotter corona

Flare
Sudden release of energy in the form of radiation



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#SolarOrbiter #WeAreAllSolarOrbiters