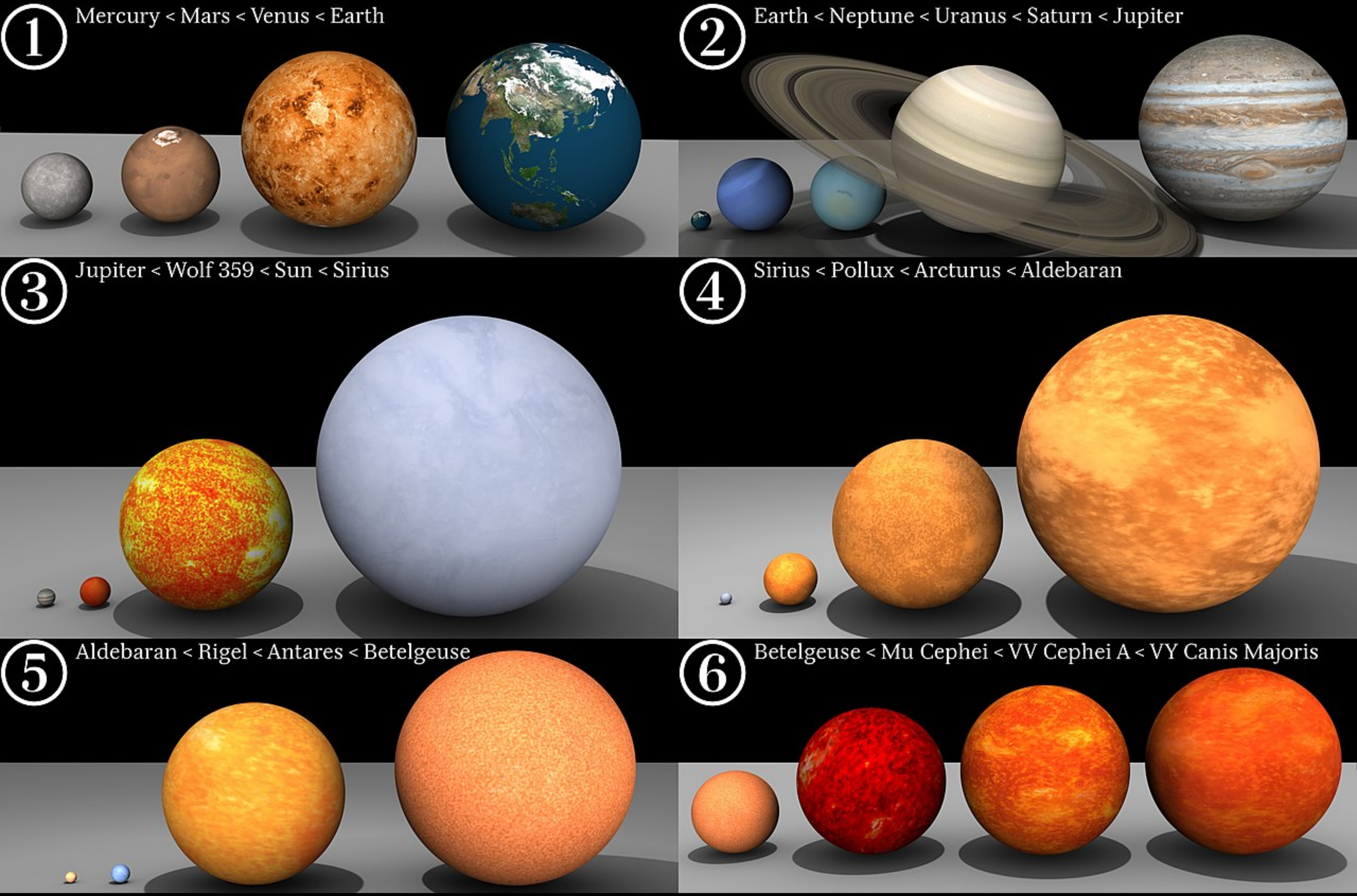
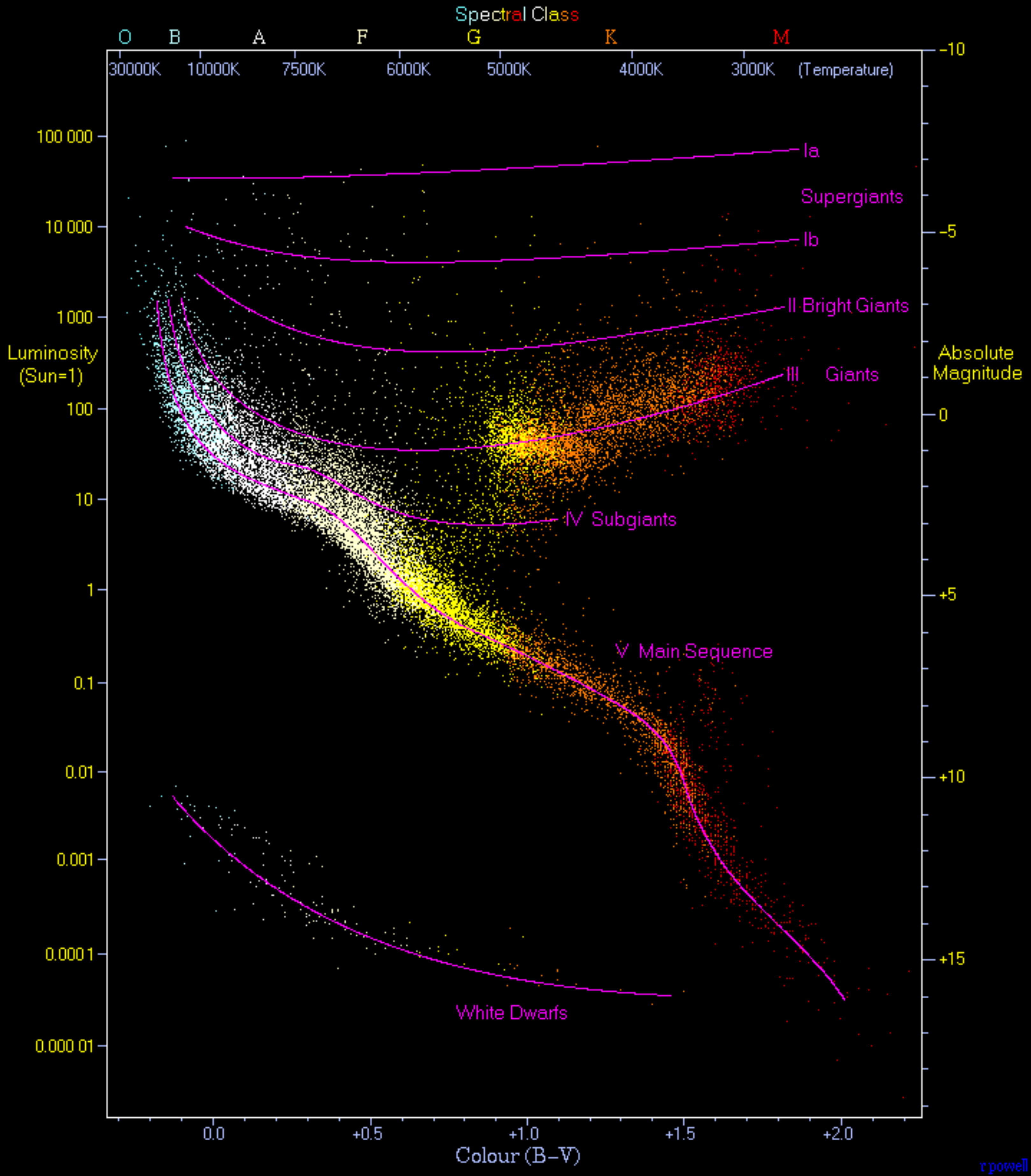
Chap 1: Taking the Measure of Stars



Chap 1: Key Diagram

log(luminosity) vs. log(flux ratio)

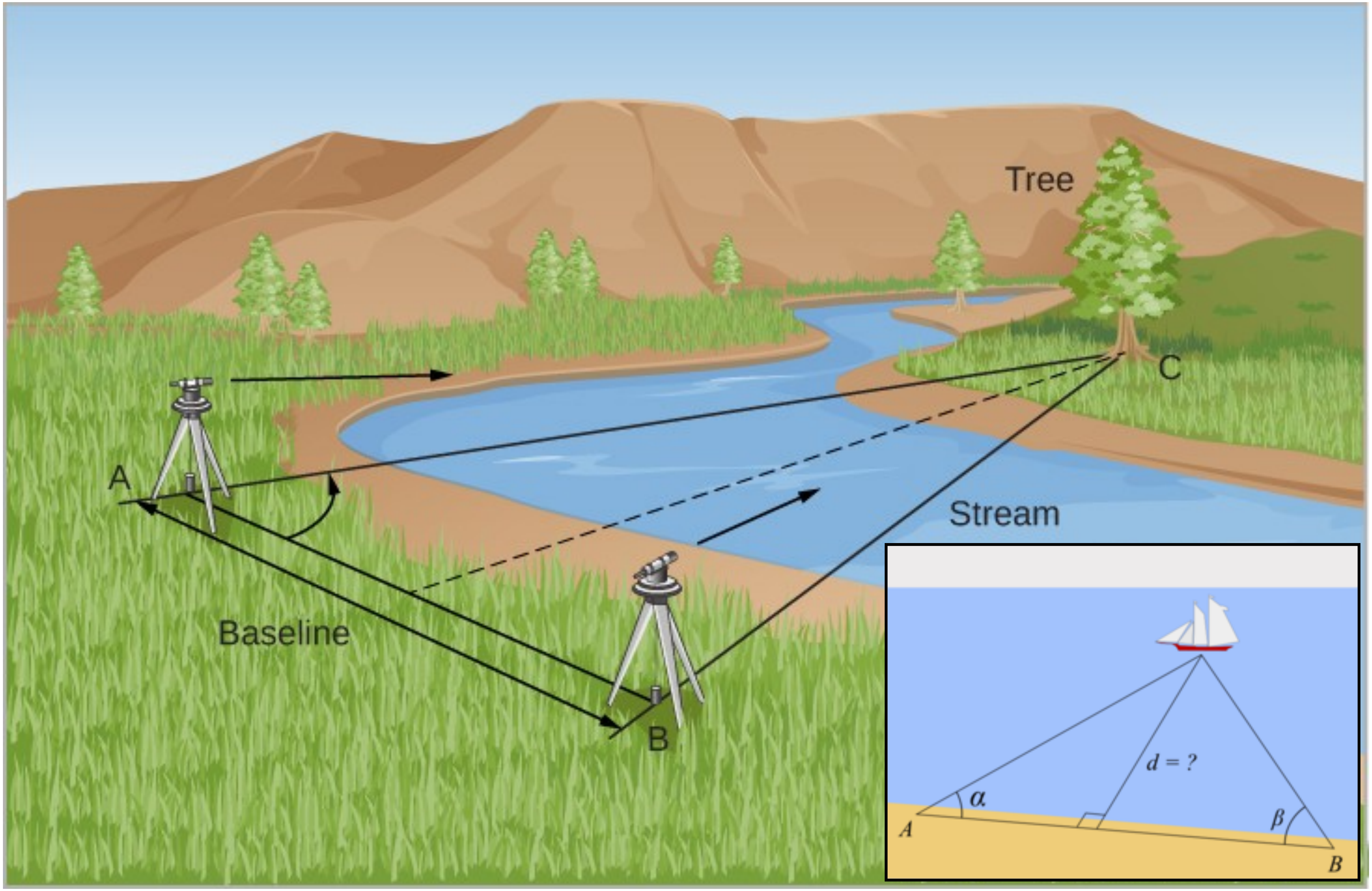


Chap 1: Taking the Measure of Stars

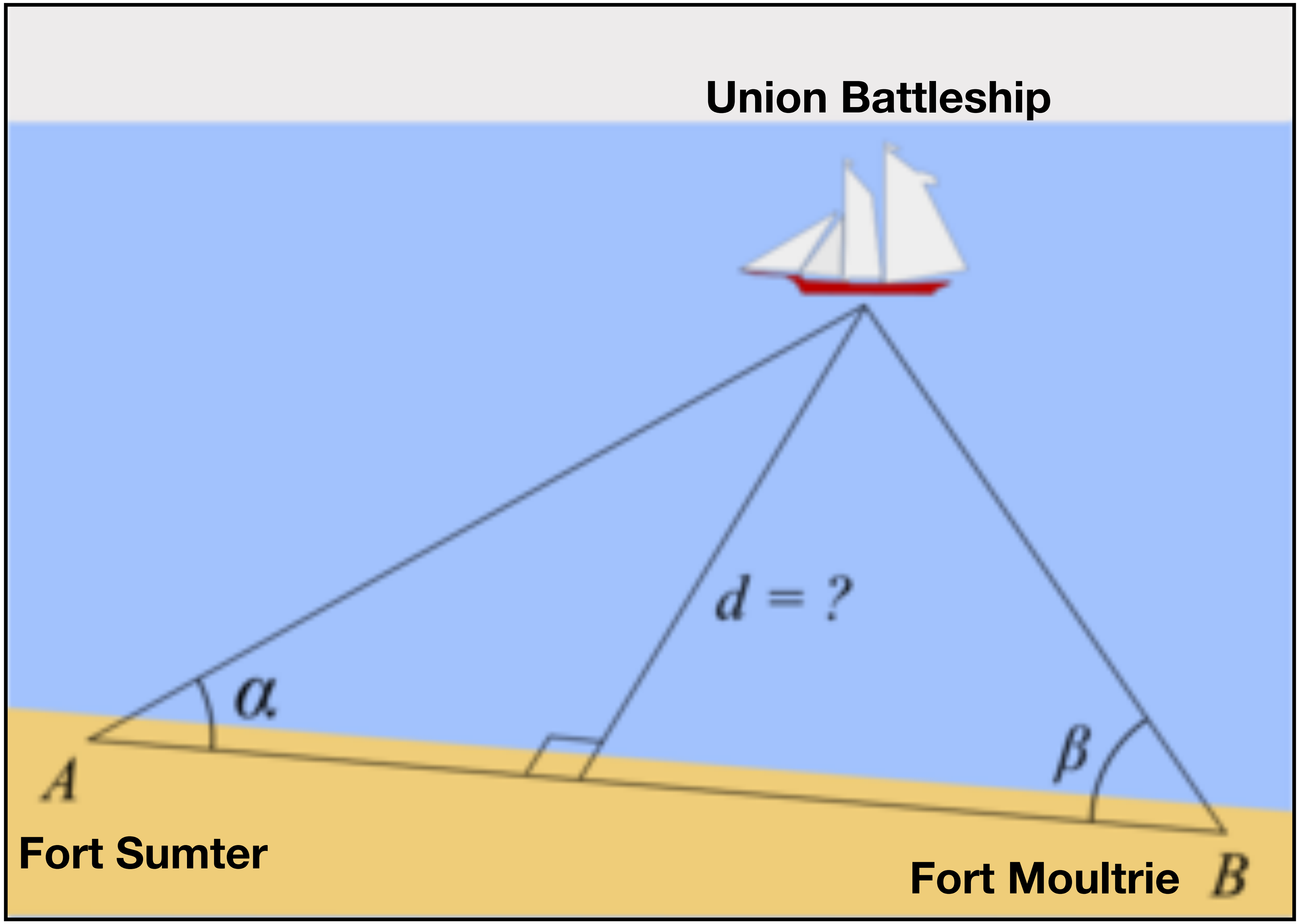
- How do we use parallax to determine distance? Astrometry.
- How do we measure brightness? Photometry.
- How do we combine distance (d) with brightness (apparent magnitude, m) to determine luminosity (absolute magnitude, M)?
- How do we measure temperature (T)? color index
- The Hertzsprung-Russell (H-R) diagram: M vs. color index
- Key concepts:
  - **parallax, magnitude system, distance modulus**
  - **H-R diagram and the distribution of stars on the diagram**
- Other measurements: size & mass of stars

Distance Measurements: Parallax

Geological Survey Method



Geological Survey Method: Naval Defense



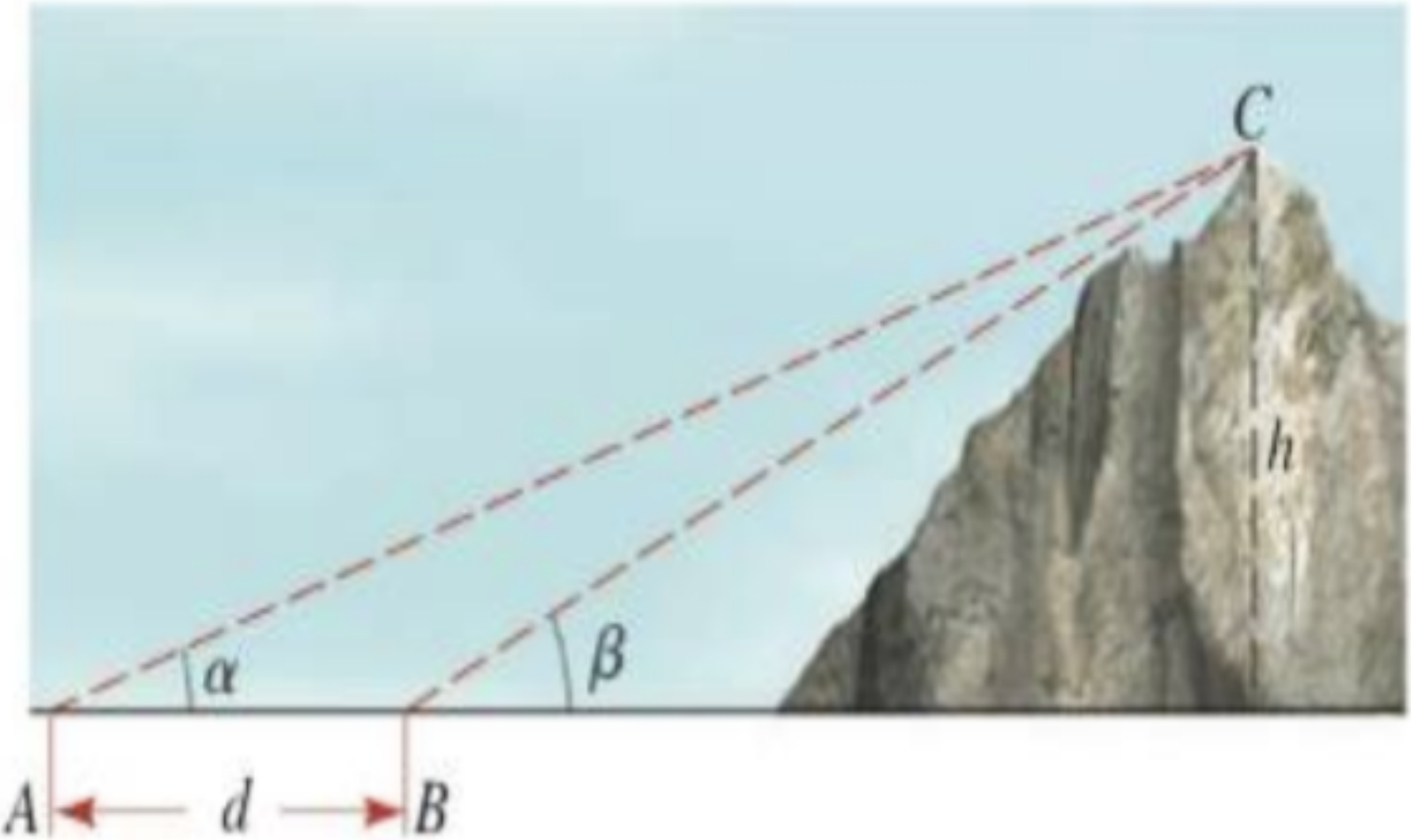
Distance can be calculated from the baseline length ($l = AB$) and the two angles (α, β):

$$\ell = d\left(\frac{\cos\alpha}{\sin\alpha} + \frac{\cos\beta}{\sin\beta}\right)$$

Fort Sumter, Charleston, SC

Geological Survey Method: Height of a Mountain

Derive the height of the mountain based on trigonometry, express h with α, β, d

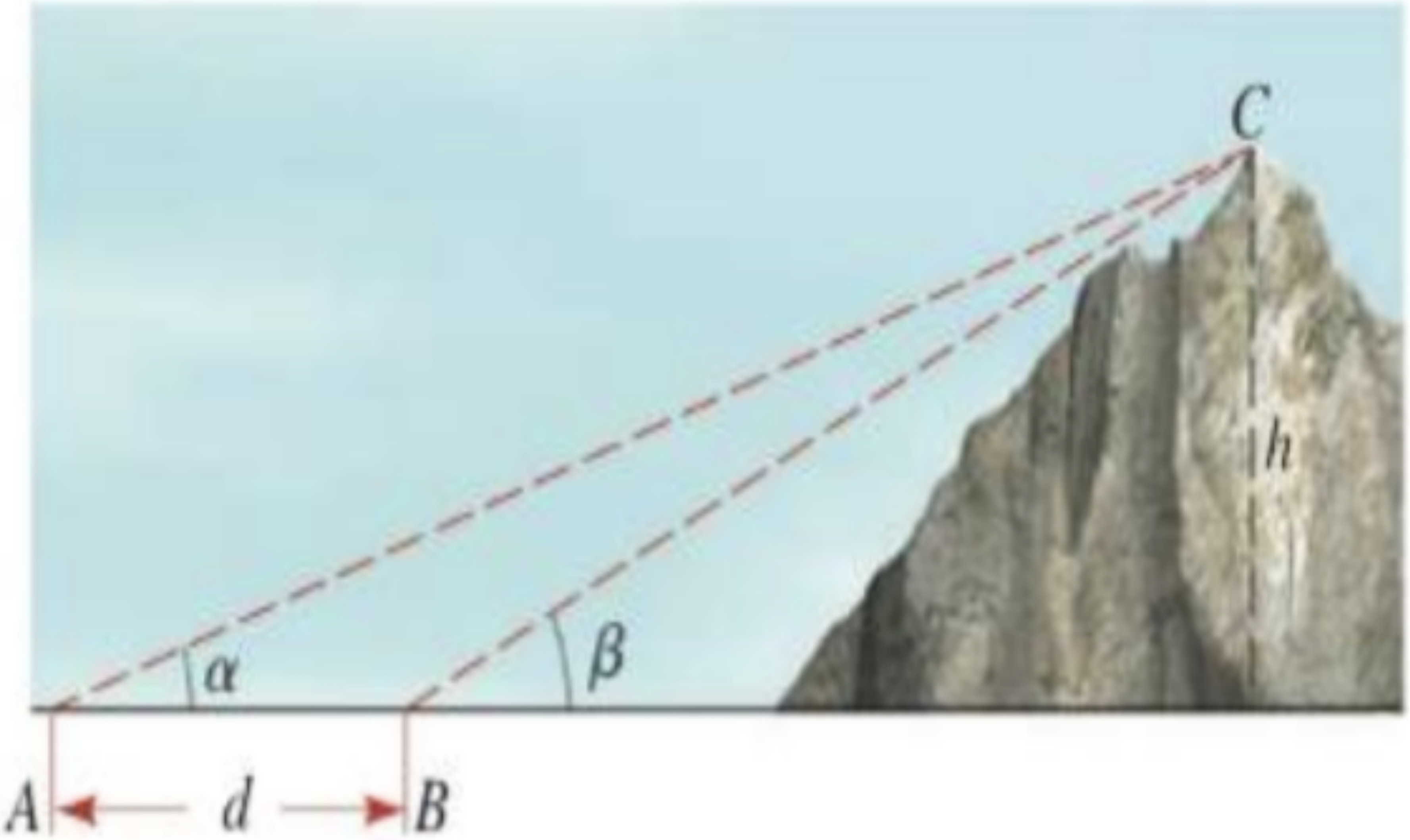


Doing Trigonometry without knowing how to calculate Sine and Cosine

- We have derived that the unknown height of the mountain is related to three known quantities:

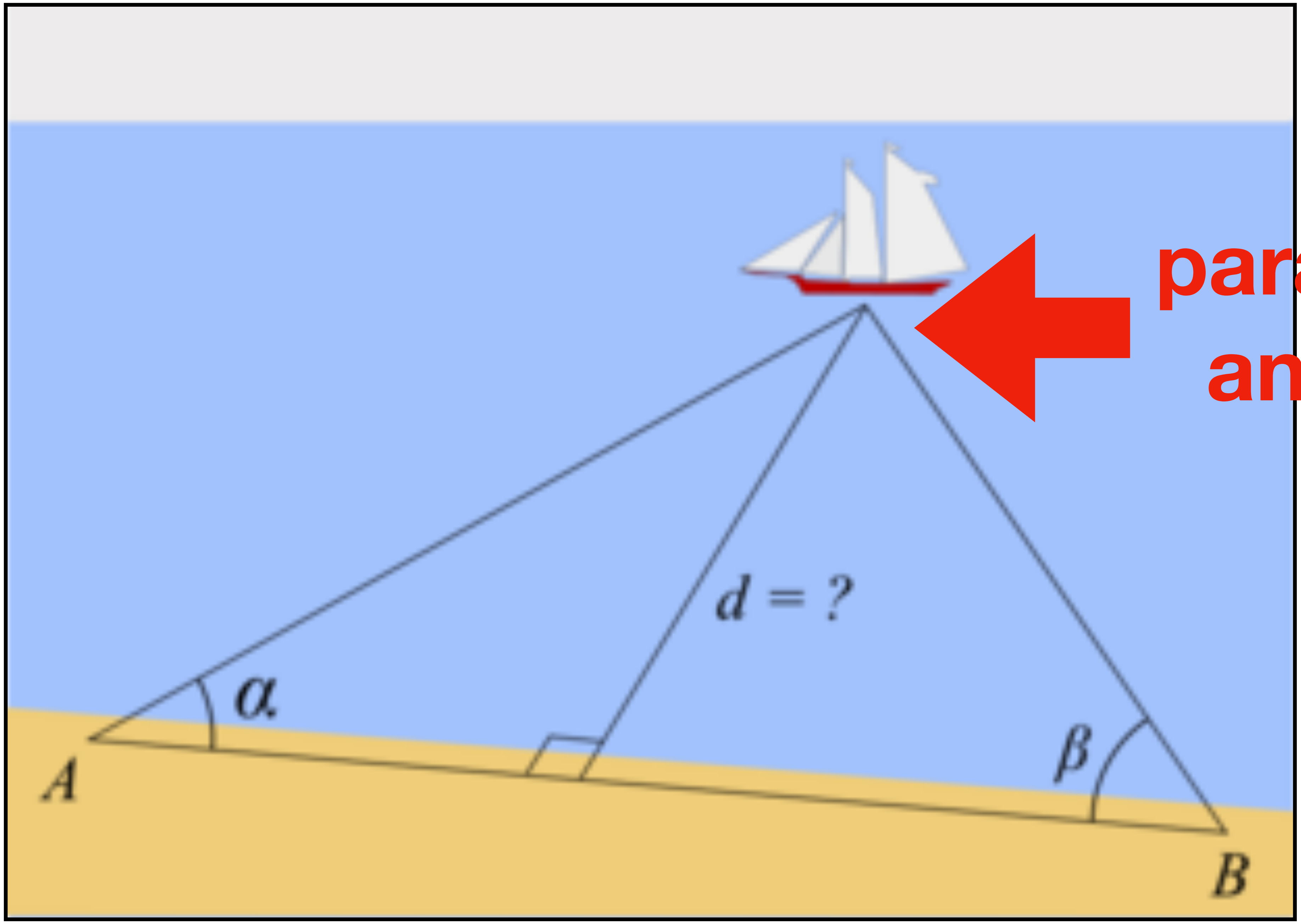
$$h = \frac{l}{\cot\alpha - \cot\beta}$$

- What if you don’t have a calculator (as the soldiers in 1863), how do you calculate the distance?



Limitation of the Geological Survey Method

What would the angles become when distance is much greater than baseline?

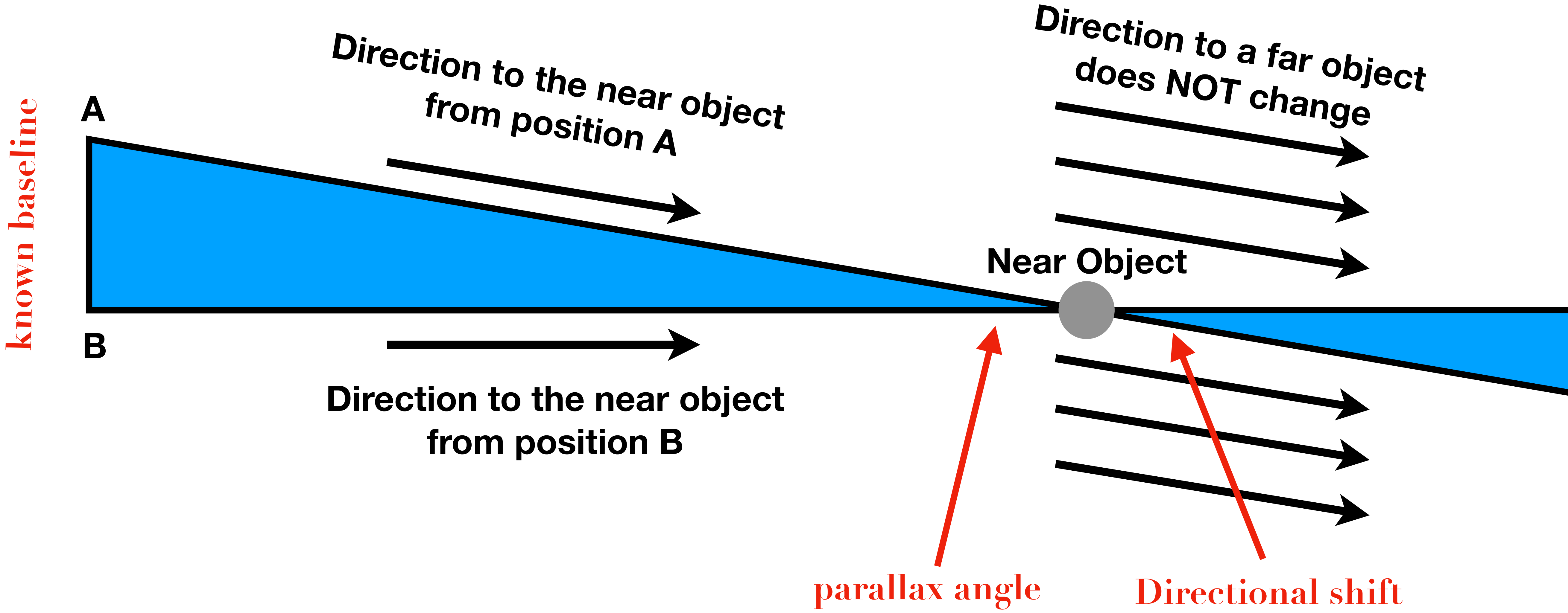


parallax angle

To measure greater distances, we need:
(1) longer baselines and (2) the ability to measure the top angle

So, how do we measure the top angle (i.e., the **parallax** angle)?

We need to measure the **directional shift** of
a foreground (**near**) object relative to a background (**far**) object
as we travel from location A to location B

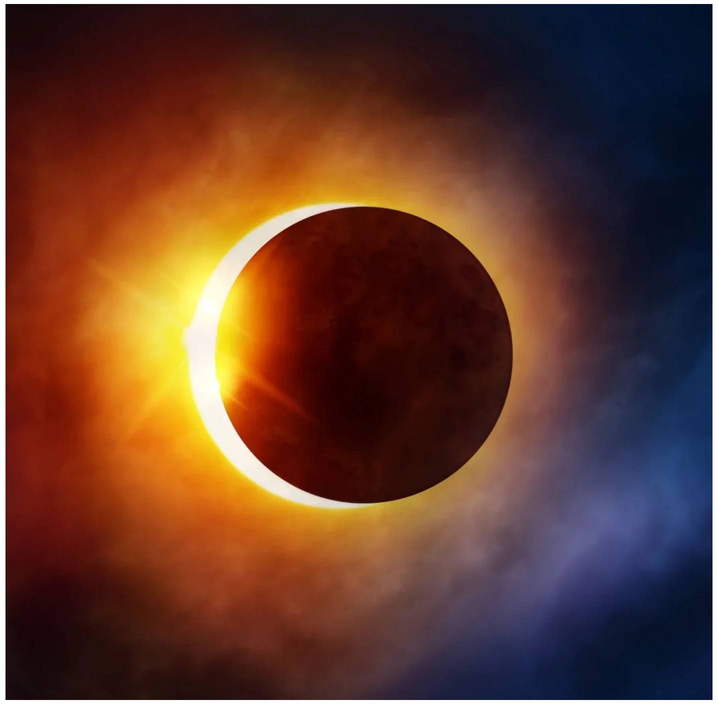


$$\text{distance} \approx \frac{\text{baseline length}}{\tan(\text{parallax angle})}$$

The Earliest Parallax Measurement by Hipparchus (~150 BC):

Maximum in Hellespont (100% obscured)

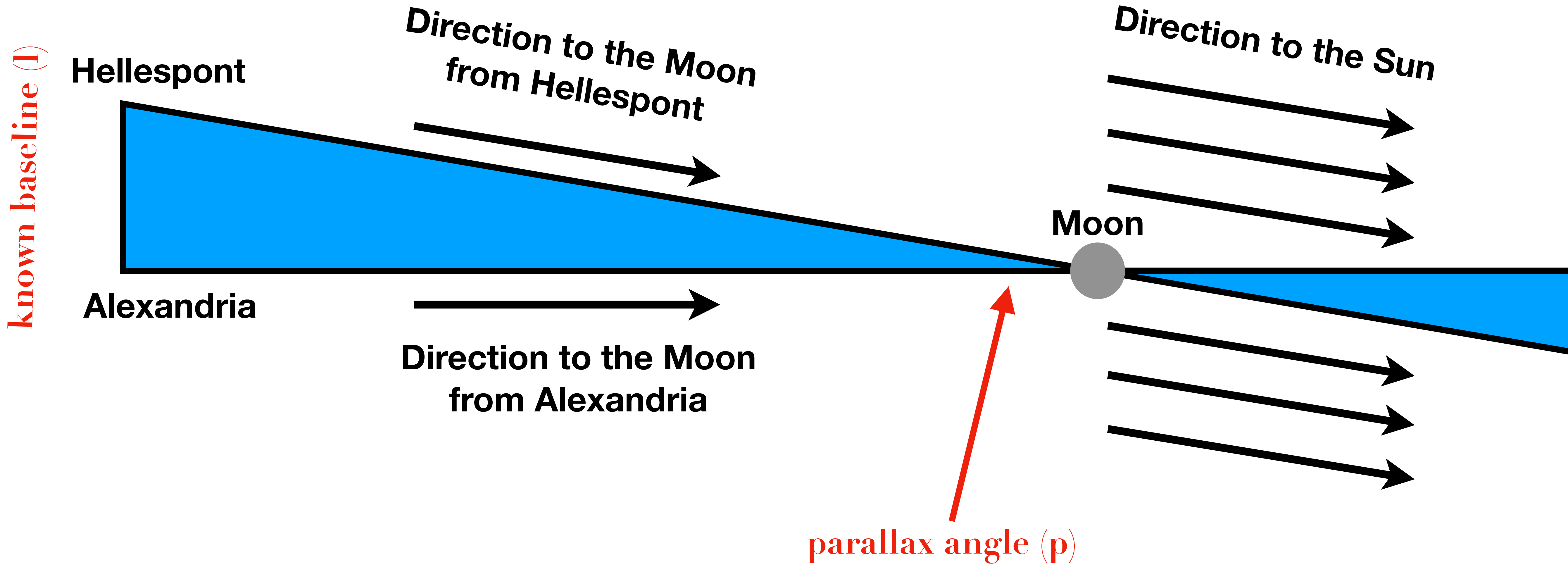
Maximum in Alexandria (80% obscured)



The Solar Eclipse on Mar 14, 190 BC

The Geometry of Hipparchus' Experiment

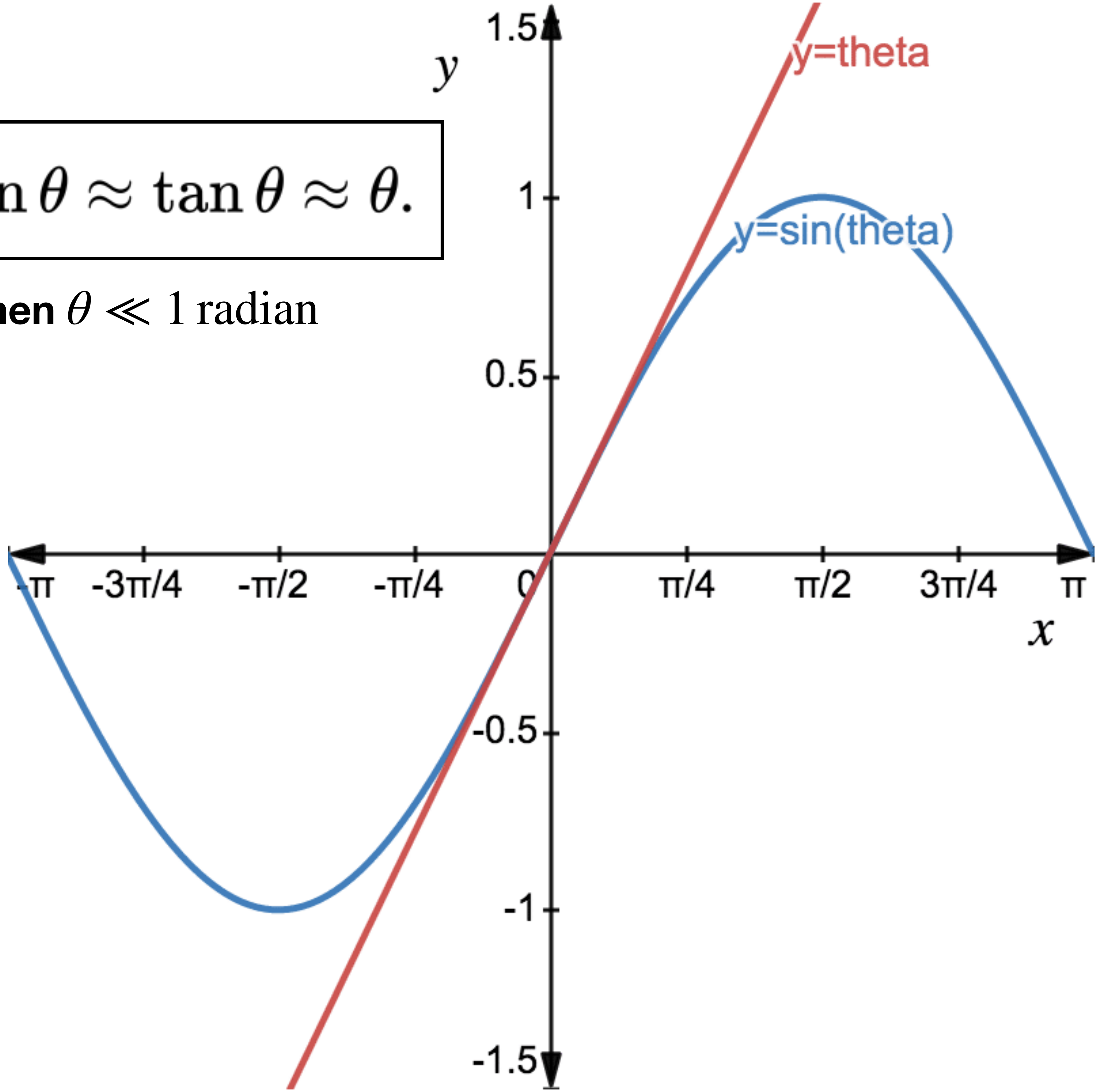
Hipparchus measured the **directional shift** of
the Moon relative to the Sun
based on a solar eclipse recorded by two observers



Small Angle Approximation: Graphical Illustration

$$\boxed{\sin\theta \approx \tan\theta \approx \theta.}$$

When $\theta \ll 1\,\text{radian}$



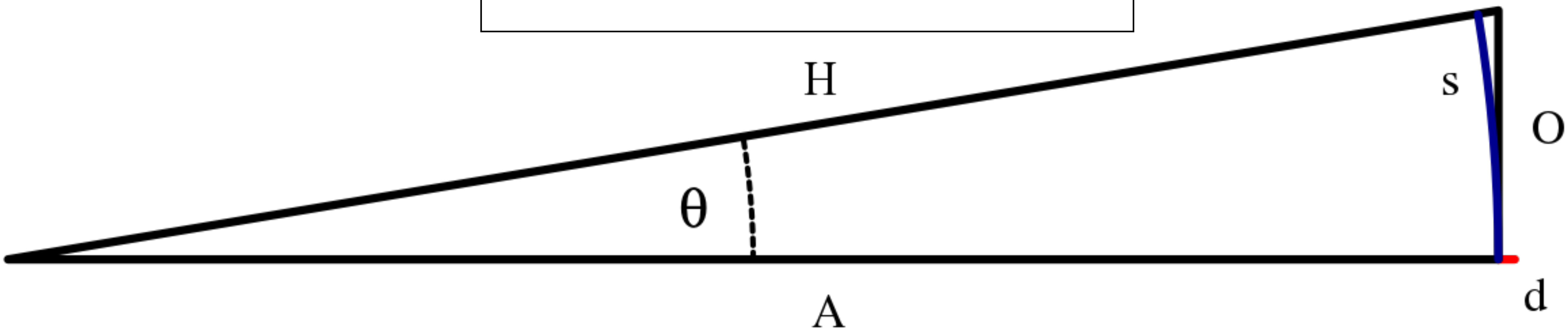
Radian: The Natural Unit of Angles

360 deg = 2π radian
so 1 deg = 1/57.3 radian
1 radian = 206,265 arcsec

arc length on a circle = radius of the circle $\times\,\theta$

Small angle approximation, θ must be in unit of radian and $\theta \ll 1$:

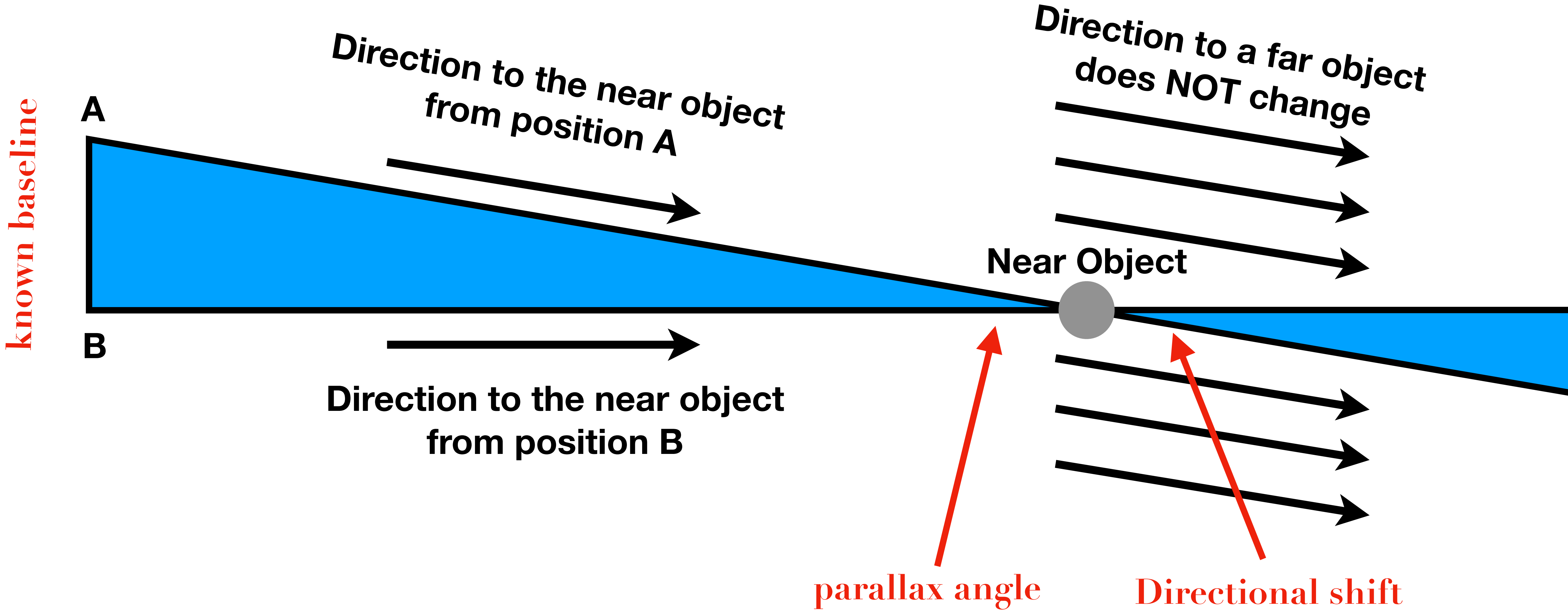
$$\sin\theta \approx \tan\theta \approx \theta.$$



Geometric illustration: $O \sim s$ when $\theta \ll 1$

Small-Angle Distance-Parallax Equation

$$\text{distance} \approx \frac{\text{baseline length}}{\tan(\text{parallax angle})} \approx \frac{\text{baseline length}}{\text{parallax angle (radian)}} \text{ or } d = l/p$$



known baseline

A

B

Direction to the near object from position A

Direction to the near object from position B

Direction to a far object does NOT change

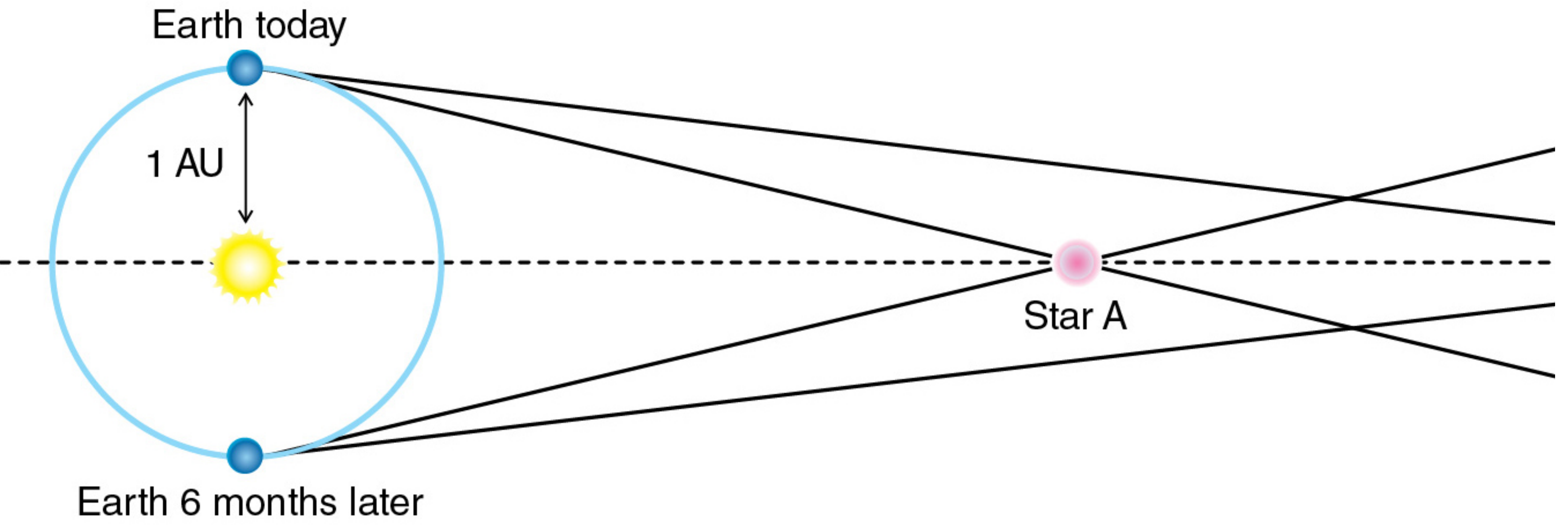
Near Object

parallax angle

Directional shift

Extend the Baseline from Earth Size to Earth's Orbit Size: Stellar Parallax

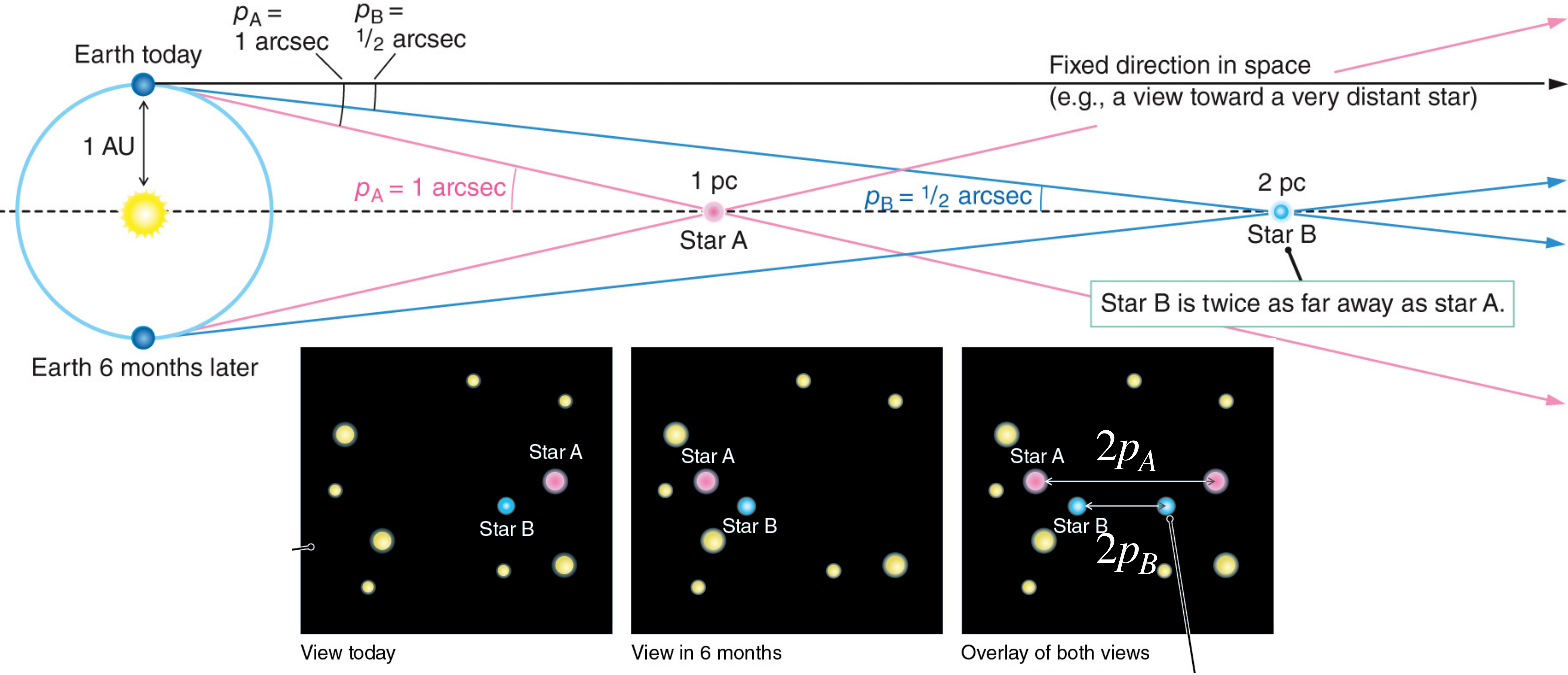
Parallax distance can be measured given the baseline length (l) and the parallax angle (p); by using the orbit of the Earth, *the baseline length is increased by 23500 times from 2x 6400 km to 2x 149.6 million km.*



The Definition of Parallax in Stellar Astronomy

Any directional shift due to a positional shift is a parallax effect, but in **stellar astronomy**, **parallax** is defined as **half (!!) of the maximum directional shift** due to Earth's orbital motion.

From this diagram, it's clear that **parallax** is inversely proportional to **distance**: $p \sim 1/d$



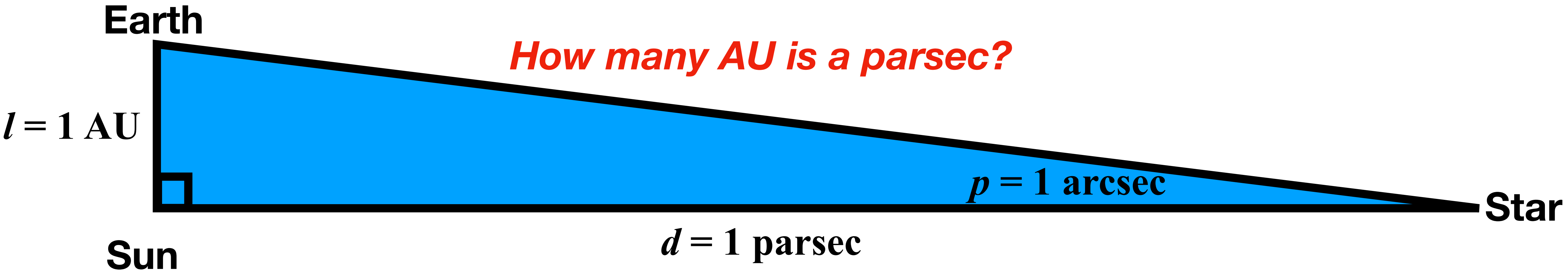
Definition of the unit parsec: the distance at which *p* = 1 arcsec

Let p be the parallax in arcseconds.

Let d be the distance in parsecs; the unit **parsec** is defined as the distance at which ***p* = 1 arcsec**

Given this definition we have:

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p}\right)$$



From the diagram above, we derived:

1 parsec = 206,205 AU since $l/d = \tan p \sim p$ (in radian)

Practice: convert parallax to distance

The greater the parallax, the smaller the distance.

A star with a parallax of 1 **arcsecond (arcsec)** is at a distance of 1 **parsec (pc)**.

- 1 arcsec = 1/3,600 degree
- 1 pc = 3.26 light-years (o*nly useful when talking to non-astronomers*)

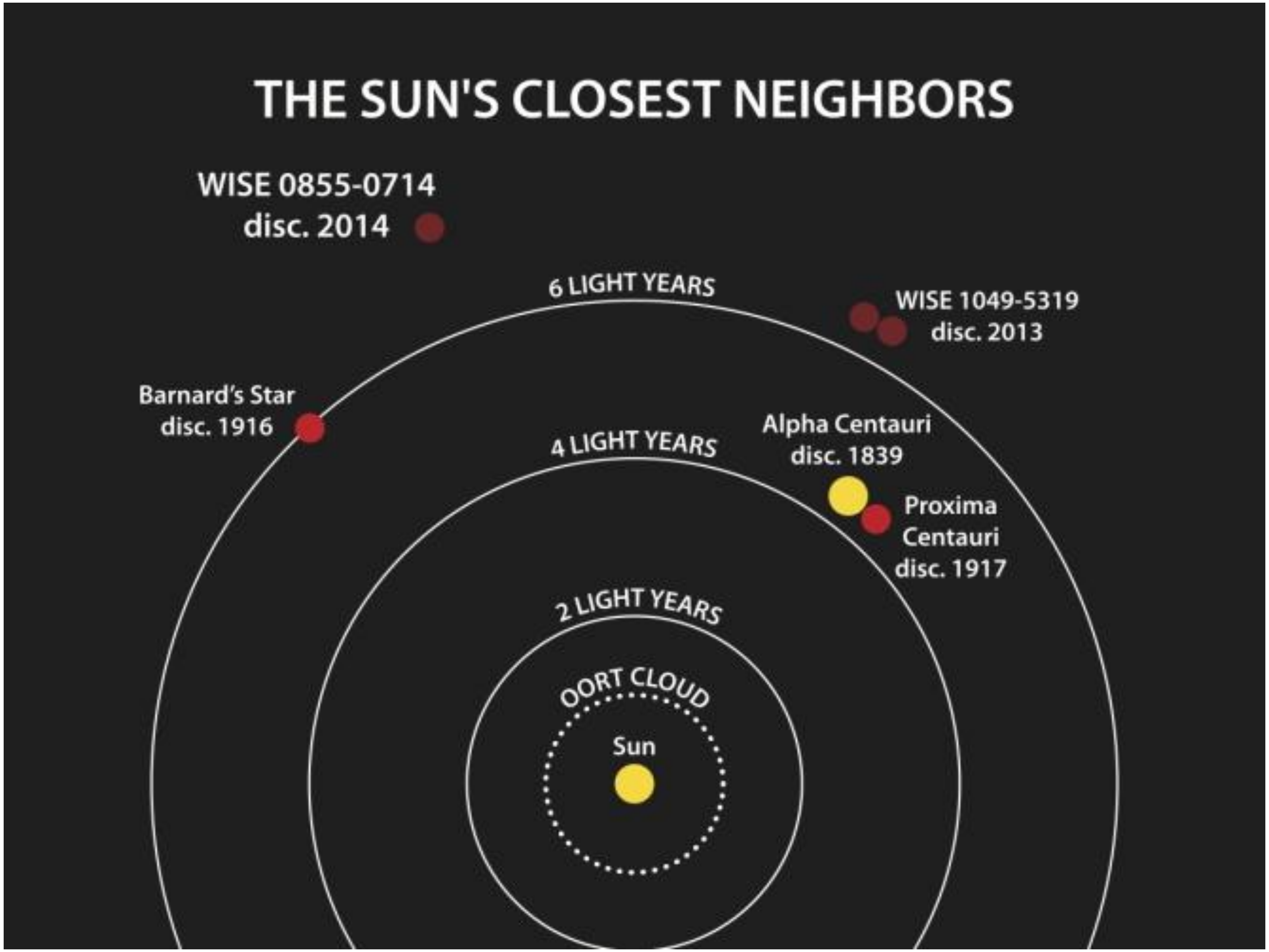
Parallax angles have been measured for >1 billion stars.

The first star with measured parallax was 61 Cygni by Friedrich Bessel in **1838**.
It had a parallax of **0.314 arcsec**, what is its distance in parsec and light-year?

Bessel functions in Mathematics are named after him.

Practice: Convert distance to parallax

Let's try a reversed problem. After the Sun, the closest star to Earth is Proxima Centauri, which is 4.24 light-years away. What is the star's parallax in arcsec? (1 pc = 3.26 ly)



Practice: Convert distance to parallax

Let’s try a reversed problem. After the Sun, the closest star to Earth is Proxima Centauri, which is 4.24 light-years away. What is the star’s parallax in arcsec?

First, we convert light-years to parsecs:

$$d = 4.24 \text{ light–years } \times \frac{1 \text{ parsecs}}{3.26 \text{ light–years}} = 1.30 \text{ parsecs}$$

Then, we plug in to find the distance:

$$p\ (\text{arcsec}) = \frac{1}{1.30 \text{ pc}} = 0.77 \text{ arcsec}$$

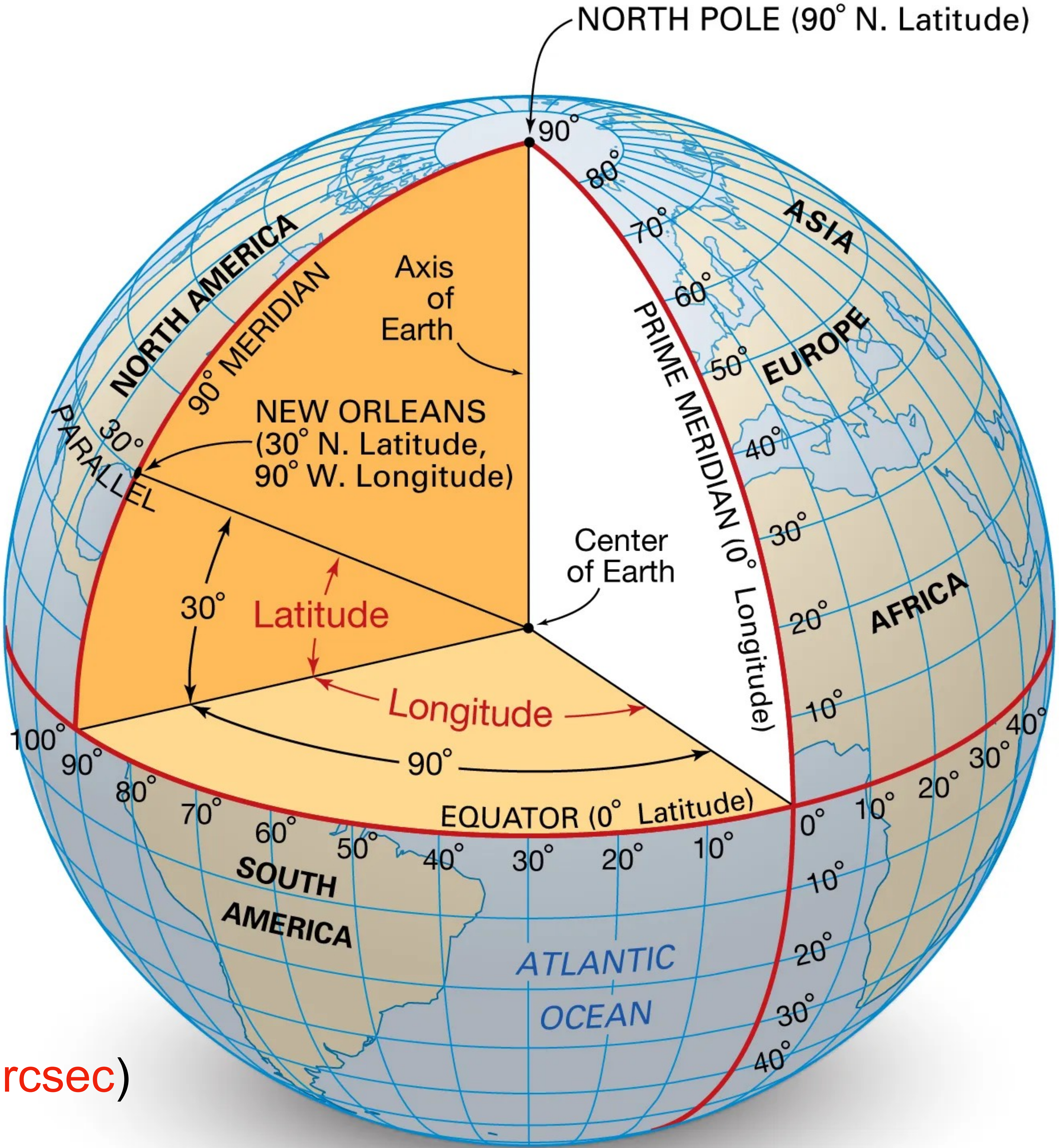
The closest star to the Sun has a parallax smaller than 1 arcsec!

How to Plan Parallax Observations?

Given a star’s position in equatorial coordinates (RA, Dec), how to decide when to make the two observations to detect the maximum parallax effect?

Review:

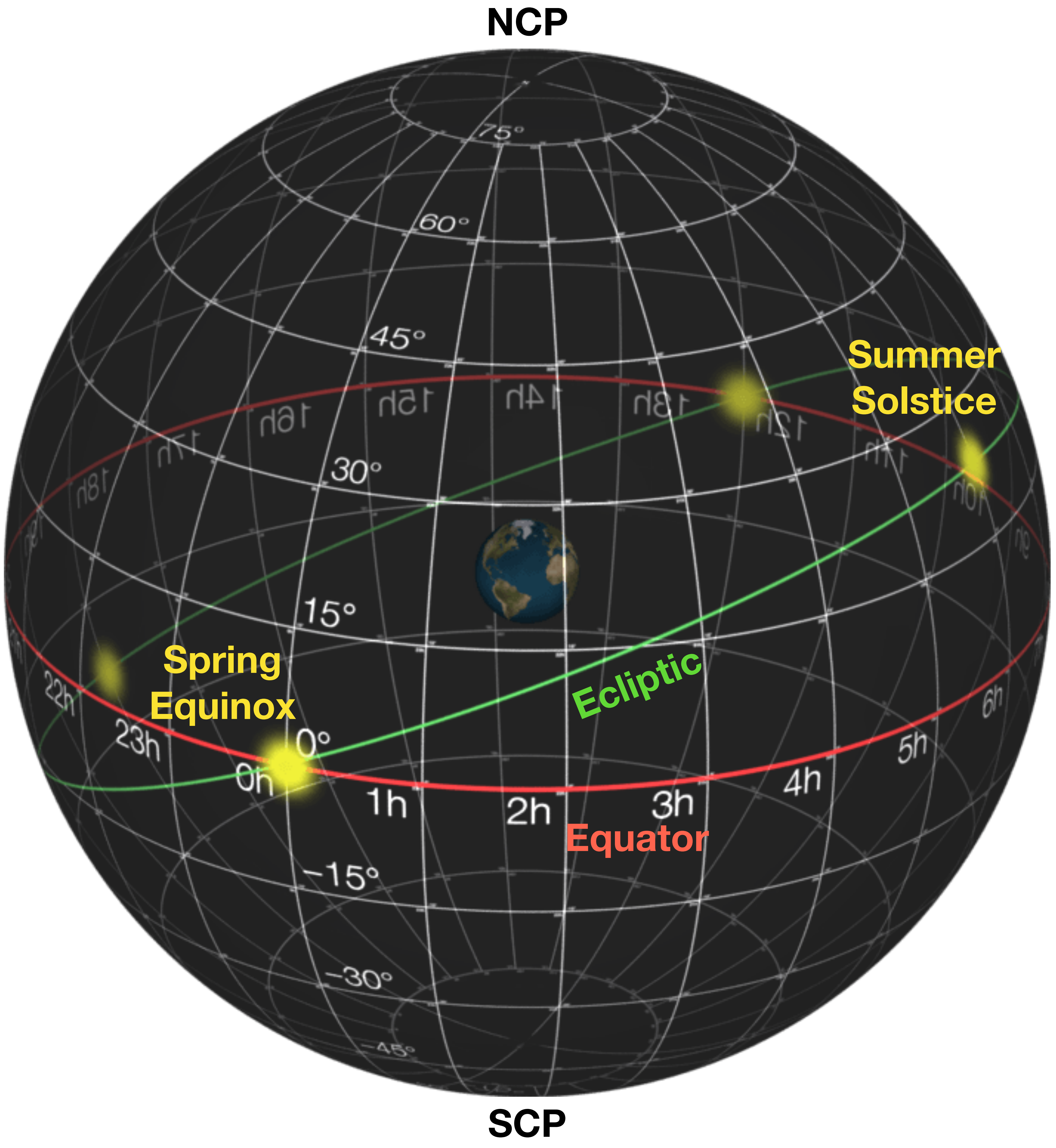
Celestial Coordinates are similar to the **Longitude** and **Latitude** system on Earth's surface



Longitude & Latitude units: (deg, arcmin, arcsec)

Equatorial coordinates

right ascension (RA)
declination (Dec)



RA's units
(hour, minute, second)

Dec's units:
(deg, arcmin, arcsec)

Equatorial coordinates

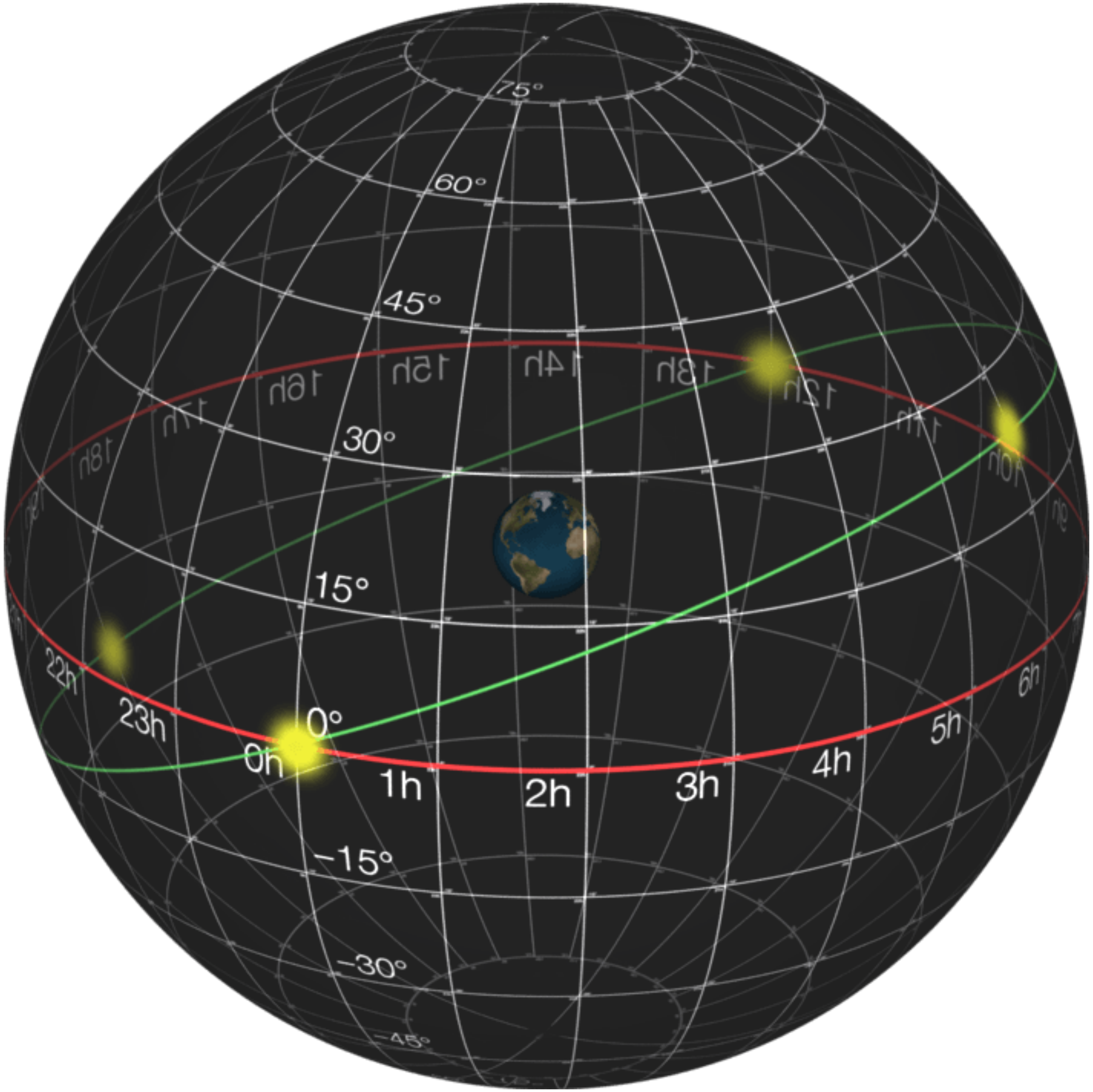
right ascension (RA)
declination (Dec)

Ecliptic coordinates

Longitude
Latitude

Ecliptic Longitude ~ RA
~ means "roughly equal"

|Ecliptic Latitude - Dec|
< 23.5 degrees



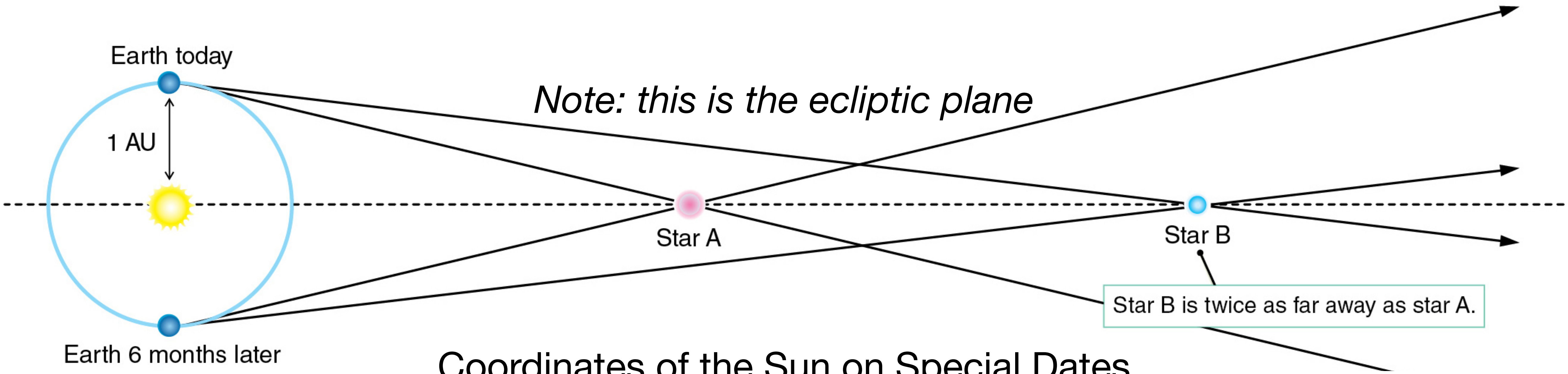
The Equatorial and Ecliptic Coordinates of the Sun

- In the course of a year, the Sun travels on the Ecliptic from Spring Equinox, to Summer Solstice, to Fall Equinox, to Winter Solstice, and back to Spring Equinox

	RA	*Dec*	*Ecliptic Longitude*	*Ecliptic Latitude*	*Notes*
Spring Equinox (Mar 20)	***0 hr***	***0 deg***	***0 hr*** ***0 deg***	***0 deg***	***Coordinates Origin***
Summer Solstice (Jun 21)	***6 hr***	***+23.5 deg***	***6 hr*** ***90 deg***	***0 deg***	***longest day in a year***
Fall Equinox (Sep 22)	***12 hr***	***0 deg***	***12 hr*** ***180 deg***	***0 deg***	***equal day and night***
Winter Solstice (Dec 21)	***18 hr***	***-23.5 deg***	***18 hr*** ***270 deg***	***0 deg***	***longest night in a year***

Stellar Parallax: Observational Considerations

- To see maximum parallax effect, you must choose two nights when the **Ecliptic Longitudes** of the target is **6 hrs (90 deg)** away from the **Sun**.



Note: this is the ecliptic plane

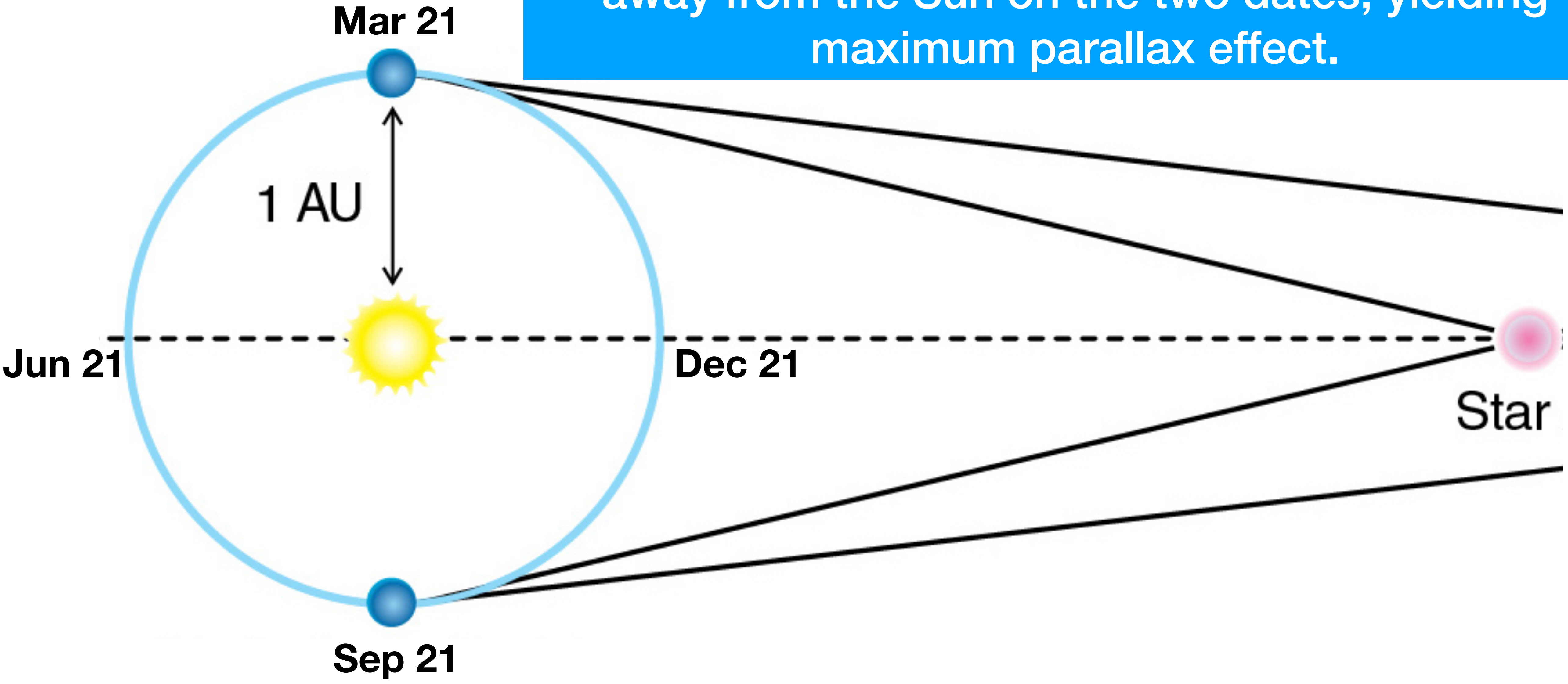
Coordinates of the Sun on Special Dates

	RA	*Dec*	*Ecliptic Longitude*	*Ecliptic Latitude*	*Notes*
Spring Equinox (Mar 20)	*0 hr*	*0 deg*	*0 hr*	*0 deg*	*Coordinates Origin*
Summer Solstice (Jun 21)	*6 hr*	*+23.5 deg*	*6 hr*	*0 deg*	*longest day in a year*
Fall Equinox (Sep 22)	*12 hr*	*0 deg*	*12 hr*	*0 deg*	*equal day and night*
Winter Solstice (Dec 21)	*18 hr*	*-23.5 deg*	*18 hr*	*0 deg*	*longest night in a year*

Let’s check the RA, Dec, & Dates in the previous practice example

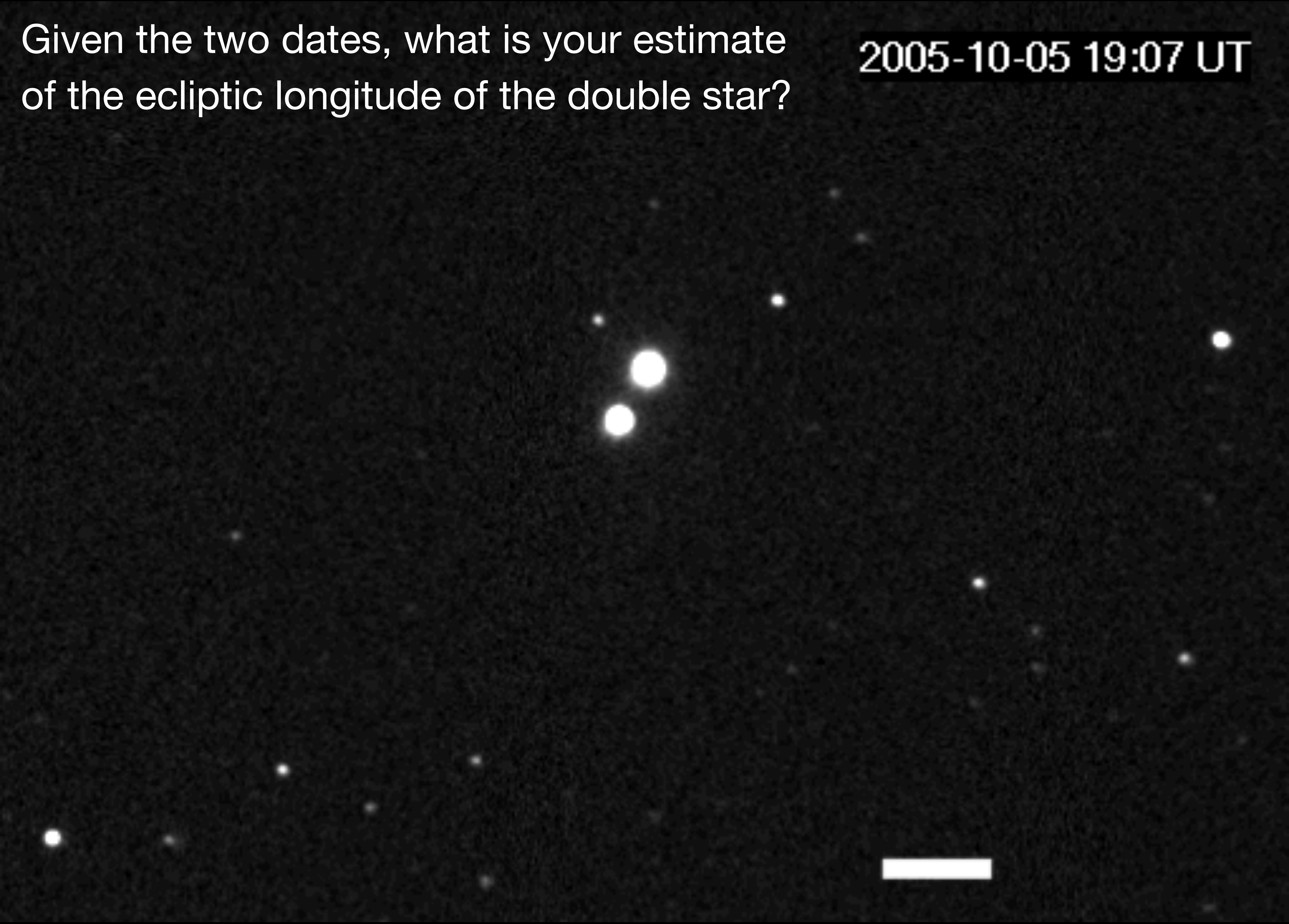
The equatorial coordinates of the star on two dates:
Mar 21 2022: 06h00m15.205s 23d29’15.155”
Sep 21 2022: 06h00m15.235s 23d29’15.160”

The star is on the ecliptic, and its RA places it 90° away from the Sun on the two dates, yielding maximum parallax effect.



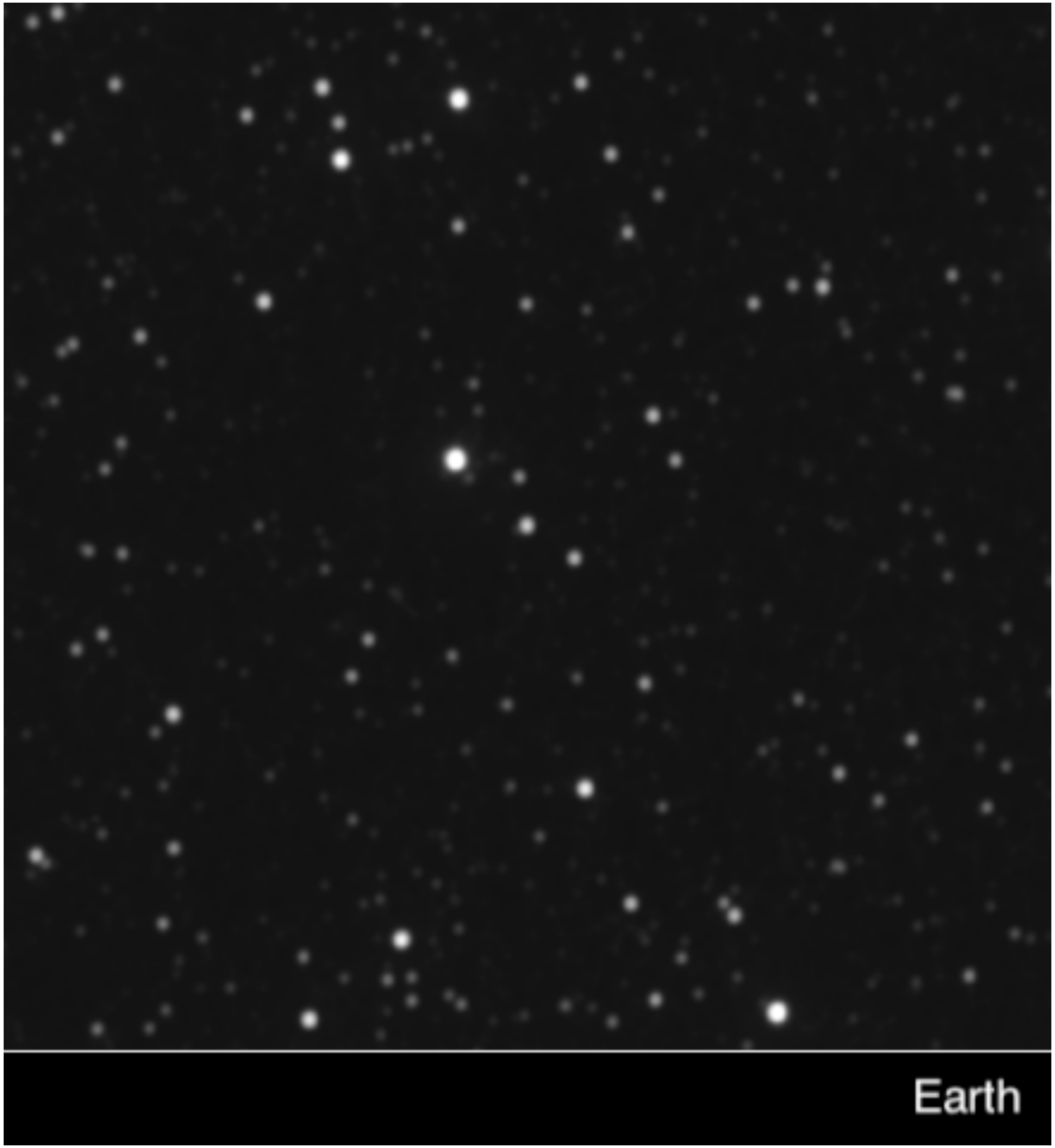
Parallax Examples

Given the two dates, what is your estimate of the ecliptic longitude of the double star?



2005-10-05 19:07 UT

Parallax of Proxima Centauri between Earth and Pluto (baseline ~= 40 Astronomical Units)

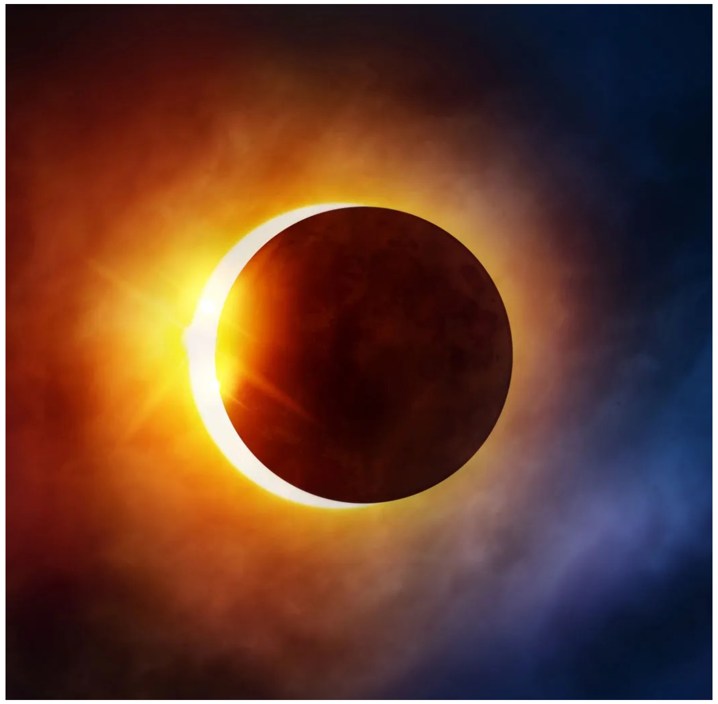


Parallax of Proxima Centauri as observed simultaneously from *New Horizons* at Pluto's orbit and the Earth.

The Earliest Parallax Measurement by Hipparchus (~150 BC): Baseline limited by the diameter of the Earth

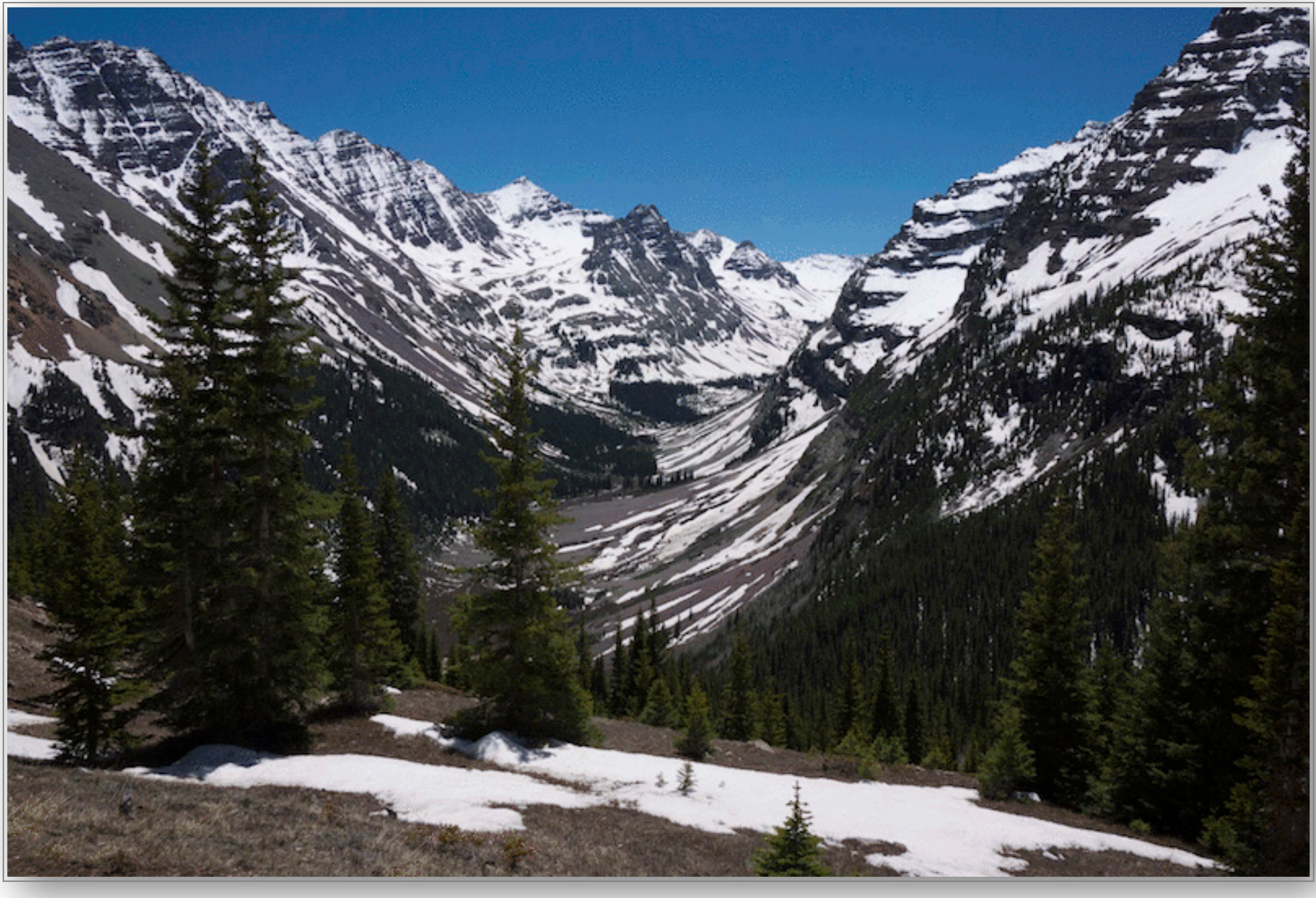
seen in Hellespont (100% obscured)

seen in Alexandria (80% obscured)



The Solar Eclipse on Mar 14, 190 BC

Maroon Bells, Aspen, Colorado

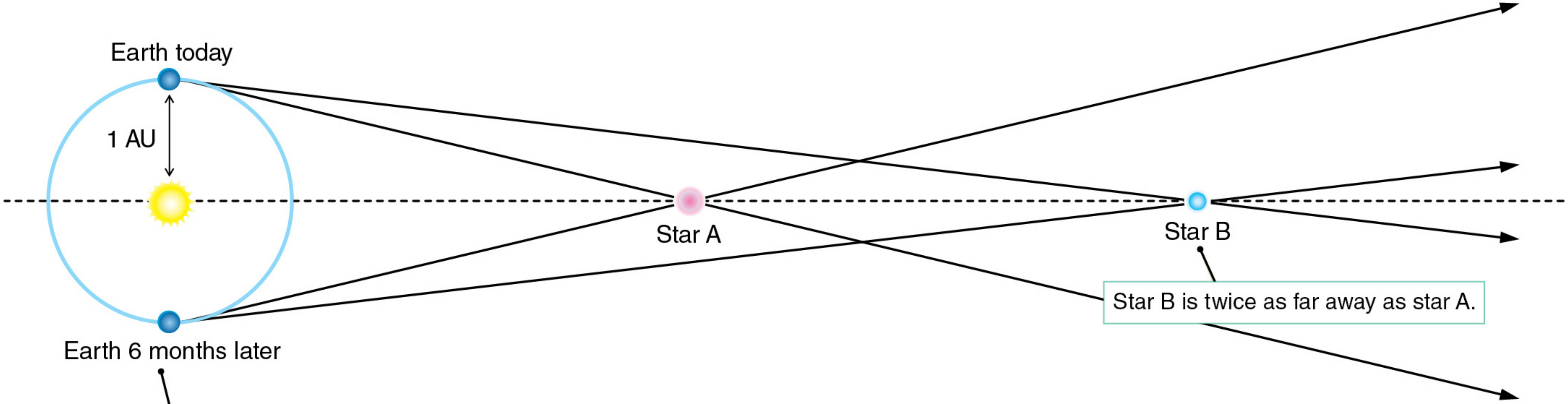


Annual Parallax Traces

What kind of pattern does a star draw on the sky due to Earth’s annual motion?
We can record this pattern if we continuously monitor its position over a year

Special case 1: sources on the ecliptic plane

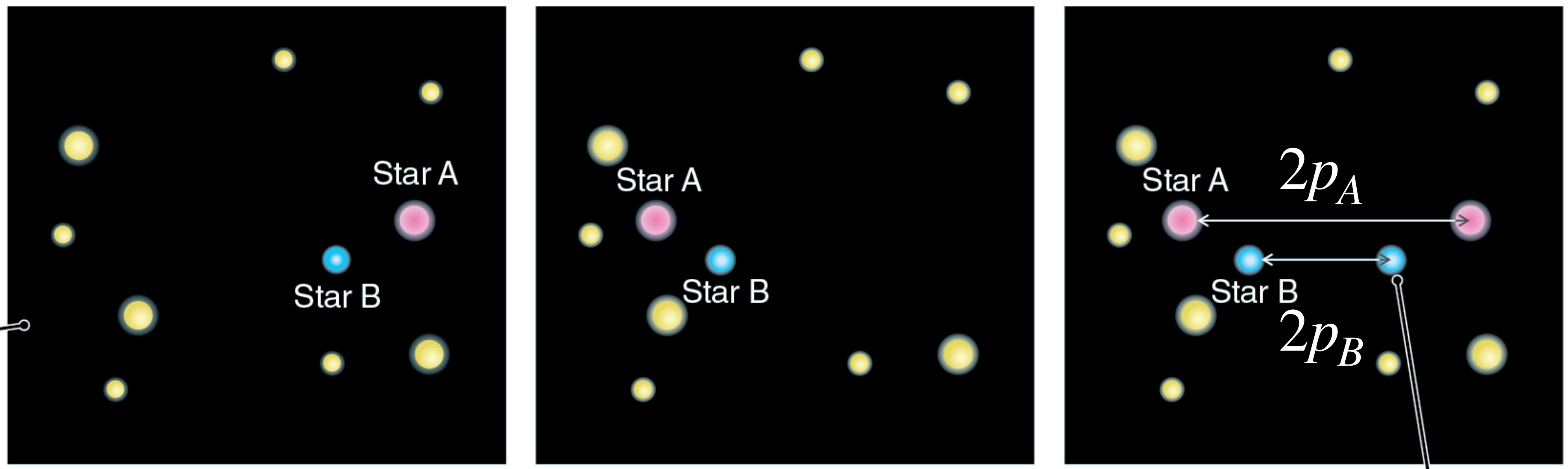
oscillating along a short line



Star B is twice as far away as star A.

Astronomers use the changing perspective of Earth through the year to measure distances to stars.

Nearby stars appear to change their positions more than distant stars do.



View today

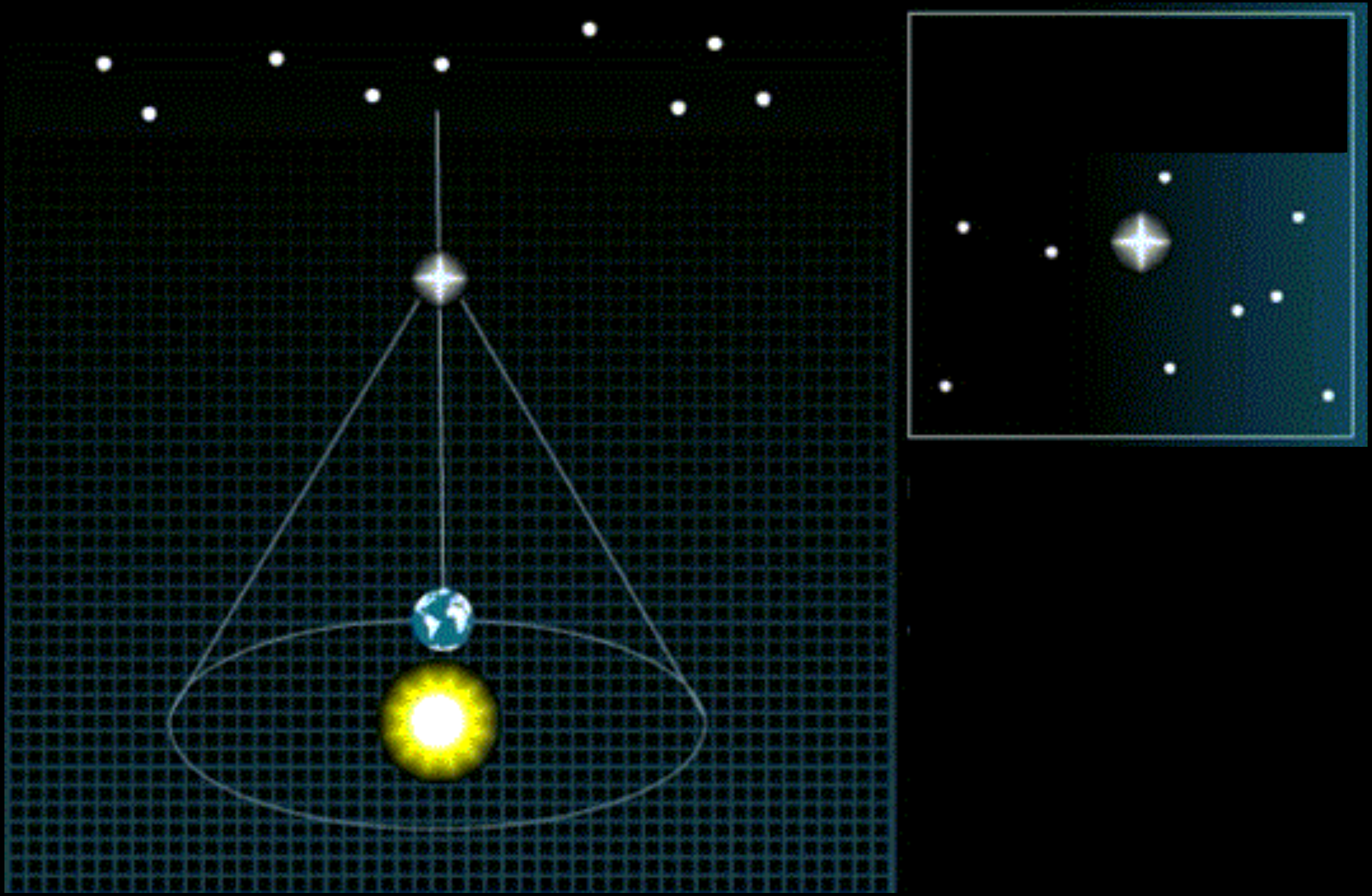
View in 6 months

Overlay of both views

Star B appears to move half as much as star A over the year.

Special case 2: sources on the ecliptic poles

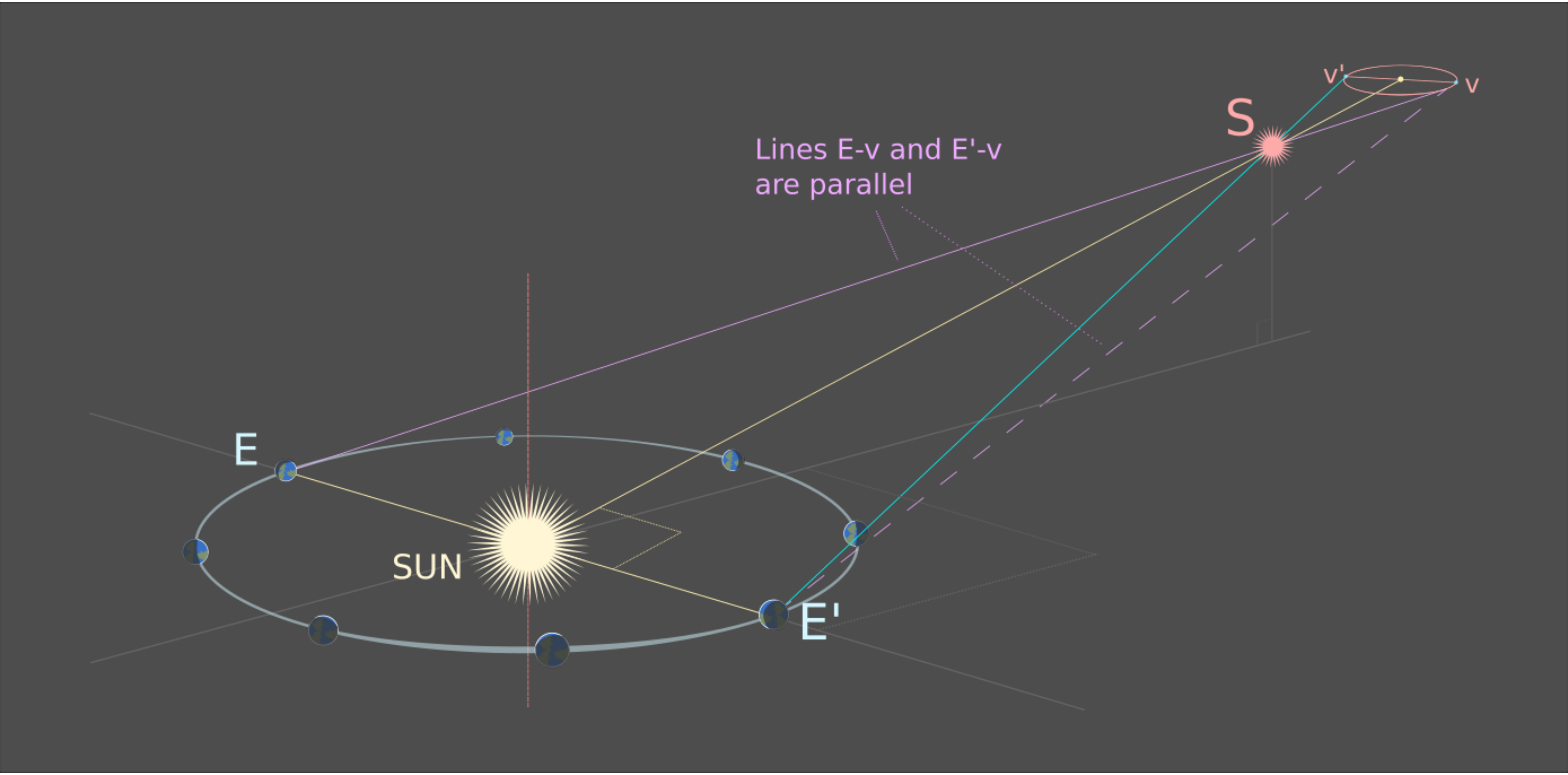
moving along a circle



Weten Schaps

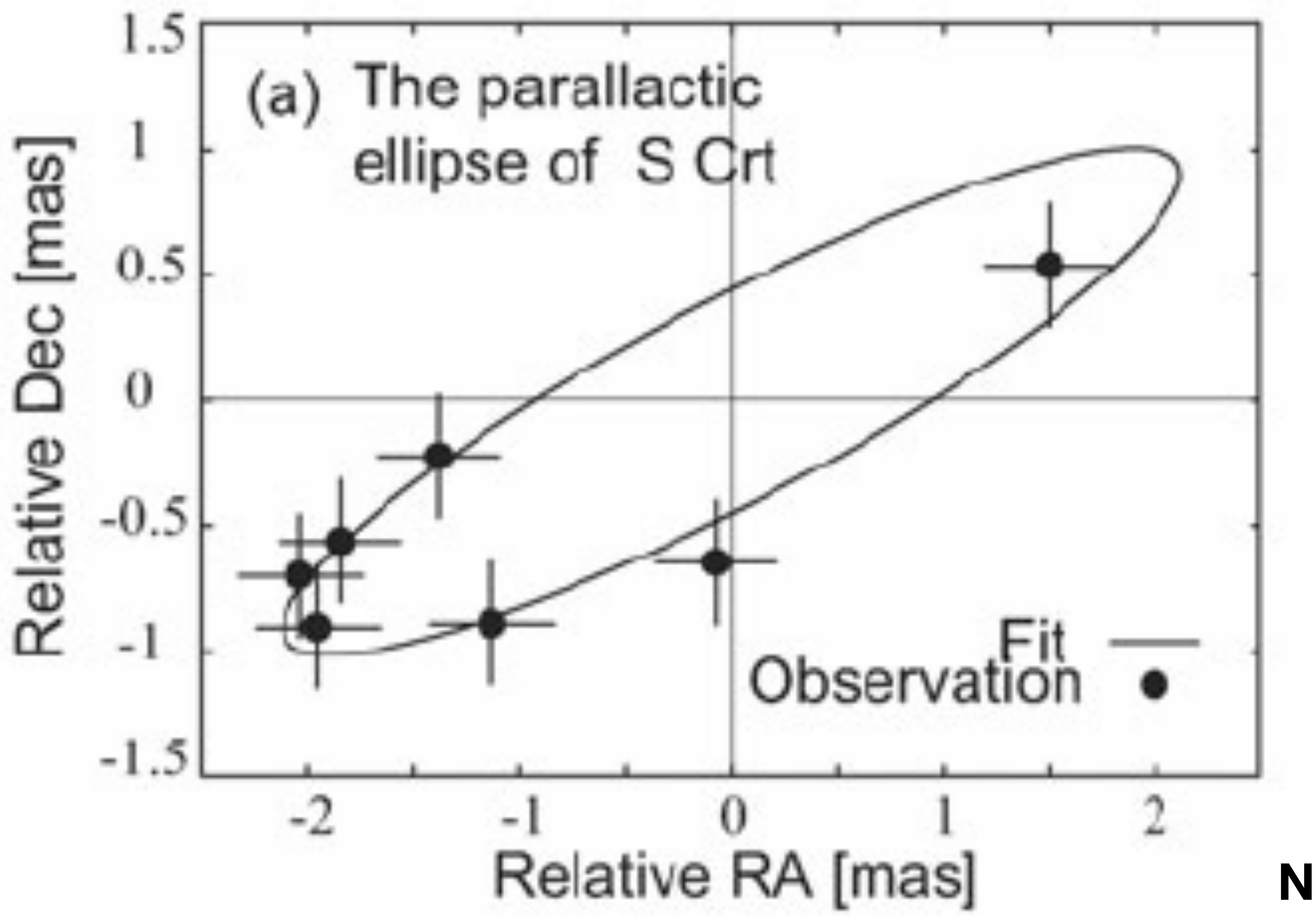
General cases: 0 < ecliptic latitude < 90 deg

moving along an ellipse



Summary: Parallactic Traces & Parallax Measurements

- Sources on the ecliptic oscillate on short lines along the ecliptic; the parallax to measure distance is half of the length of the line.
- Sources on the ecliptic poles draw parallactic circles; the parallax to measure distance is the radius.
- All other sources draw ellipses with major axes parallel to ecliptic; For a parallactic ellipse, what is the parallax to measure distance?



Nakagawa+2008

How to Calculate Parallax from Celestial Coordinates?

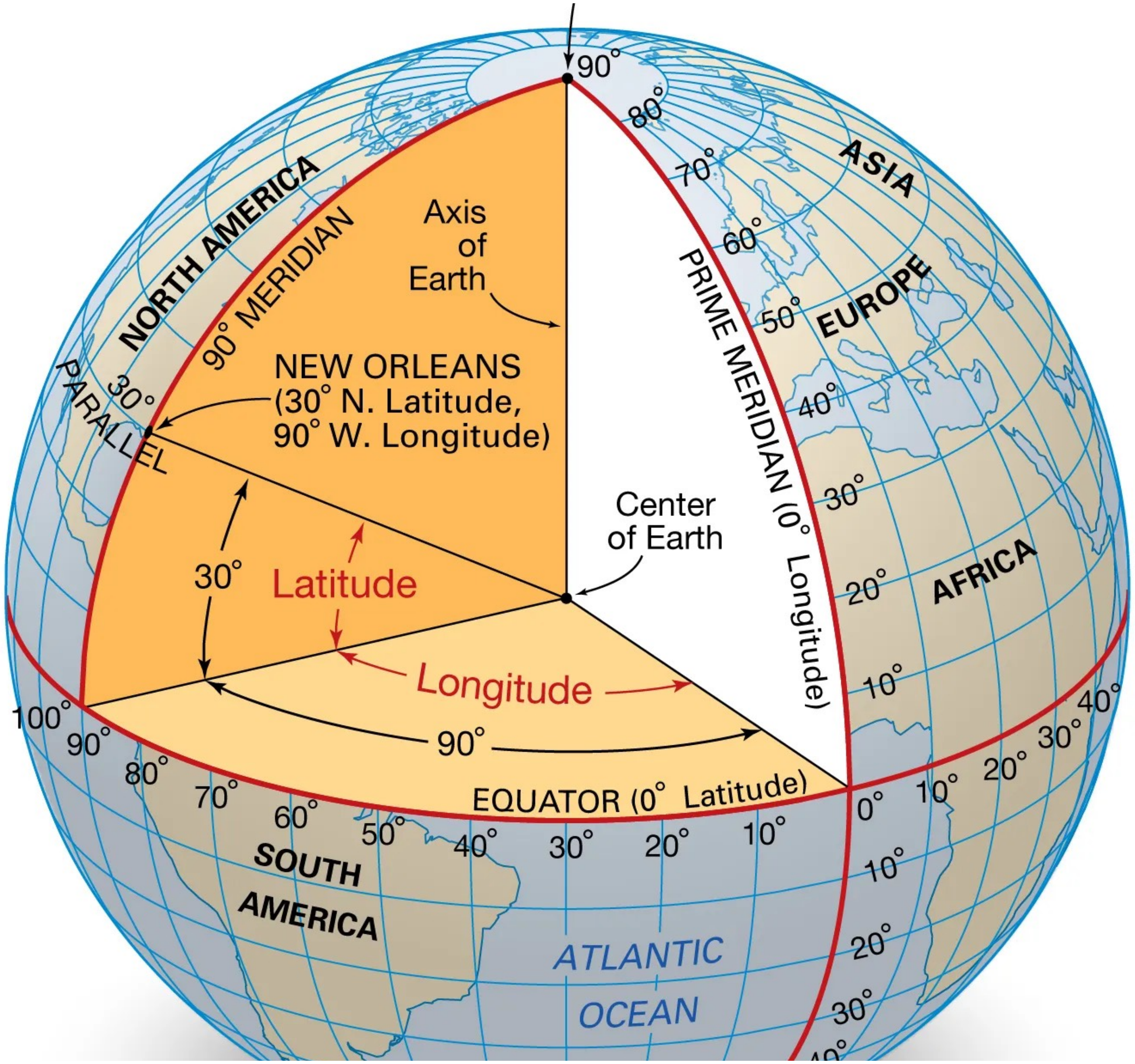
A star’s position is recorded in celestial coordinates (RA, Dec), how to calculate the angular offset between two coordinates?

Practice: Calculate the distance between locations along the same parallel

Iowa City: 41.6578° N, 91.5346° W

Des Moines: 41.5868° N, 93.6250° W

Earth’s Radius: 3960 miles

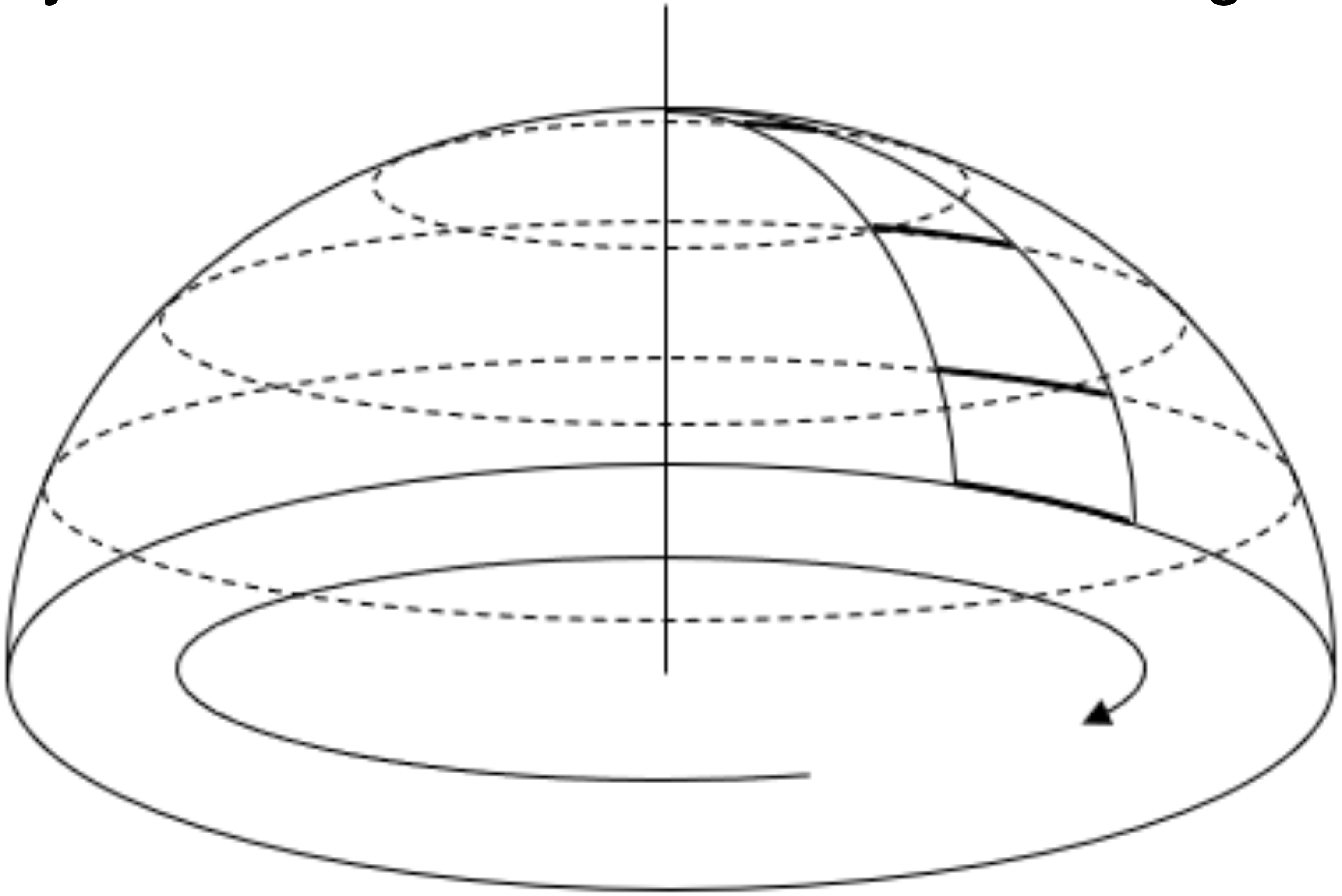


Almost the same latitude, difference in longitude about 2.1 degrees.

- convert 2.1 deg to radian: 0.0366519
- distance = arc length = $R \cdot \theta = 3960\text{miles} \times 0.0366519 = 145\text{miles}$
- Google Map says the distance is 115 miles (much less than 145 miles), why?

Practice: at fixed Declination, calculate angular offset in R.A.

- Obj 1: RA = 2 hr, Dec = 60 deg, Obj 2: RA = 3 hr, Dec = 60deg; what's their angular distance in **degrees**?
- Note that you'll need to first convert hours to degrees …



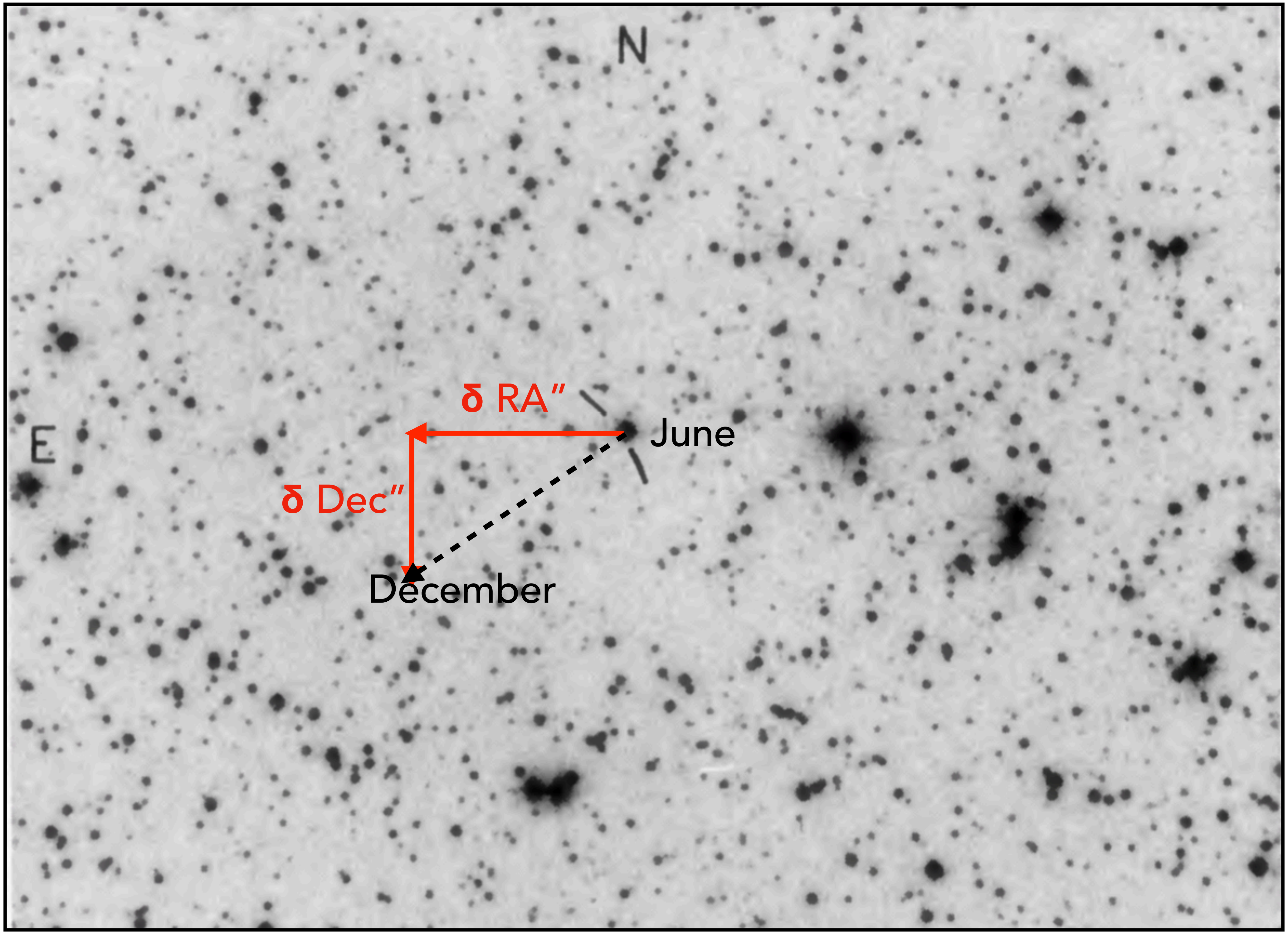
$$\delta\mathrm{RA}^\circ = (\mathrm{RA}_1^h - \mathrm{RA}_2^h)\cdot\cos(\mathrm{Dec}_1^\circ)\cdot 15^\circ/\mathrm{hour}$$

$$\delta\mathrm{Dec}^\circ = \mathrm{Dec}_1^\circ - \mathrm{Dec}_2^\circ$$

Offsets in both RA and Dec, how to calculate the total offset?

When the two coordinates are close together, we can use plane trigonometry to approximate spherical trigonometry:

$$\Delta'' = \sqrt{\delta\mathrm{RA}''^2 + \delta\mathrm{Dec}''^2}$$



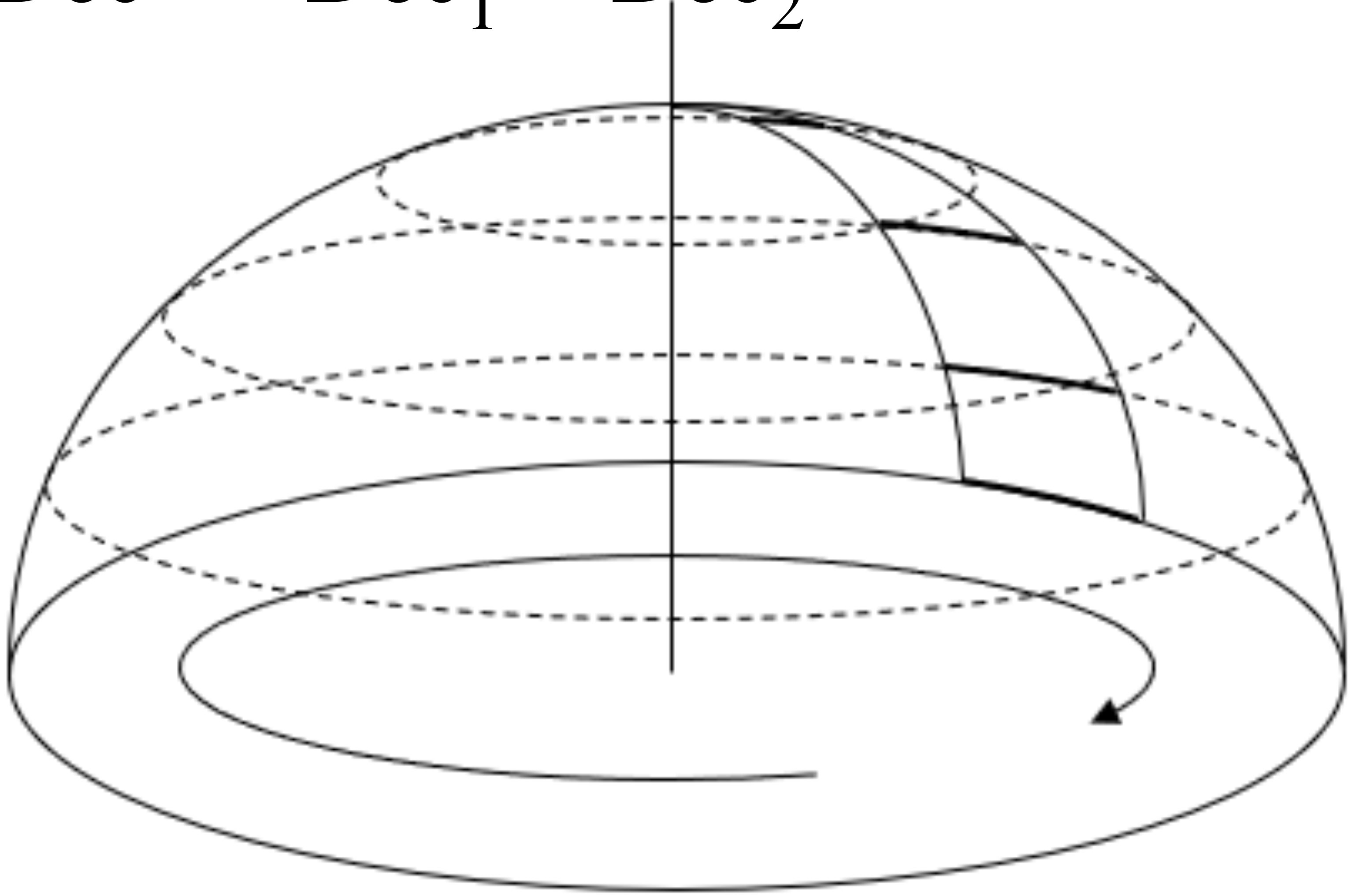
Given two (RA, Dec) coordinates, calculate their angular offset

$$\Delta'' = \sqrt{\delta \mathrm{RA}''^2 + \delta \mathrm{Dec}''^2}$$

Note that (1) RA's units are (hour, minute, second), and Dec's units are (deg, arcmin, arcsec), and (2) the angular distance between two meridians ***decreases*** from the equator to the poles. As a result, we have the following formulae to calculate both the RA offset and the Dec offset in arcsec:

$$\delta \mathrm{RA}'' = (\mathrm{RA}_1^s - \mathrm{RA}_2^s) \cdot \cos(\mathrm{Dec}^\circ) \cdot 15''/\mathrm{s}$$

$$\delta \mathrm{Dec}'' = \mathrm{Dec}_1'' - \mathrm{Dec}_2''$$



Practice: Given two (RA, Dec) coordinates, calculate their angular offset

$$\Delta'' = \sqrt{\delta\mathrm{RA}''^2 + \delta\mathrm{Dec}''^2}$$

$$\delta\mathrm{RA}'' = (\mathrm{RA}_1^s - \mathrm{RA}_2^s) \cdot \cos(\mathrm{Dec}^\circ) \cdot 15''/\mathrm{s}$$

$$\delta\mathrm{Dec}'' = \mathrm{Dec}_1'' - \mathrm{Dec}_2''$$

dRA = 0.03 * cos(23.5 deg) * 15 = 0.413”
dDec = 0.005”
=> p = 0.413”/2 => d = 2.4*2 parsec

A star’s coordinates have been recorded based on images taken on the following dates:

Mar 21 2022: 06h00m15.205s 23d29’15.155”
Sep 21 2022: 06h00m15.235s 23d29’15.160”

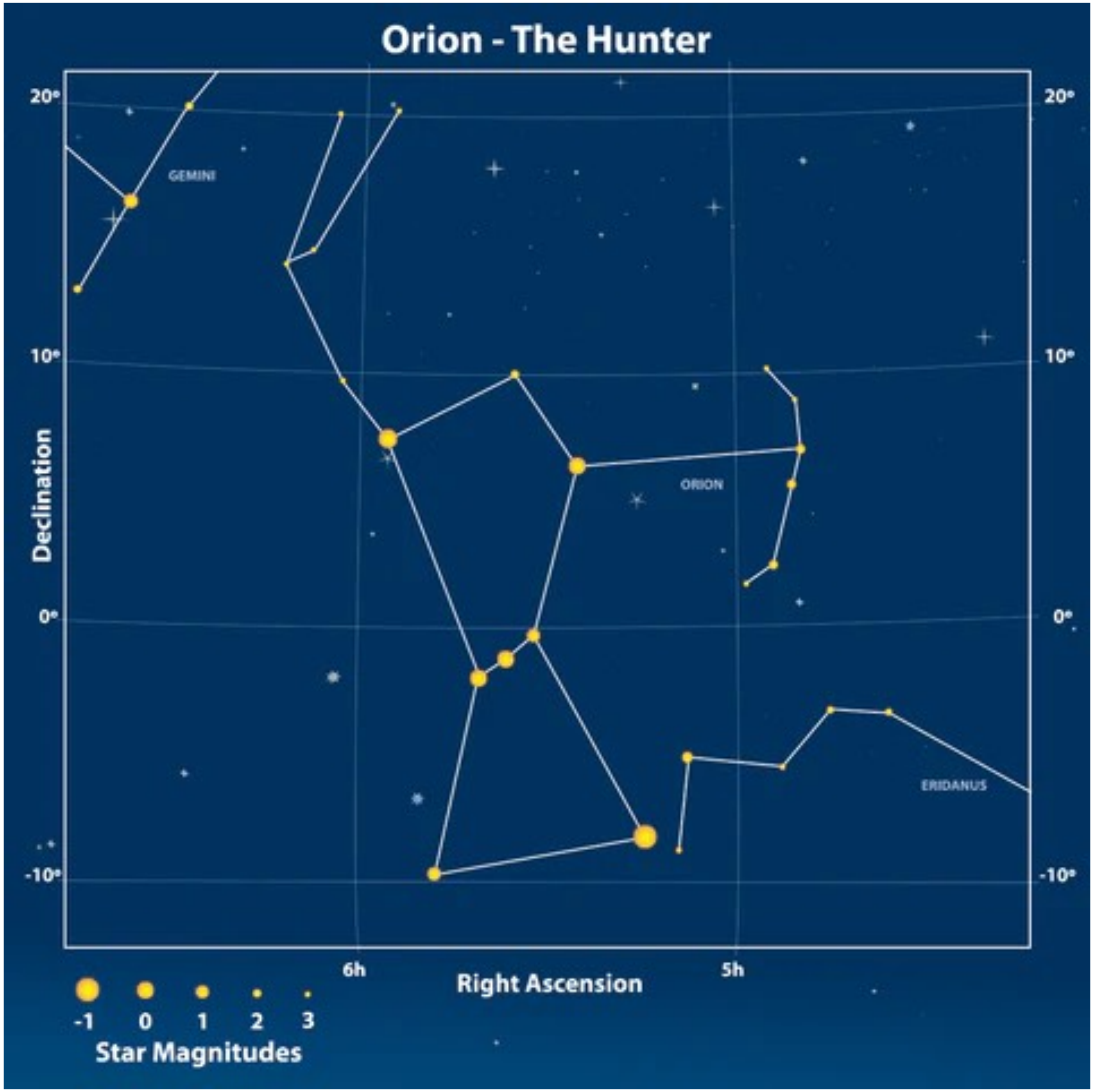
- How far has the star moved in RA & in Dec (both in arcsec)?
- How large is the parallax? What’s the distance in parsec?

Brightness Measurements: Apparent Magnitude

Visual classification of brightness: The Greek Magnitude System

Ancient Greeks: "*the stars that appear first after sunset are the 1st magnitude stars, the stars that appear second are the 2nd magnitude stars, and so on ……*"

129 BC, first formally introduced by **Hipparchus**, then refined by **Ptolemy** in **150 AD**: visual classification of stars into 6 classes, **brightest as being of 1st magnitude, faintest of 6th magnitude**



Observed Brightness of Stars show a HUGE range

- The Sun is the brightest star, which dominates the sky during the day, rendering it impossible to see any other stars
- The faintest star your eye can see is **10^{13}** fainter than the Sun
- The faintest star that can be detected by the Hubble space telescope is **10^{20}** fainter than the Sun.

- How do we deal with such a large range? We put everything on a logarithmic scale similar to that used by the Greeks, thus preserving the history started from Hipparchus in 129 BC.
- As a result, brighter stars still have lower magnitudes (*a minor annoyance astronomy students have to live with*).

- Mathematically we have the Pogson's ratio (1856):

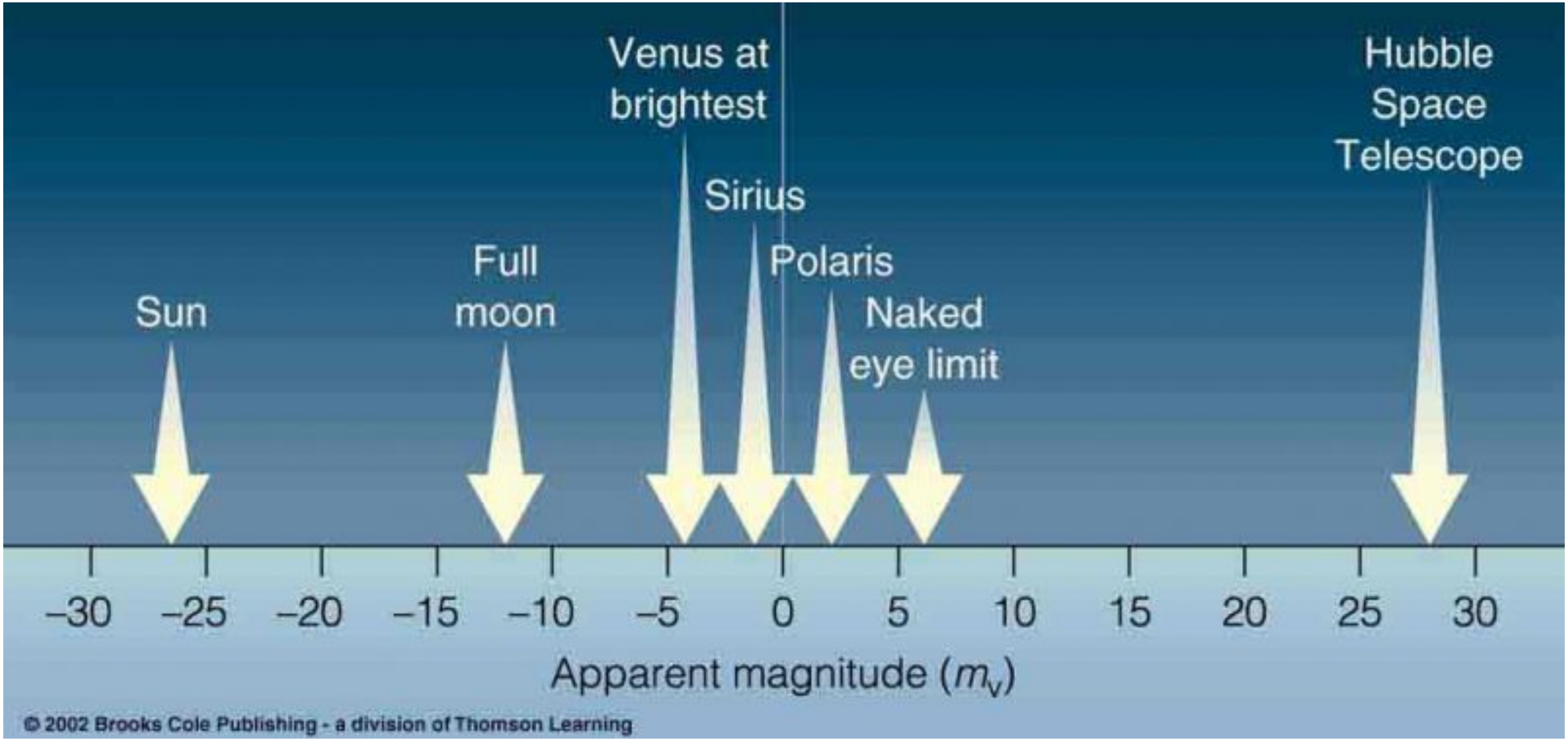
$$m_{\lambda,1} - m_{\lambda,2} = -\,2.5\log(f_{\lambda,1}/f_{\lambda,2})$$

  to tell the **magnitude difference** between two objects, but how do we put the magnitudes on a universal scale so that a given magnitude means the same flux to all astronomers?

The universal magnitude system based on reference objects

$$m_\lambda - m_{\lambda,0} = -2.5\log(f_\lambda/f_{\lambda,0})$$

- where **_0** indicate the chosen **reference source**'s magnitude and flux at wavelength lambda. In optical wavelengths, the reference star is **Vega**.



The universal magnitude system based on Vega

$$m_\lambda - m_{\lambda,0} = -2.5\log(f_\lambda/f_{\lambda,0})$$

- Normally in the optical wavelengths, the reference star is **Vega**.
- For simplicity, **Vega's** magnitude is set to be **zero** at all wavelengths
- As a result, we have the Vega magnitude defined in the following equation:

$$\text{Vega magnitude}: m_\lambda = -2.5\log(f_\lambda/f_{\lambda,\text{Vega}})$$

Practice: From flux ratio to apparent magnitude relative to Vega

- What's the magnitude of a star that is 50x fainter than Vega at 500nm?
- What's the magnitude of a star that is 30x fainter than Vega?

m(50x fainter) = 4.25
m(30x fainter) = 3.69

Practice: From apparent magnitude to flux ratio

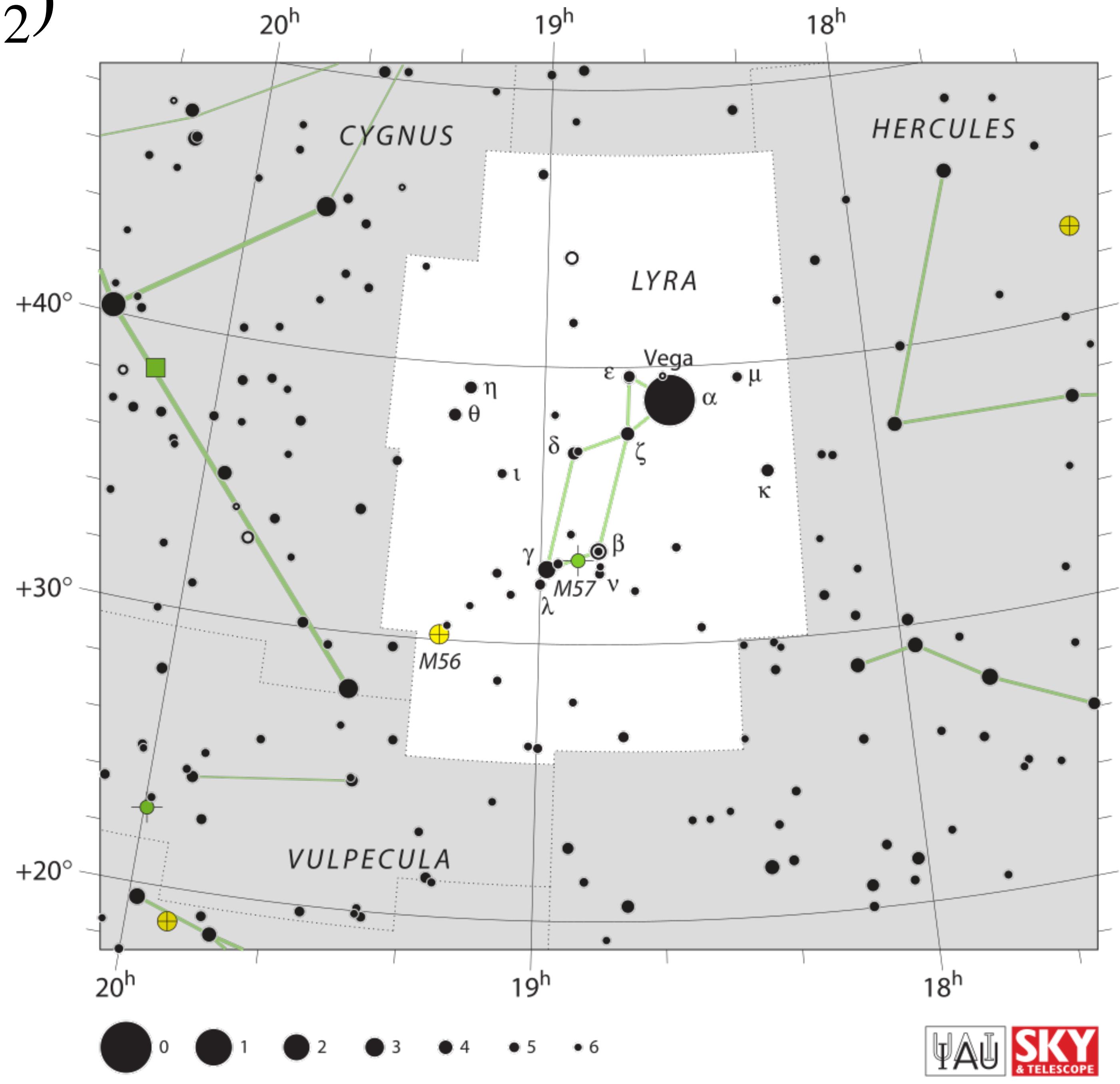
$$\text{Pogson's ratio}: m_{\lambda,1} - m_{\lambda,2} = -2.5\log\left(\frac{f_{\lambda,1}}{f_{\lambda,2}}\right)$$

$$\Rightarrow \frac{f_{\lambda,1}}{f_{\lambda,2}} = 10^{-0.4(m_{\lambda,1}-m_{\lambda,2})}$$

- **δ Lyrae** has an apparent magnitude of **4.2** in V-band (551 nm), how many times fainter is it compared to Vega (α Lyrae)?
- **17 Lyrae** has an apparent magnitude of **5.2** in V-band, how many times fainter is it compared to **δ Lyare**?

10^(0.4*4.2) = 47.9

10^(0.4*(5.2-4.2)) = 2.512



Summary: Apparent Magnitude and Flux Ratio

- 100x in flux ratio corresponds to a magnitude difference of 5
- 1 magnitude difference corresponds to 2.514x (=10^0.4) difference in flux

- To determine the magnitude of one source, you must know the magnitude and flux of another source (reference or standard) and compare the fluxes of the two sources

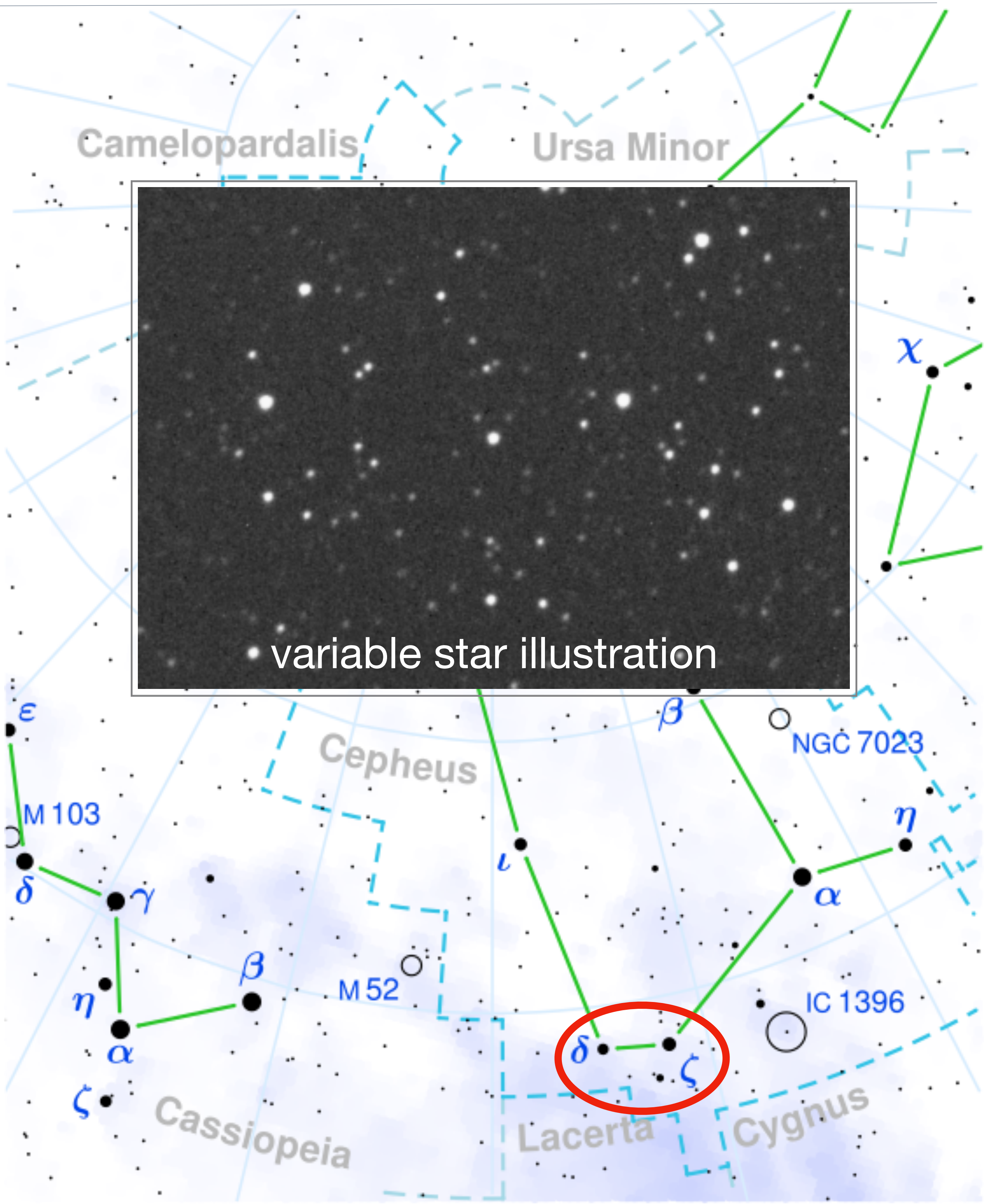
$$m_{\lambda,1} - m_{\lambda,2} = -2.5\log\left(\frac{f_{\lambda,1}}{f_{\lambda,2}}\right)$$

$$\Rightarrow \frac{f_{\lambda,1}}{f_{\lambda,2}} = 10^{-0.4(m_{\lambda,1}-m_{\lambda,2})}$$

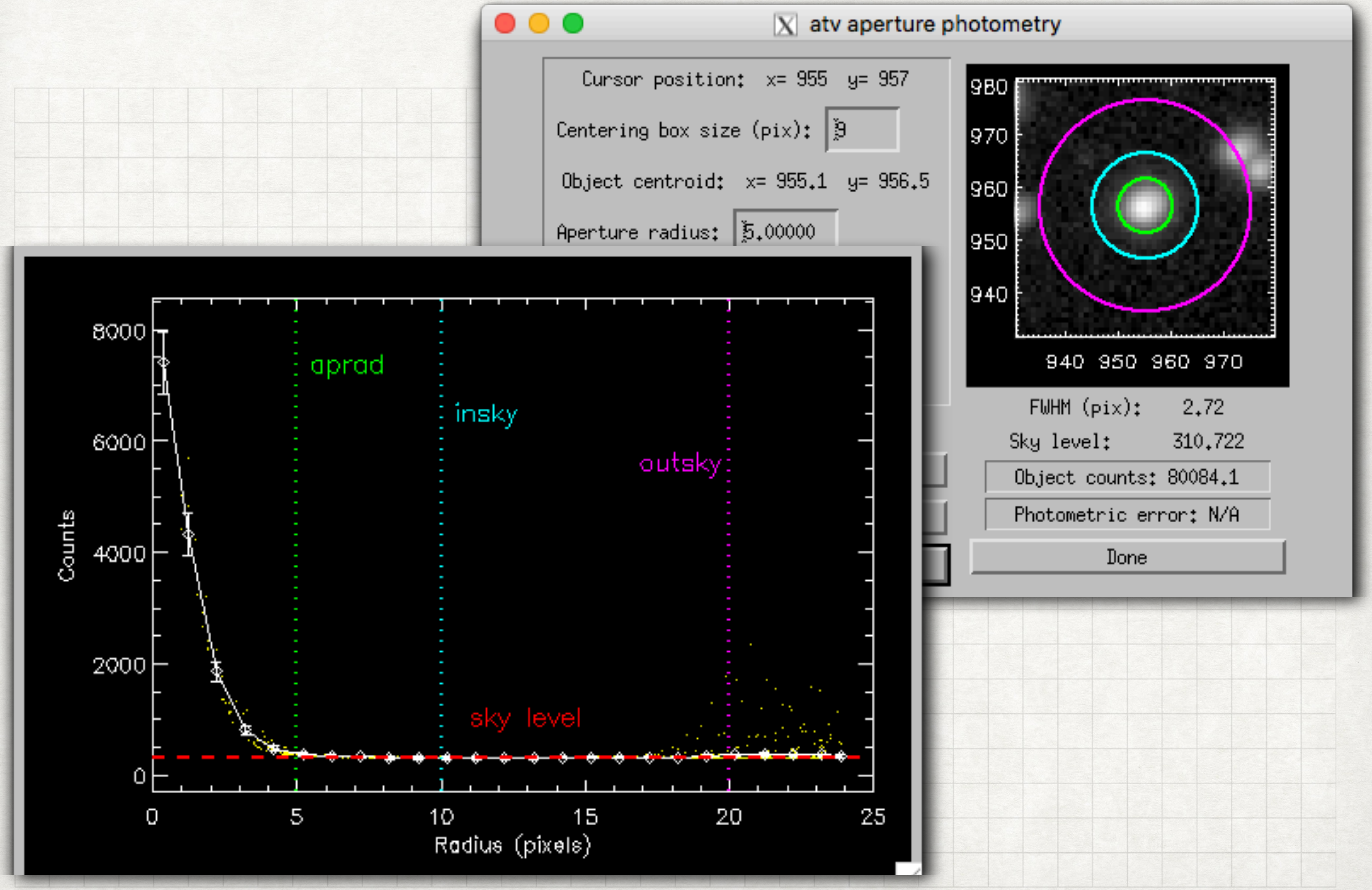
Differential Photometry

CCD Photometry: e- Count Rates to Magnitudes

- We can point the same telescope at **two different sources *simultaneously*** and measure the **ratio** of their **count rates**.
- This approach is easier because all instrumental effects in the two measurements cancel out.
- If we know the magnitude from one of the sources, we can infer the magnitude of the other source using this relative measurement.



TO COUNT ELECTRONS FROM A SOURCE, WE USE APERTURES



atv aperture photometry

Cursor position: x= 955 y= 957

Centering box size (pix): 9

Object centroid: x= 955.1 y= 956.5

Aperture radius: 5.00000

FWHM (pix): 2.72

Sky level: 310.722

Object counts: 80084.1

Photometric error: N/A

Definition of Magnitudes is based on Differential Photometry

$$m_\lambda = m_{\lambda\mathrm{ref}} - 2.5\log(f_\lambda / f_{\lambda,\mathrm{ref}})$$

the reference source here does not need to be Vega, it just needs to be a relatively stable source with a known magnitude

Count rates to magnitude difference

$$m_a - m_b = -2.5\log\left(\frac{F_a}{F_b}\right) = -2.5\log\left(\frac{Q_a/t_a}{Q_b/t_b}\right)$$

where object a is your science target and object b is the reference source with known magnitudes.

Practice: from count rates to magnitude

$$m_a - m_b = -2.5\log\left(\frac{F_a}{F_b}\right) = -2.5\log\left(\frac{Q_a/t_a}{Q_b/t_b}\right)$$

where object a is your science target and object b is the standard star with known magnitudes.

Your standard star has a magnitude of 10.5 mag in V-band, you took a CCD image of the standard star with a V-band filter and you got a total of 1500 counts in 10 seconds.

Next, you slew the telescope to take a V-band image of your science target, say a random galaxy far away, and with 30 min exposure, you could barely see it. The total count from the galaxy is 50.

What's the V-band magnitude of the galaxy?

$V_{galaxy} = 10.5 - 2.5\log((50/1800)/(1500/10)) = 19.83$

MAGNITUDE (FLUX) MEASUREMENTS

A BRIEF HISTORY

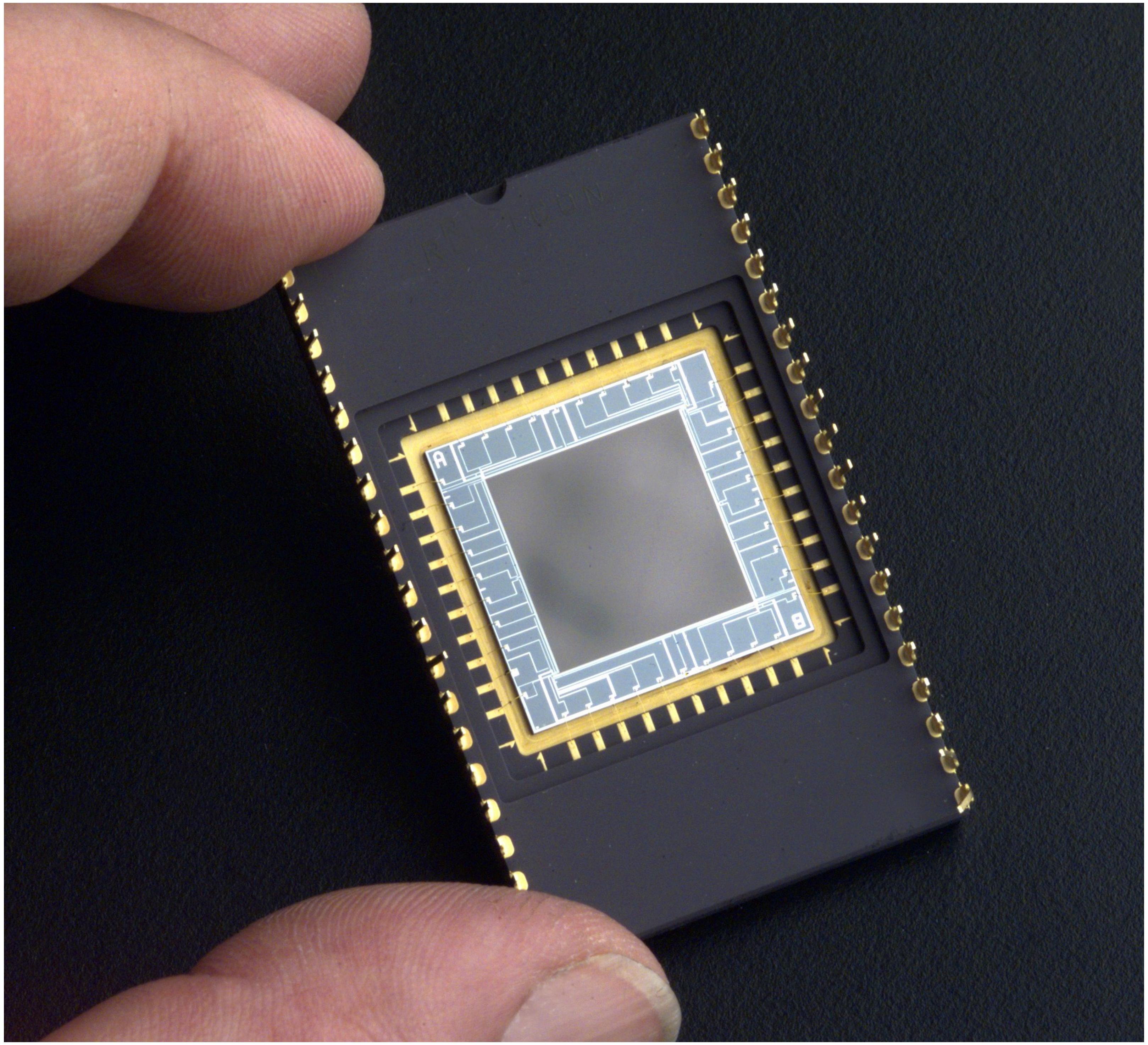
- 129 BC, first **Hipparchus**, then refined by **Ptolemy** in 150 AD: visual classification of stars into 6 classes, brightest as being of 1st magnitude, faintest of 6th magnitude
- 1856, **Pogson**: 5 magnitude difference = 100x in **energy flux**, while preserving historically classified 6th mag stars, some brightest stars have negative magnitudes (e.g., **Sirius**, V-band mag = -1.5). Summarized in an equation, we have **Pogson's ratio**:

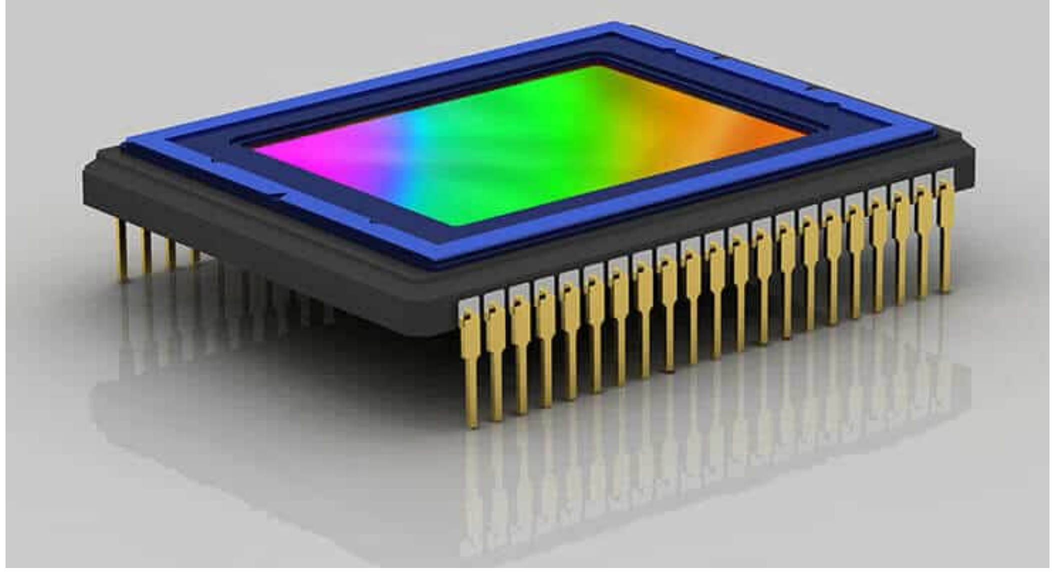
$$m_{\lambda,1} - m_{\lambda,2} = -2.5\log(f_{\lambda,1}/f_{\lambda,2})$$

- 1850s - 1990s: photographic glass plates
- 1940s, photoelectric cells, tubes, photomultipliers
1959, **monolithic integrated circuit** on silicon, **Bob Noyce** (1949 Grinnell College, IA)
- 1969, Boyle & Smith: CCD detectors (2009 Nobel Prize for Physics). First used in astronomy in 1976 at U. of Arizona

Charge-Coupled Device (CCD)

- 1969, Boyle & Smith: CCD detectors (2009 Nobel Prize for Physics). Invented for astronomy in 1976 at U. of Arizona





Anatomy of a Charge Coupled Device (CCD)

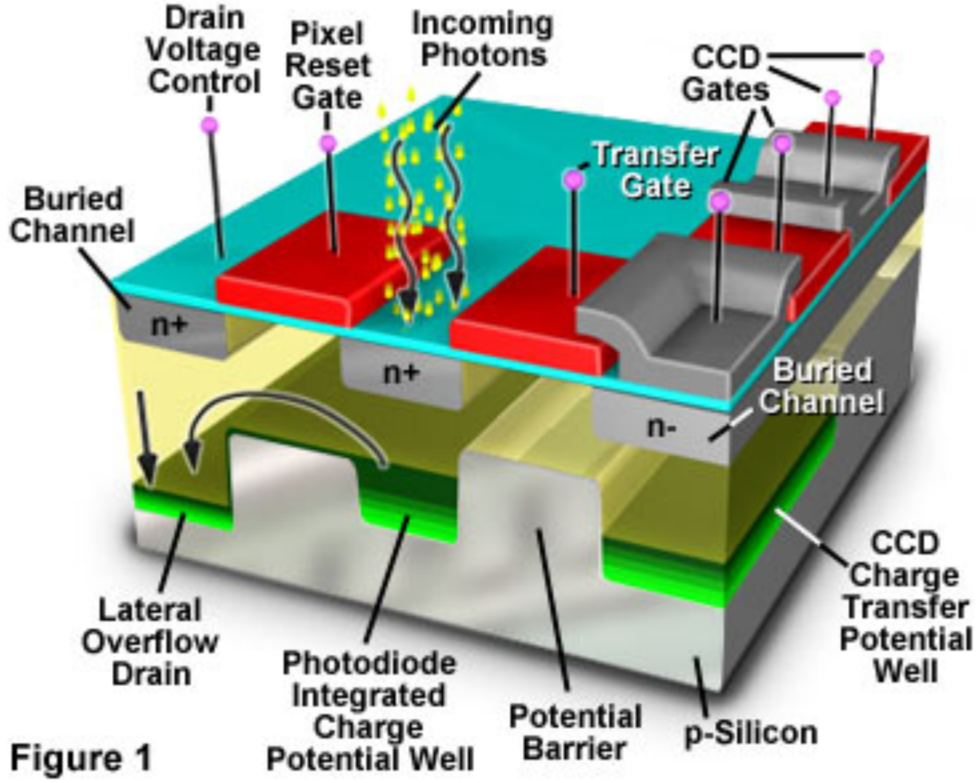


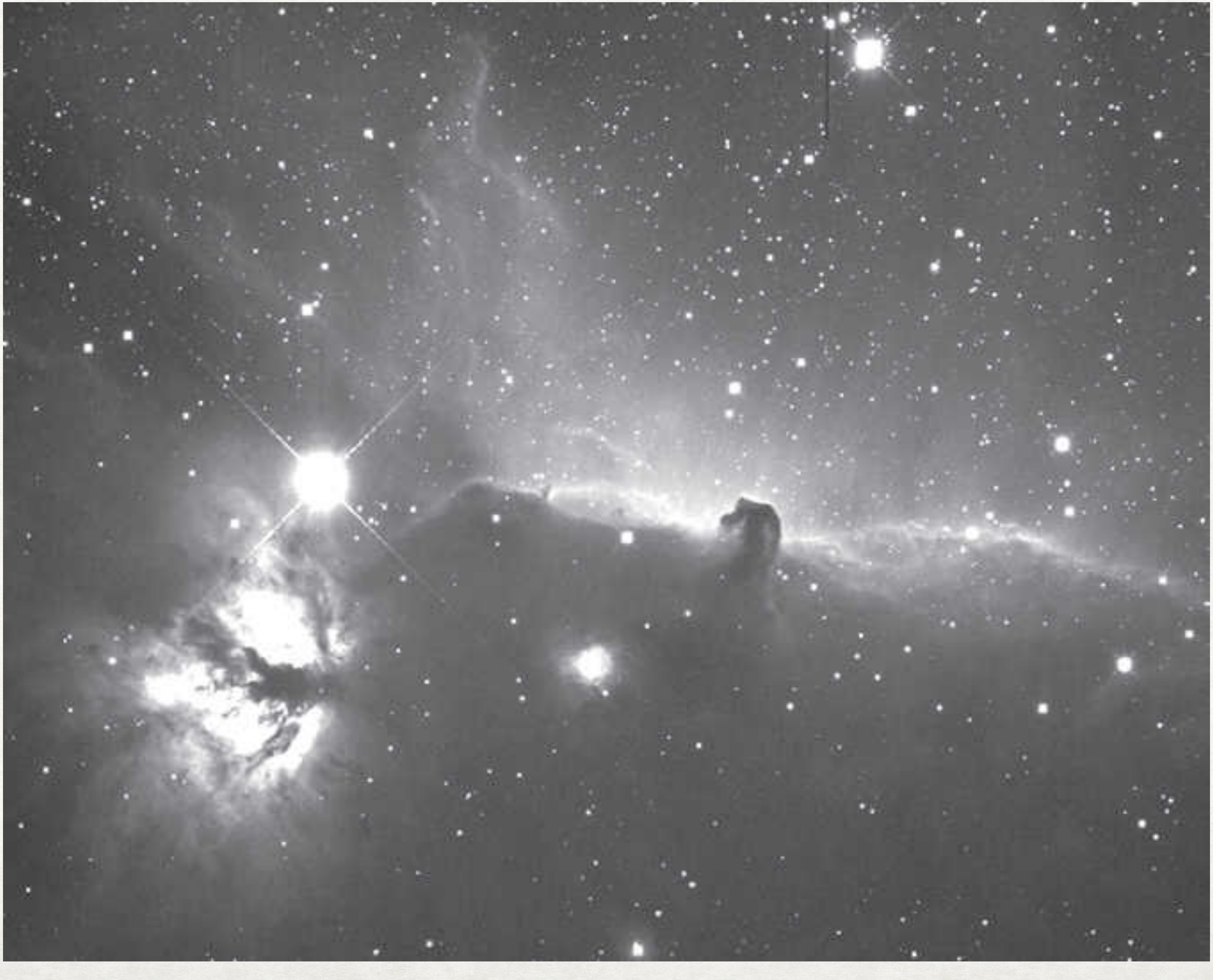
Figure 1

2009 NOBEL PRIZE IN PHYSICS

Willard S Boyle and George E Smith (1969 invention at Bell Labs)

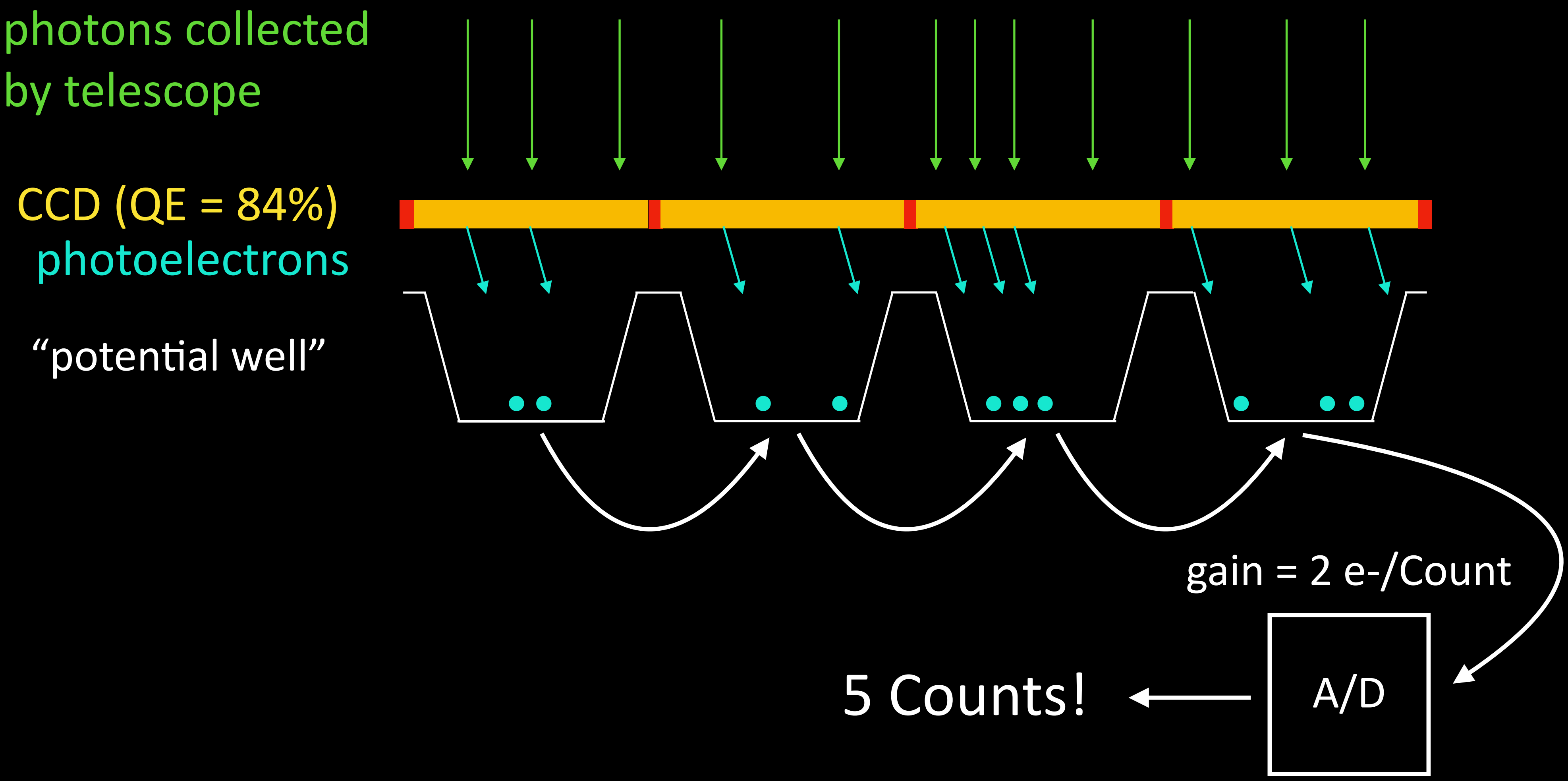
The charge-coupled device (CCD) provided the first way for a light-sensitive silicon chip to store an image and then digitize it, opening the door to the creation of digital images**.**

HOW A DIGITAL IMAGE IS RECORDED AND SAVED?



A SIMPLIFIED PICTURE

How the charge coupled device works?



photons collected
by telescope

CCD (QE = 84%)
photoelectrons

“potential well”

gain = 2 e-/Count

A/D

5 Counts!

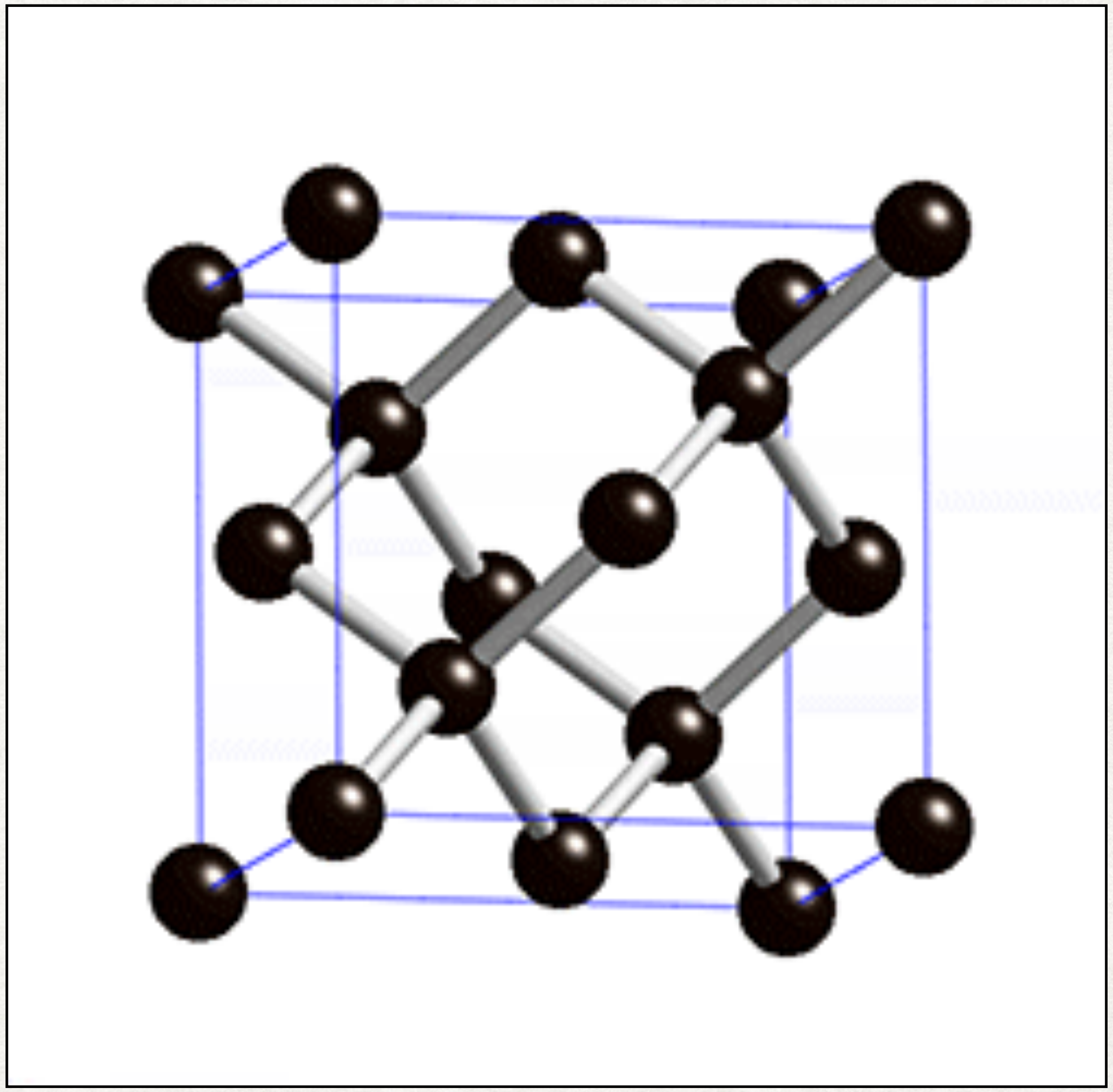
record total exposure time (2 seconds)
—> raw instrument flux is 2.5 counts/s

SILICON: CRYSTAL LATTICE STRUCTURE

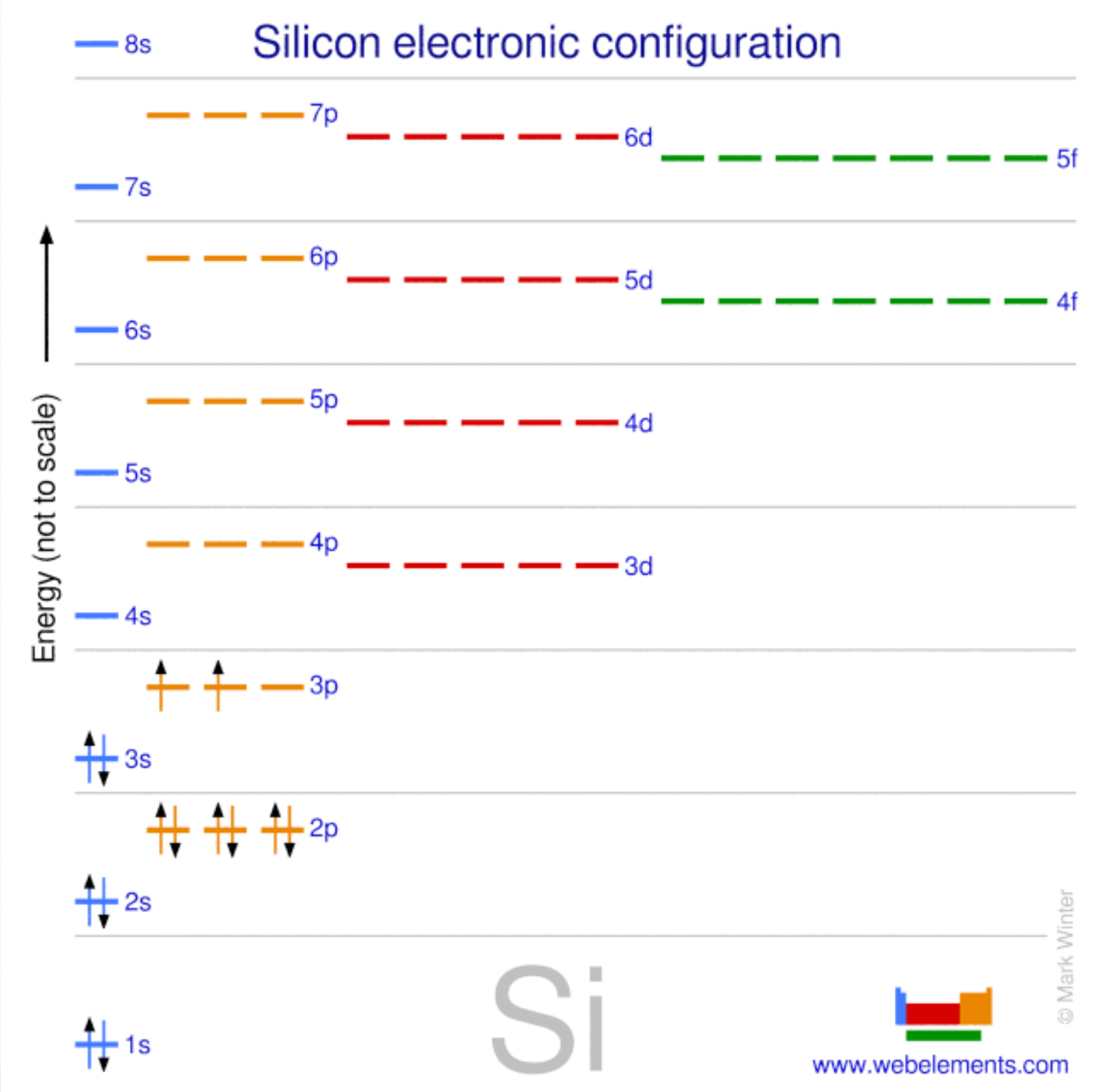
diamond cubic structure with a nearest neighbor interatomic spacing of 2.35 A (1 Angstrom= 1e-10 m),

for comparison, the atomic radius of Silicon is 1.11 A

What happens to the e-’s energy levels when we pack atoms so close together?



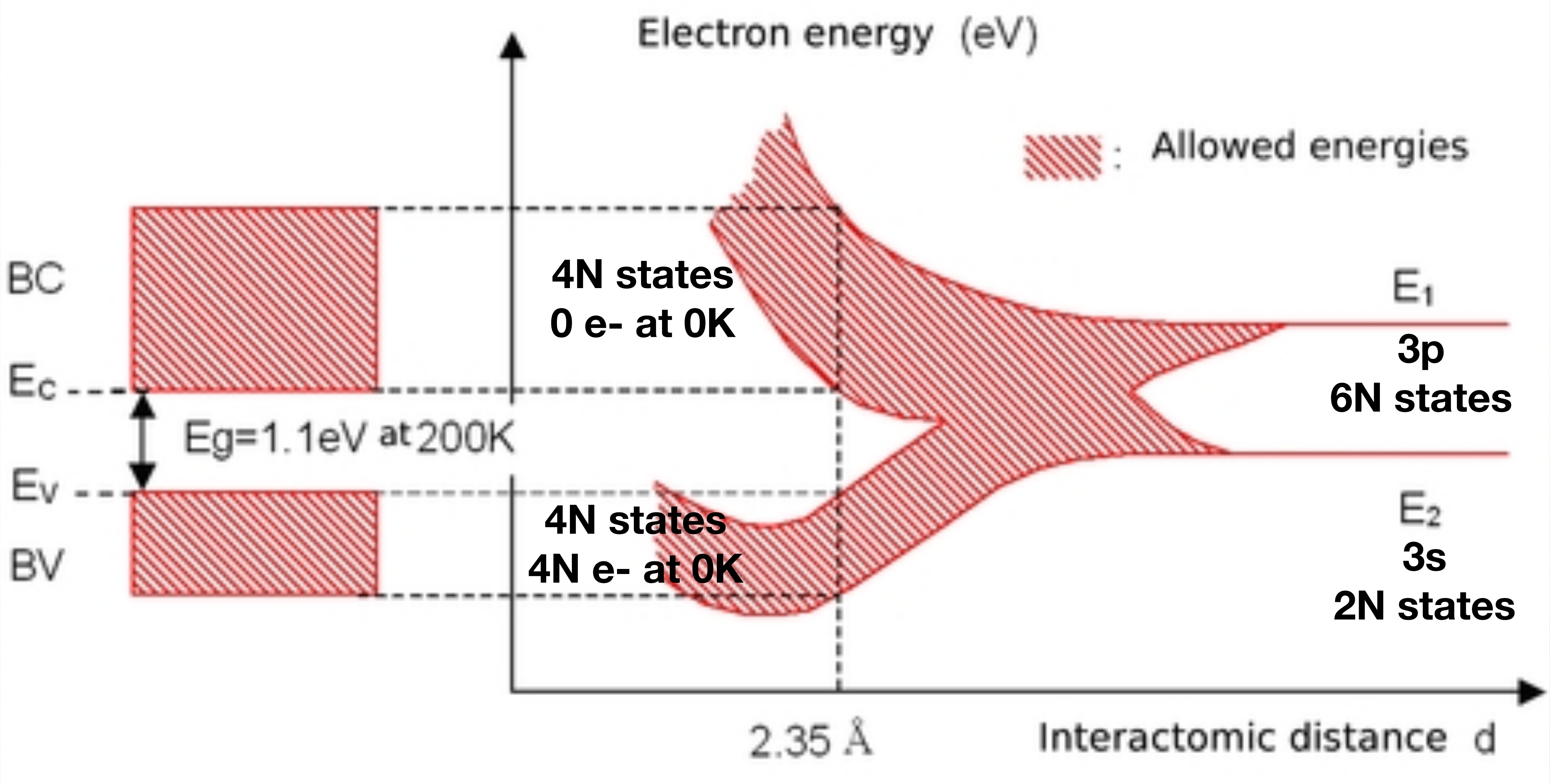
SILICON: ELECTRON ENERGY LEVELS



ENERGY GAP OF SILICON CRYSTALS

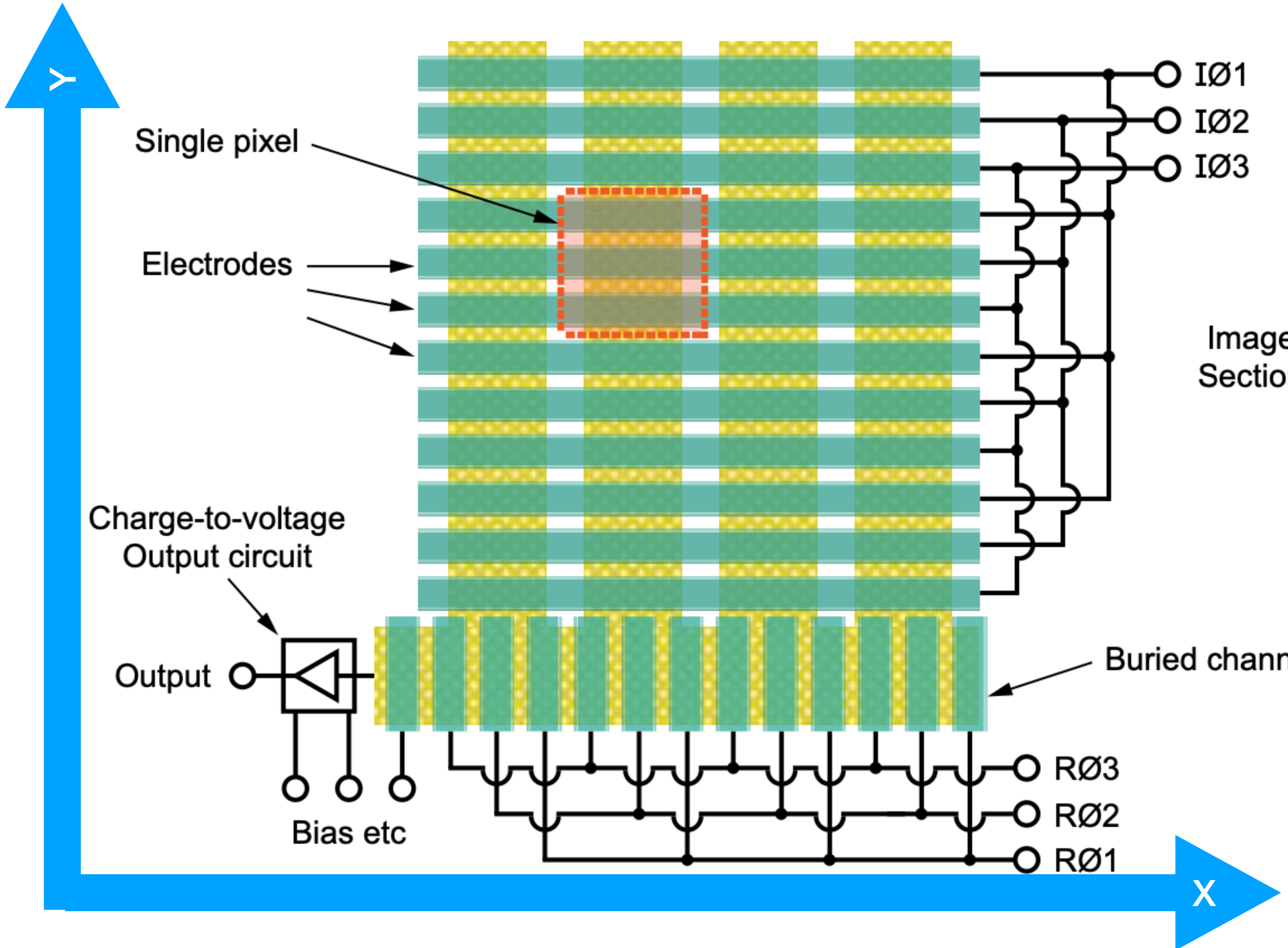
Si (Z=14): $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$ (electronic configuration)

At the actual interatomic spacing, silicon crystals develop an inaccessible energy band gap of 1.1 eV between a lower **valence band** and an upper **conduction band** **in the outermost n=3 shell**

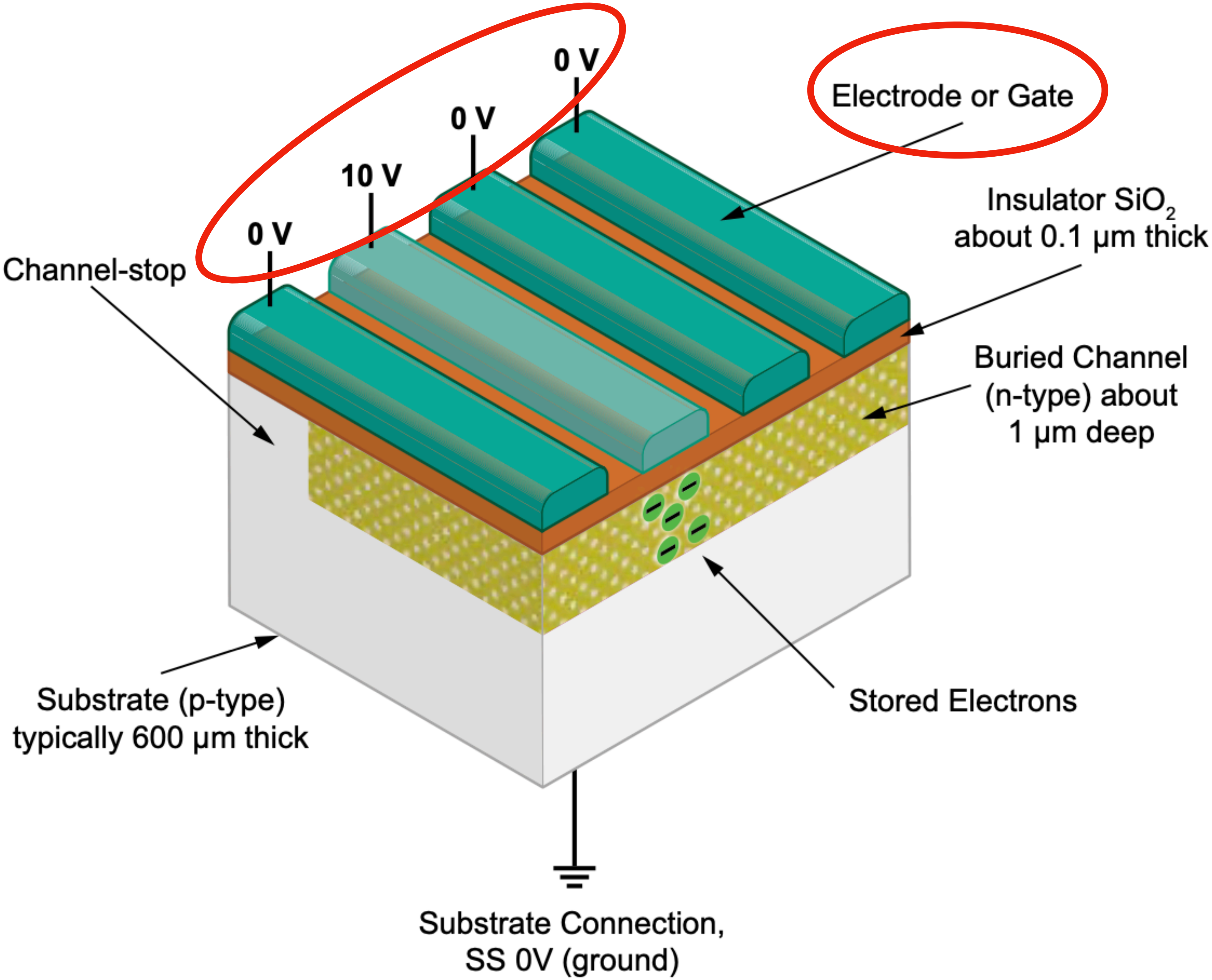


“Pixels” are constructed by channels and electrodes (gates)

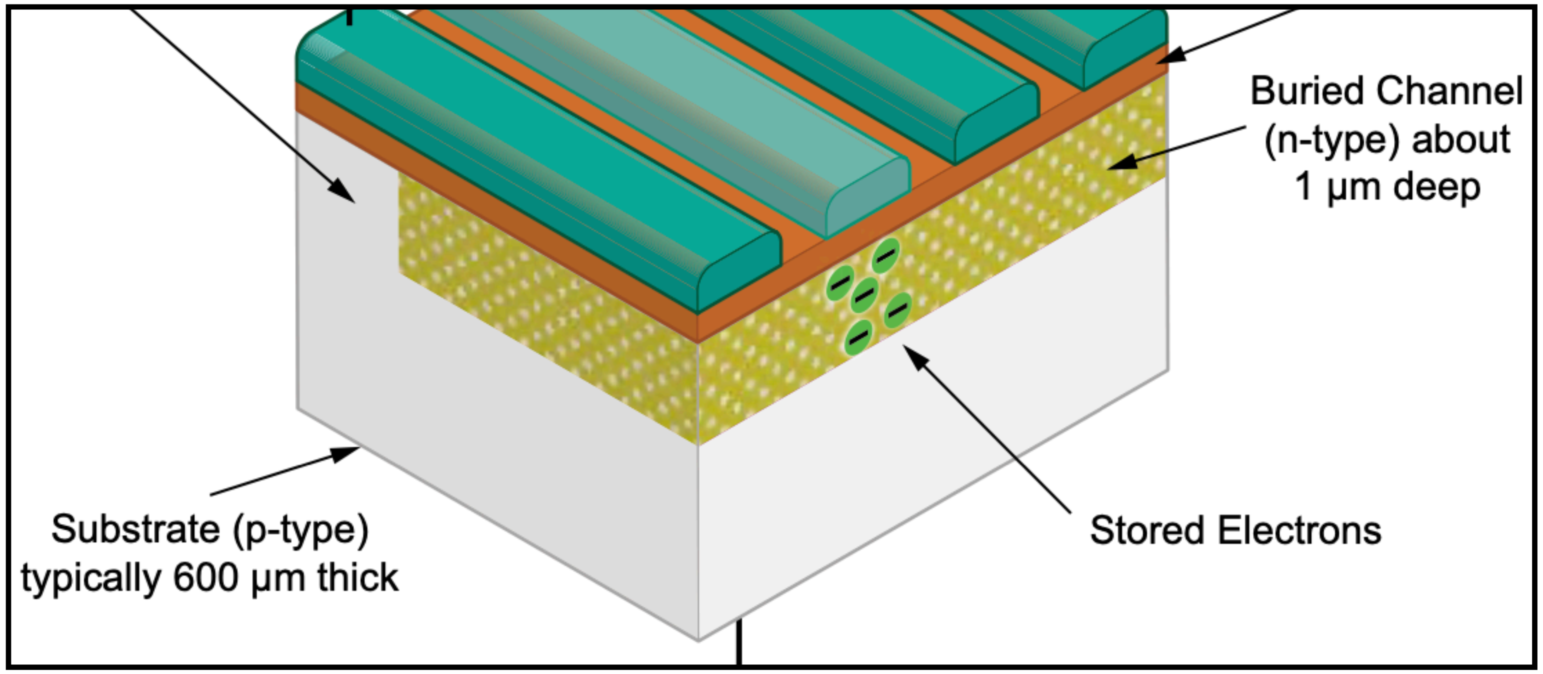
Photon absorption causes electrons in valence band to move to conduction band, our device needs to **hold** these electrons in a **bucket**



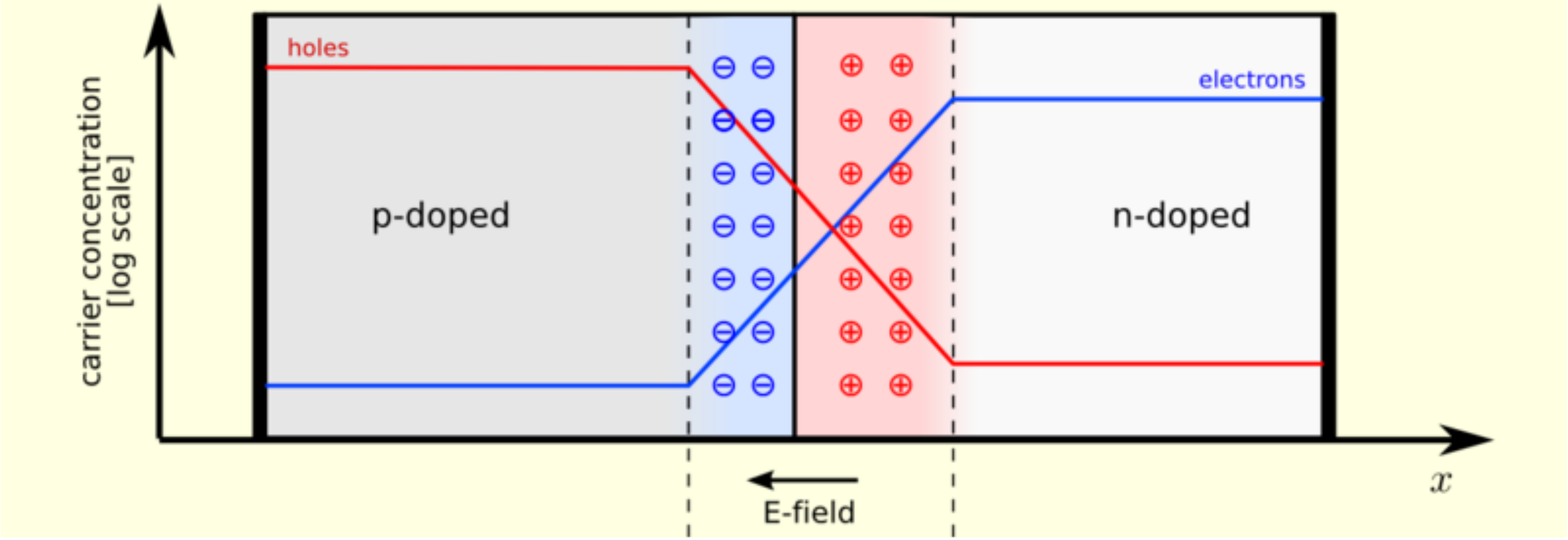
To keep electrons at a fixed Y-position, CCDs use electrodes



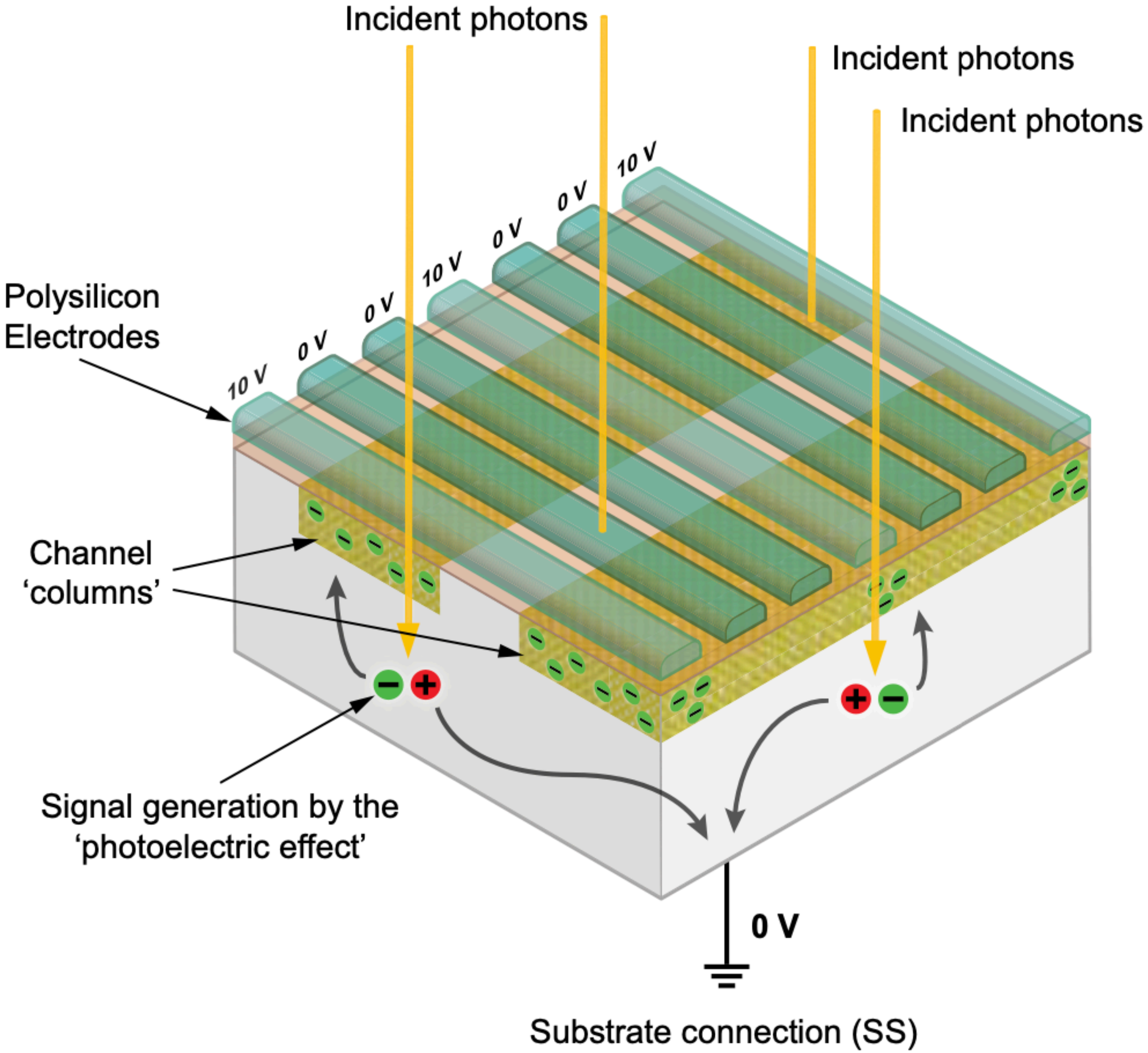
To keep electrons at a fixed X-position, CCD uses p-n interfaces



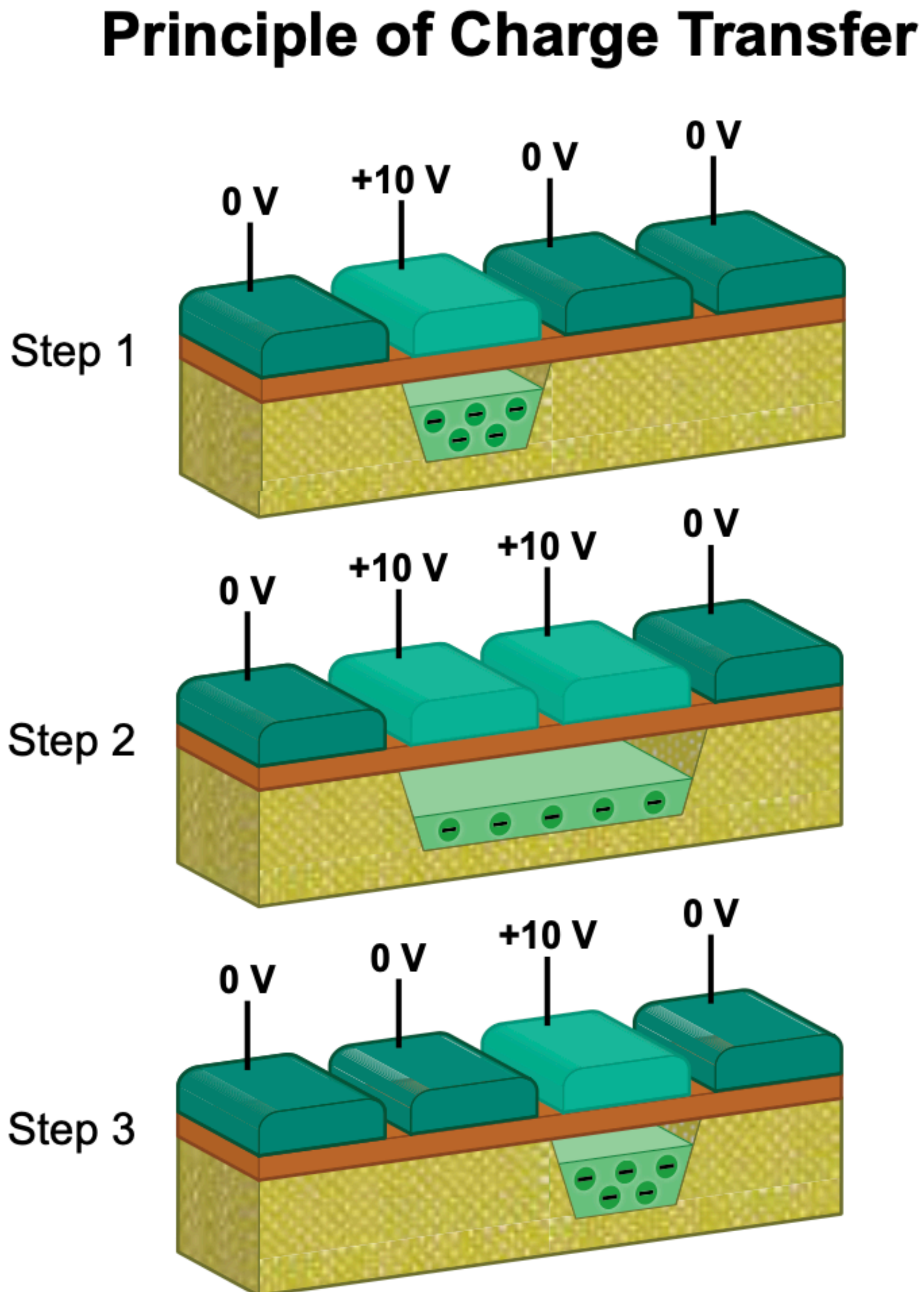
Thermal conduction-band electrons in **n-type** diffuse into **p-type** and combine with the holes in **p-type**; this diffusion forms an electric field near the interface, preventing future electrons in **n-type** from leaking into the **p-type** substrate

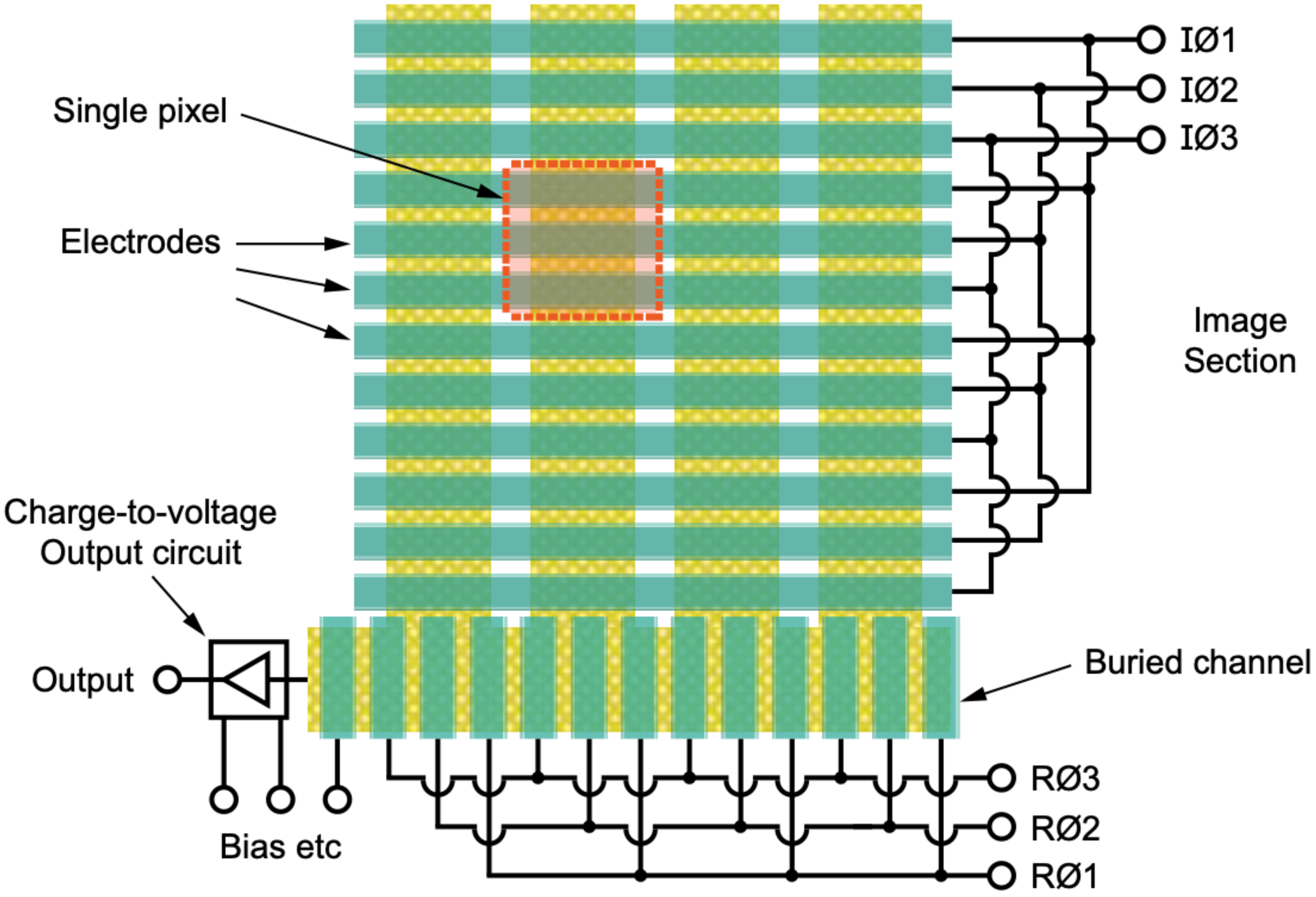


Section of a CCD Array with Electrodes and Channel 'Columns'

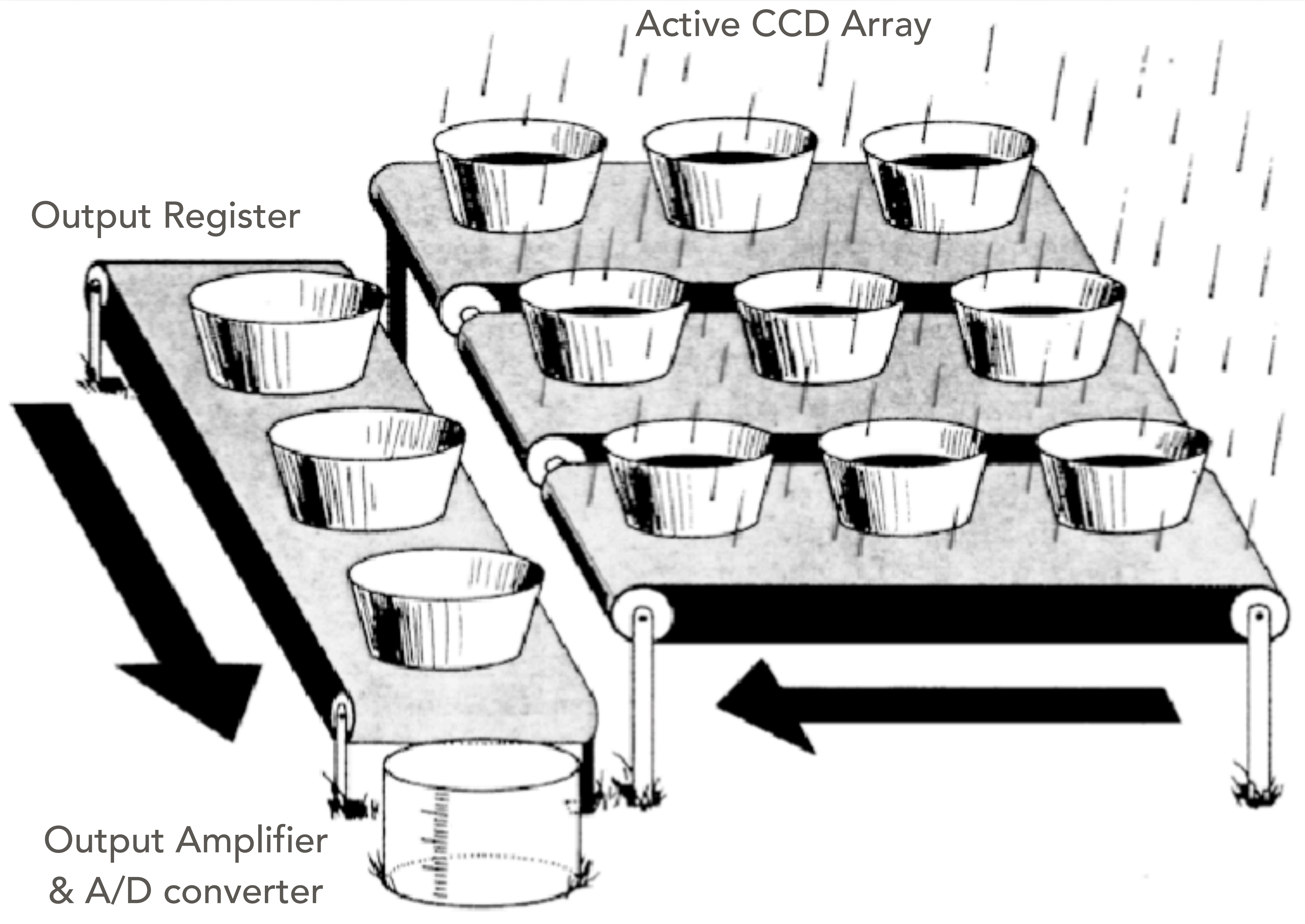


Charge transfer along the columns (Y-direction)

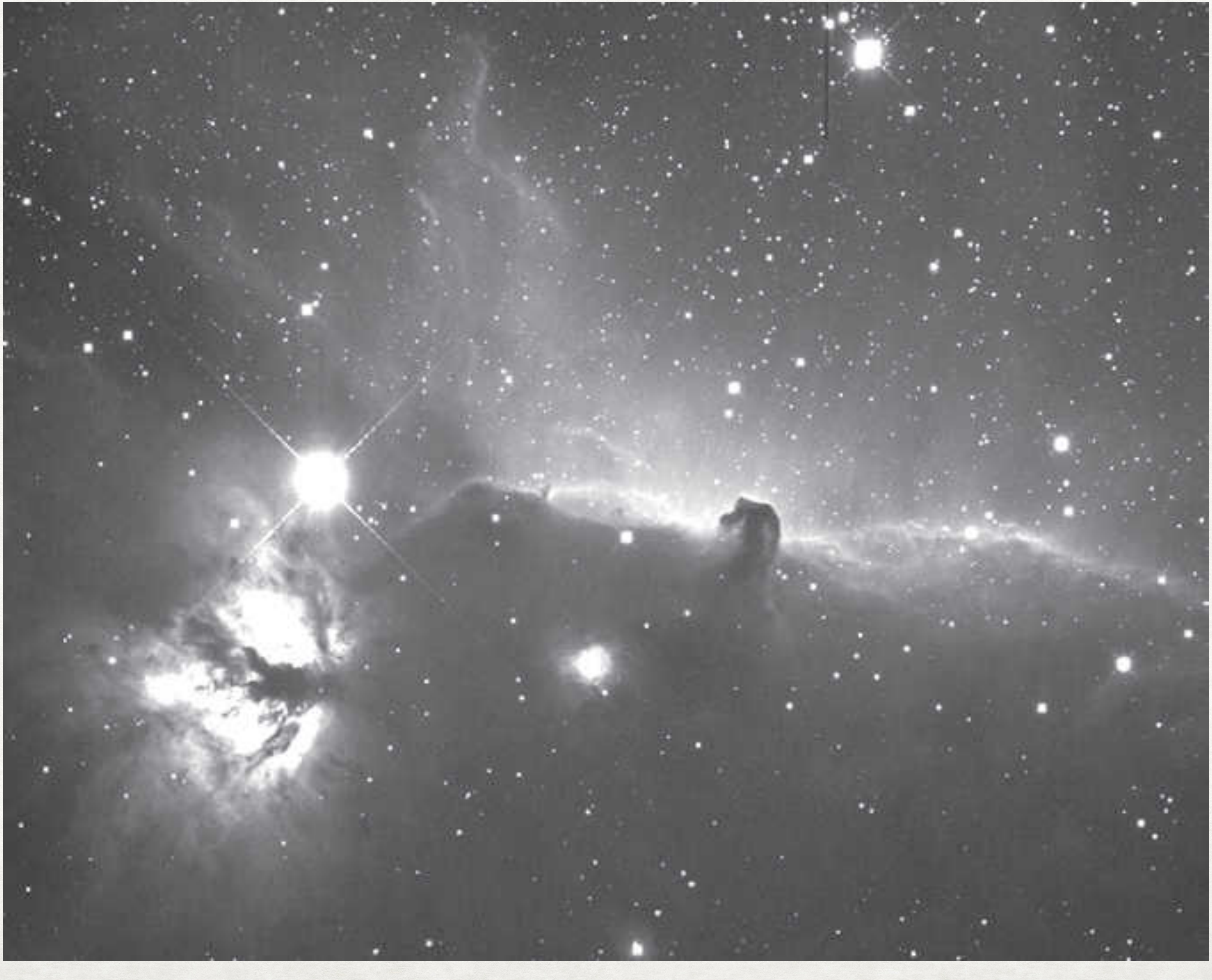




Transfer out electrons from all pixels in a data stream of a fixed order

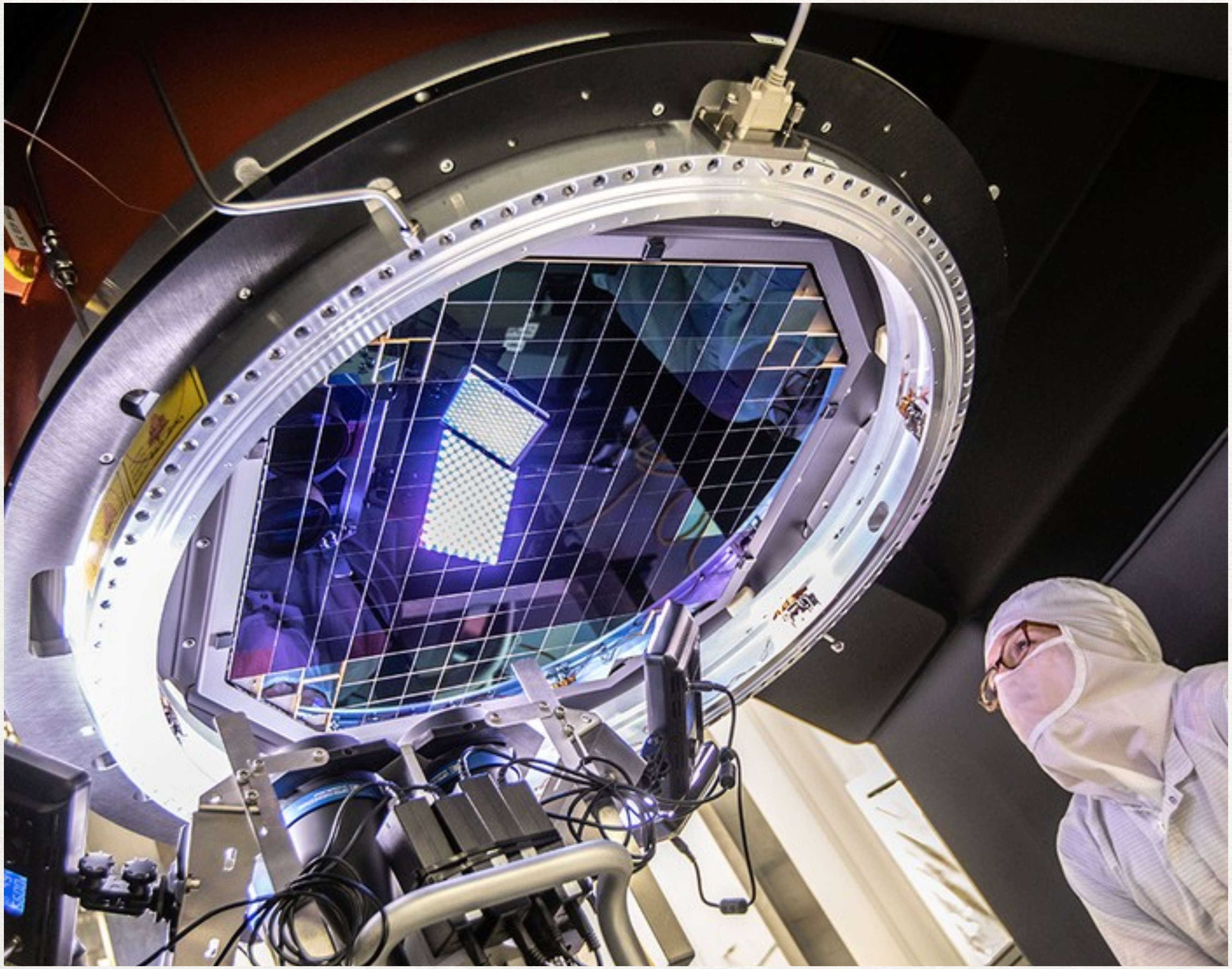


RECONSTRUCTED IMAGE (NOTE THAT THERE IS NO COLOR)



CHARGE COUPLED DEVICE (CCD): SEMICONDUCTOR LIGHT BUCKETS

The largest CCD camera today:
189 CCD detectors, each 16 megapixels
Rubin Observatory, 3.2-gigapixel camera



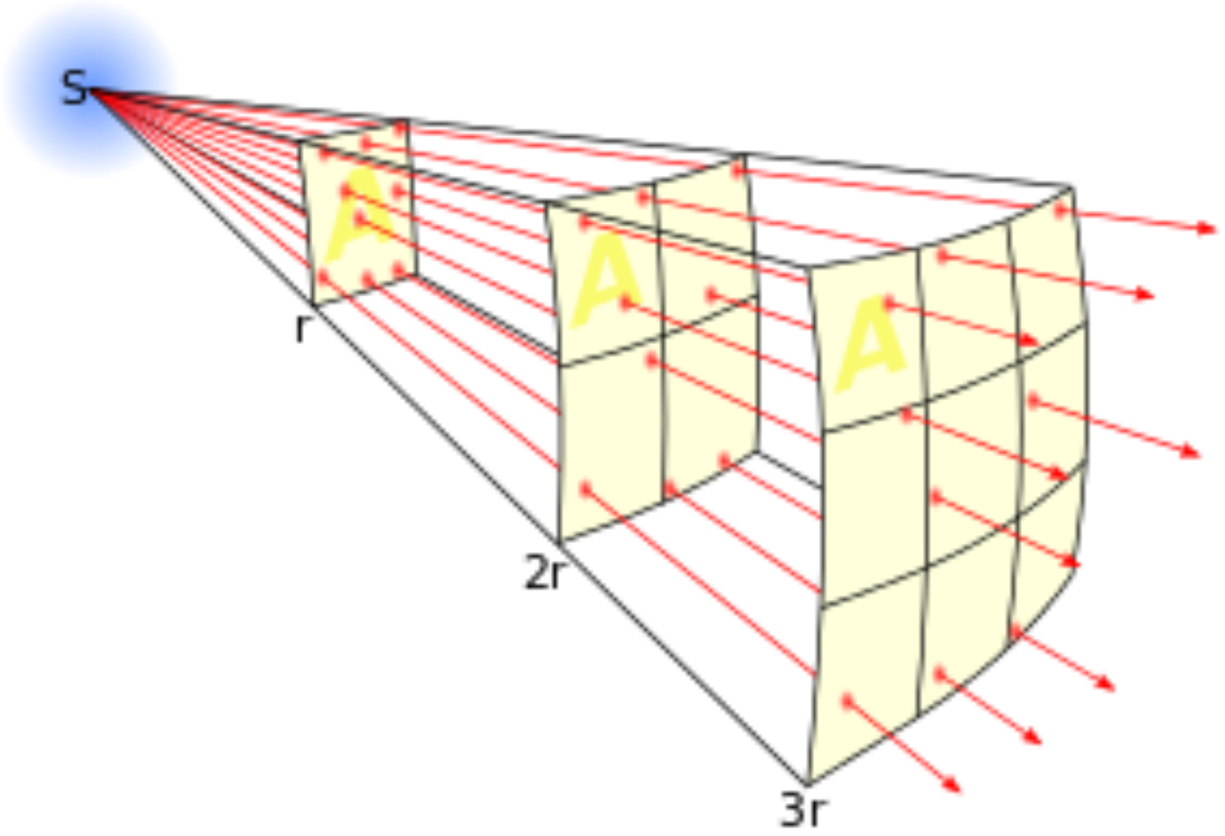
a single-crystal silicon
ingot grown by the
Czochralski method

Luminosity Measurements: Absolute Magnitude

(requires Distance & Brightness)

The Inverse Square Law of Flux

- **Luminosity** is the total amount of energy per unit time (i.e., power) emitted by the source (unit: Watt = Joule/s)
- **Flux** is the amount of arriving energy per unit time per unit area (unit: Watt/m²) at a distance *d* from source
- **Flux** decreases as the **distance** from the source increases, obeying an **inverse square law**:

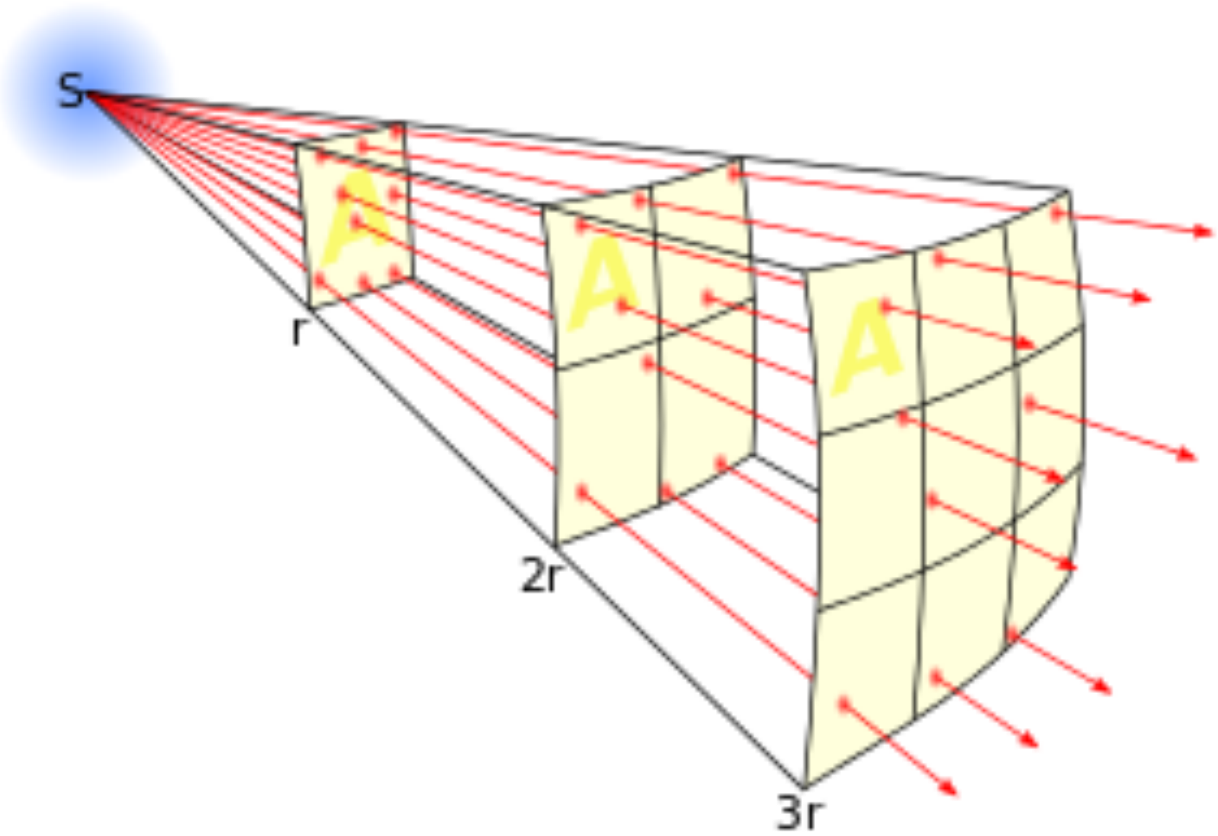


$$F = \frac{L}{4\pi d^2}$$

The Invariability of Luminosity

- **Luminosity** is the total amount of energy per unit time (i.e., power) emitted by the source (unit: Watt = Joule/s)
- **Flux** is the amount of arriving energy per unit time per unit area (unit: Watt/m^2) at a distance *d* from source
- **Flux** decreases as the **distance** from the source increases, obeying an **inverse square law,** which **preserves the luminosity**

$$L = F(d_1)4\pi d_1^2 = F(d_2)4\pi d_2^2$$



Absolute Magnitude — Apparent Magnitude — Distance Relation

- **apparent magnitude (m)** is the magnitude of the source at its actual distance (d)
- **absolute magnitude (M)** is defined as the apparent magnitude of the source if it were at a distance of 10 parsec
- because both are measurements of the same source, we can express the same luminosity (L) using its actual flux (f) and its presumed flux (F) at 10 parsec:

$$L_\lambda = 4\pi d^2 f_\lambda = 4\pi(10\ \text{parsec})^2 F_\lambda \quad \Rightarrow \frac{F_\lambda}{f_\lambda} = \frac{d^2}{(10\ \text{parsec})^2}$$

$$m_\lambda - m_{\lambda,0} = -2.5\log(f_\lambda / f_{\lambda,0})$$

$$M_\lambda - m_{\lambda,0} = -2.5\log(F_\lambda / f_{\lambda,0})$$

$$m_\lambda - M_\lambda = 2.5\log\left(\frac{d}{10\ \text{parsec}}\right)^2 = 5\ [\log\frac{d}{1\ \text{parsec}} - 1]$$

This, **m-M**, is called the **distance modulus**, because it only depends on distance

Practice: What's the absolute magnitude of the Sun?

- distance = 1 AU, V-band magnitude = -26.74
- What's its absolute magnitude in V-band?

M = -26.74-5*(log(1/206265)-1)
= 4.83

$$m_\lambda - M_\lambda = 5\ [\log d(\text{parsec}) - 1]$$

$$\Rightarrow M_\lambda = m_\lambda - 5\ [\log d(\text{parsec}) - 1]$$

Practice: Calculate absolute magnitude from p and m

- Suppose you measured a star's apparent magnitude in V-band (550 nm) to be $m_V = 10.5$
- You also measured its parallax to be $p = 5$ *mas* (milli-arcsec).
- What's its distance in parsec?

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p}\right)$$

- What's its absolute magnitude in V-band (M_V)?

$$m_\lambda - M_\lambda = 5\ [\log d(\text{parsec}) - 1]$$

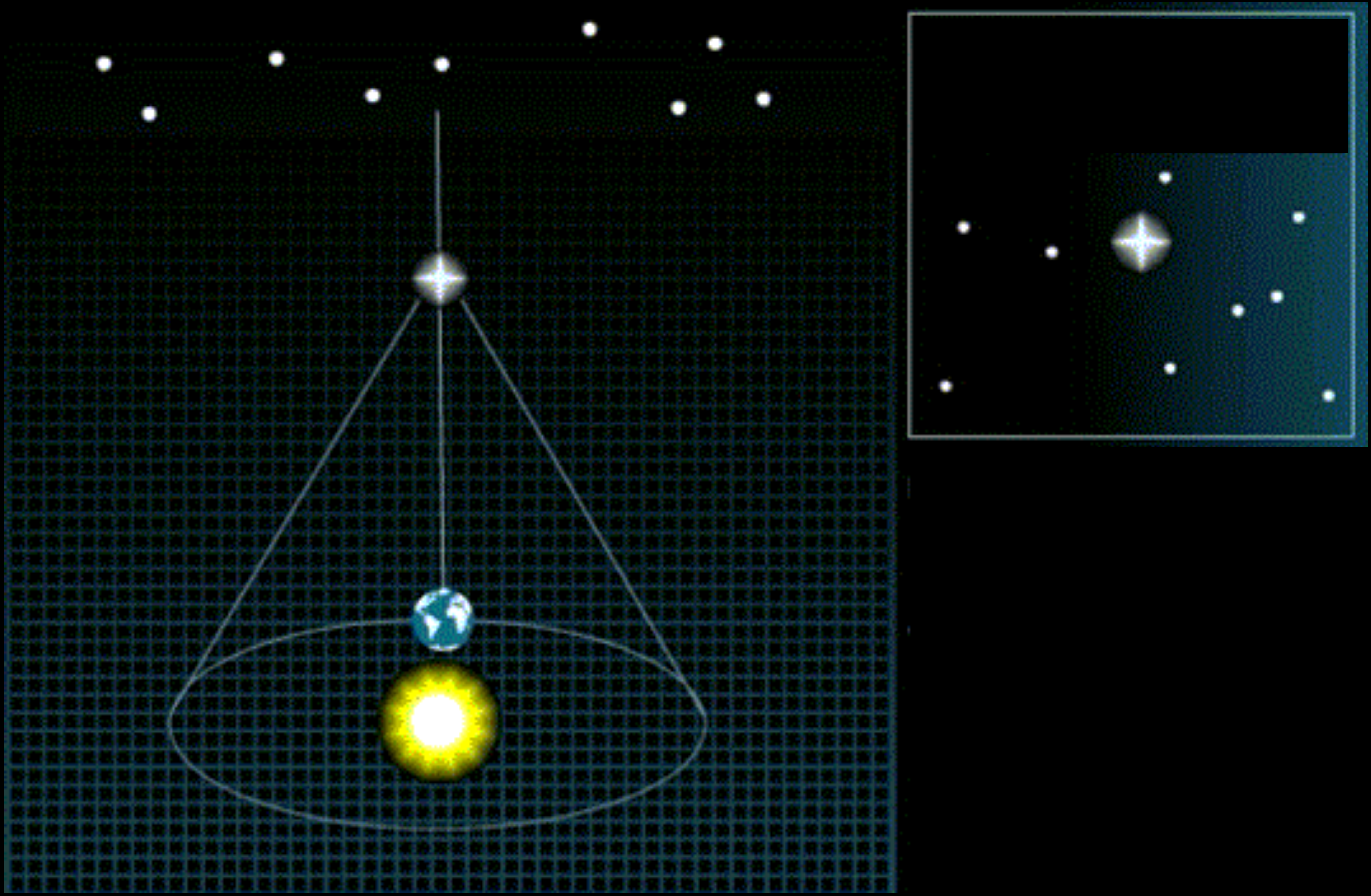
$$\Rightarrow M_\lambda = m_\lambda - 5\ [\log d(\text{parsec}) - 1]$$

d = 200 parsec
M = 10.5 - 5 * (log(200) - 1) = 4.0

Distances beyond Parallaxes:

The Standard Candle Methods

Extending the baseline with the orbit of the Earth

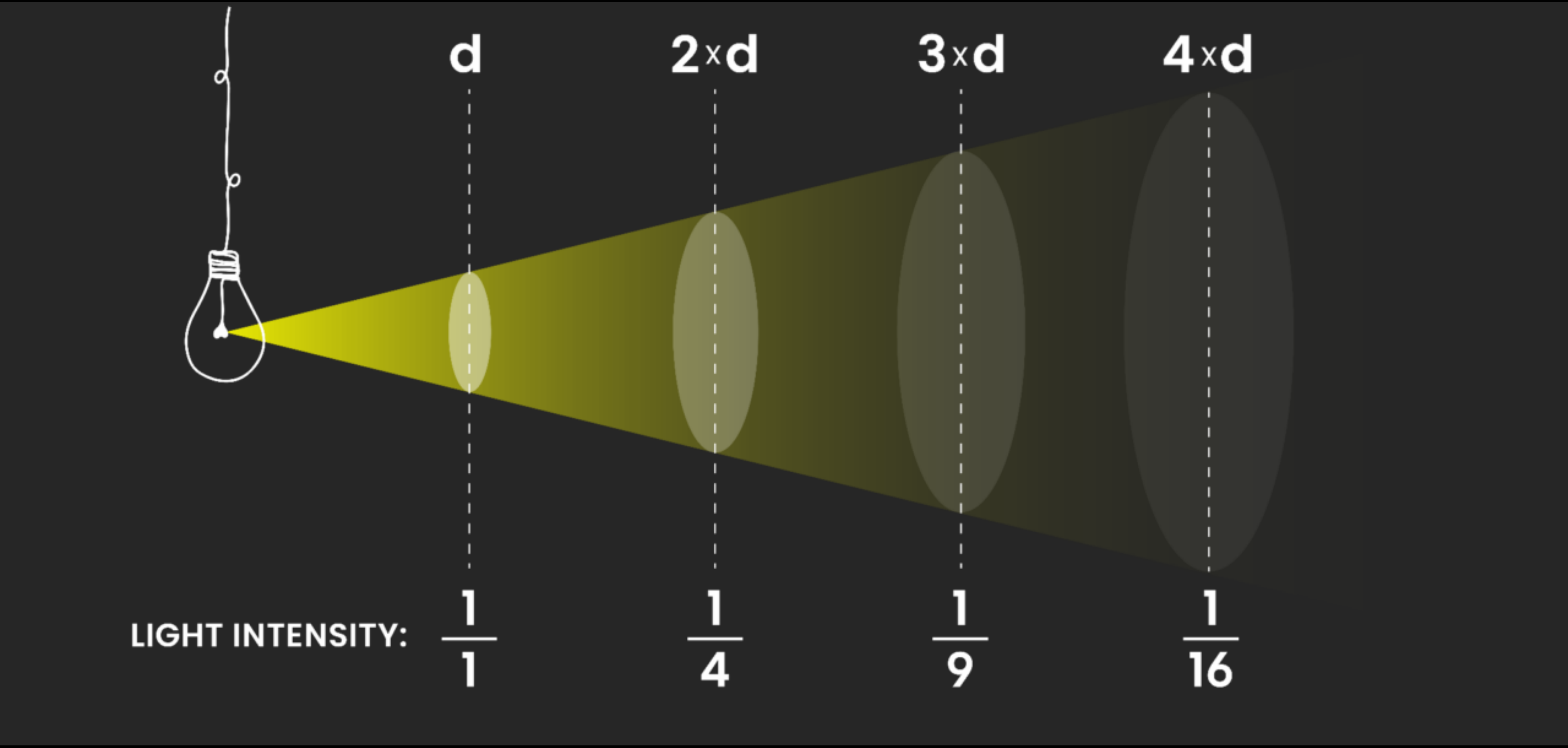


Weten Schaps

Beyond certain distance (~10 kpc or 0.1 mas), we can no longer measure parallaxes …

WHAT ABOUT OBJECTS TOO FAR TO MEASURE PARALLAX?

$$\text{brightness} = \text{luminosity} / \text{distance}^2$$



Understanding the Standard Candle Method

- Suppose you used a **laser distance measure** to measure a close lamp and got a laser distance of 40 yards.
- You can't use the laser to measure a far lamp, because it has a **range limit** of 50 yards. But based on a digital image, you found that the far lamp is **9 times fainter** than the close one, what's your conclusion of their relative distances?
- Given the laser distance to the close lamp (40 yards), how many yards are you from the far lamp?

Key Assumptions of the Standard Candle Method

- All lamps have the **same luminosity** (e.g., 100 Watts)
- The inverse distance square law is valid:

$$\text{brightness} = \text{luminosity} / \text{distance}^2$$

  the line of sight is clear: **not foggy, not dusty**

- The distances to the closest lamps can be measured directly. The **cross-calibration** allows estimation of absolute distances.

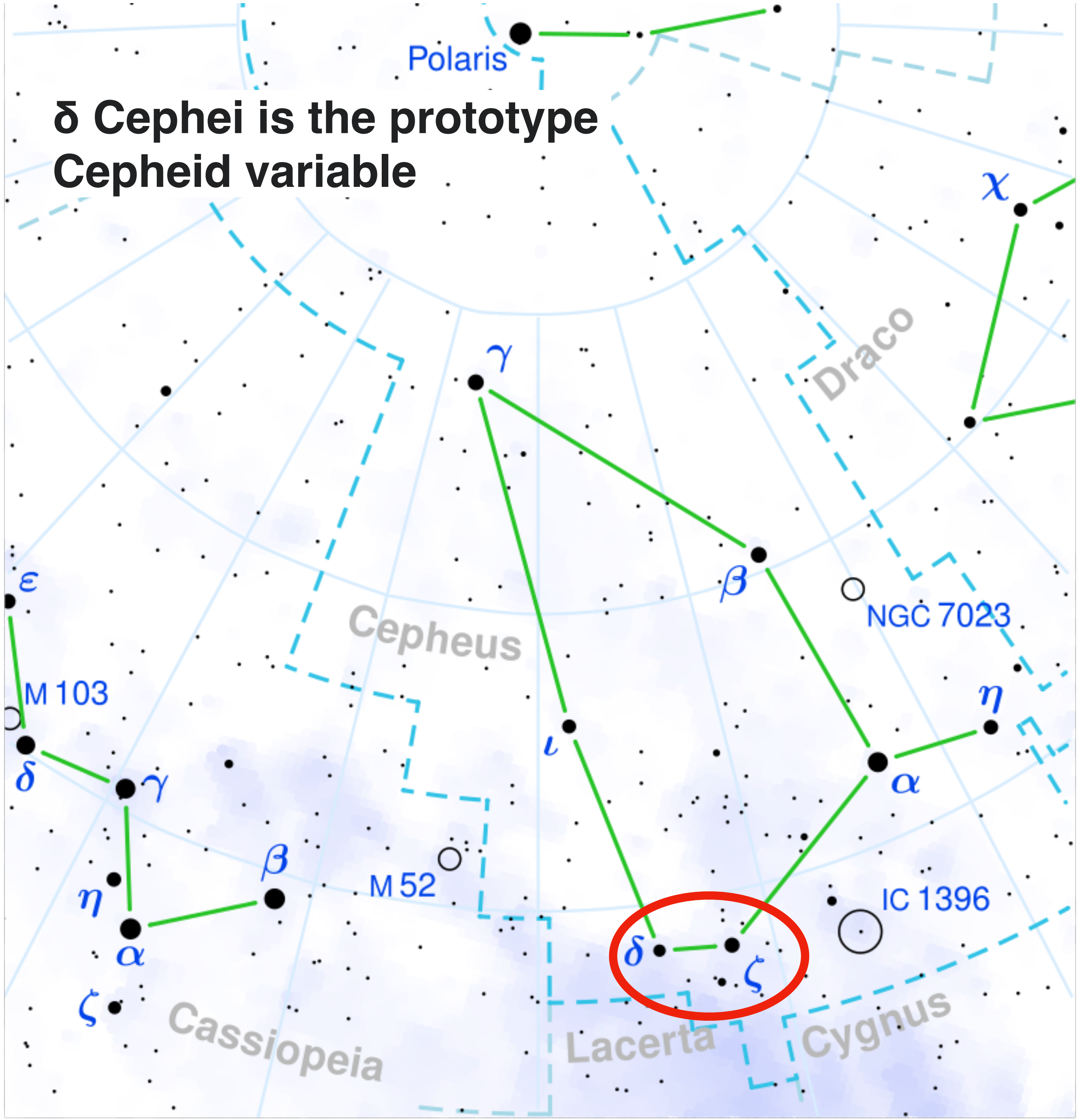
The Standard Candle Methods: Distance Modulus

- If we could infer the **absolute magnitude** of a class of astrophysical objects, we can get the **distance modulus (m-M)** from its **apparent magnitude**. The **distance modulus** and **distance** are related directly:

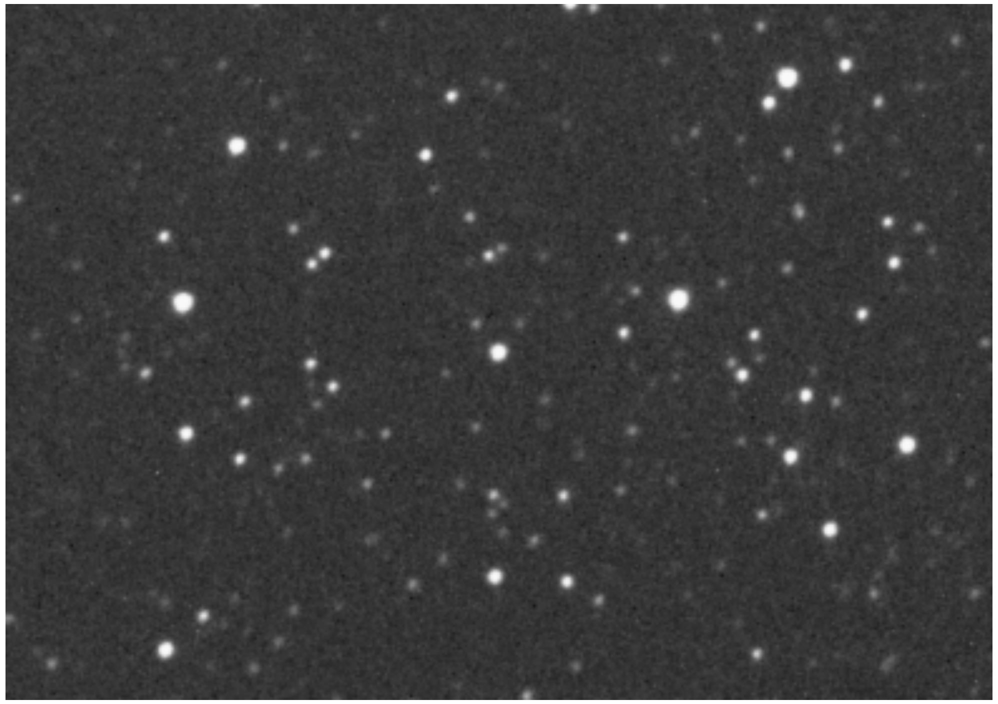
$$m - M = 5 \log d_{\mathrm{pc}} - 5 \Rightarrow d_{\mathrm{pc}} = 10^{1+0.2(m-M)}$$

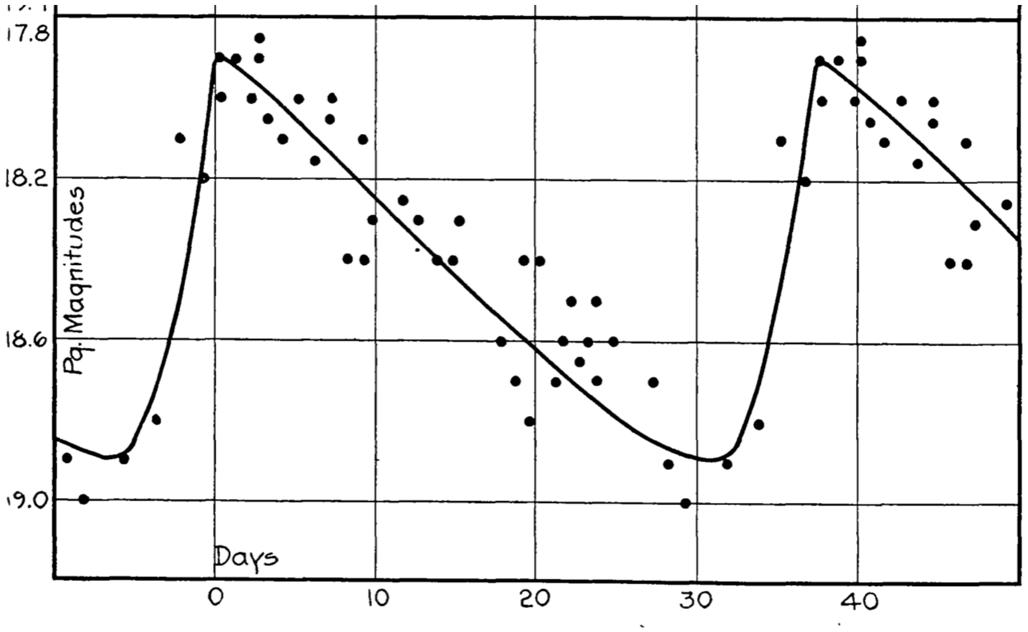
The Standard Candle Method 1 — Cepheids Variables

- All lamps have roughly the same luminosity: **Cepheids with the same pulsation period have roughly the same luminosity!**
- They are typically over **3500x** more luminous than our Sun!



δ Cephei is the prototype Cepheid variable

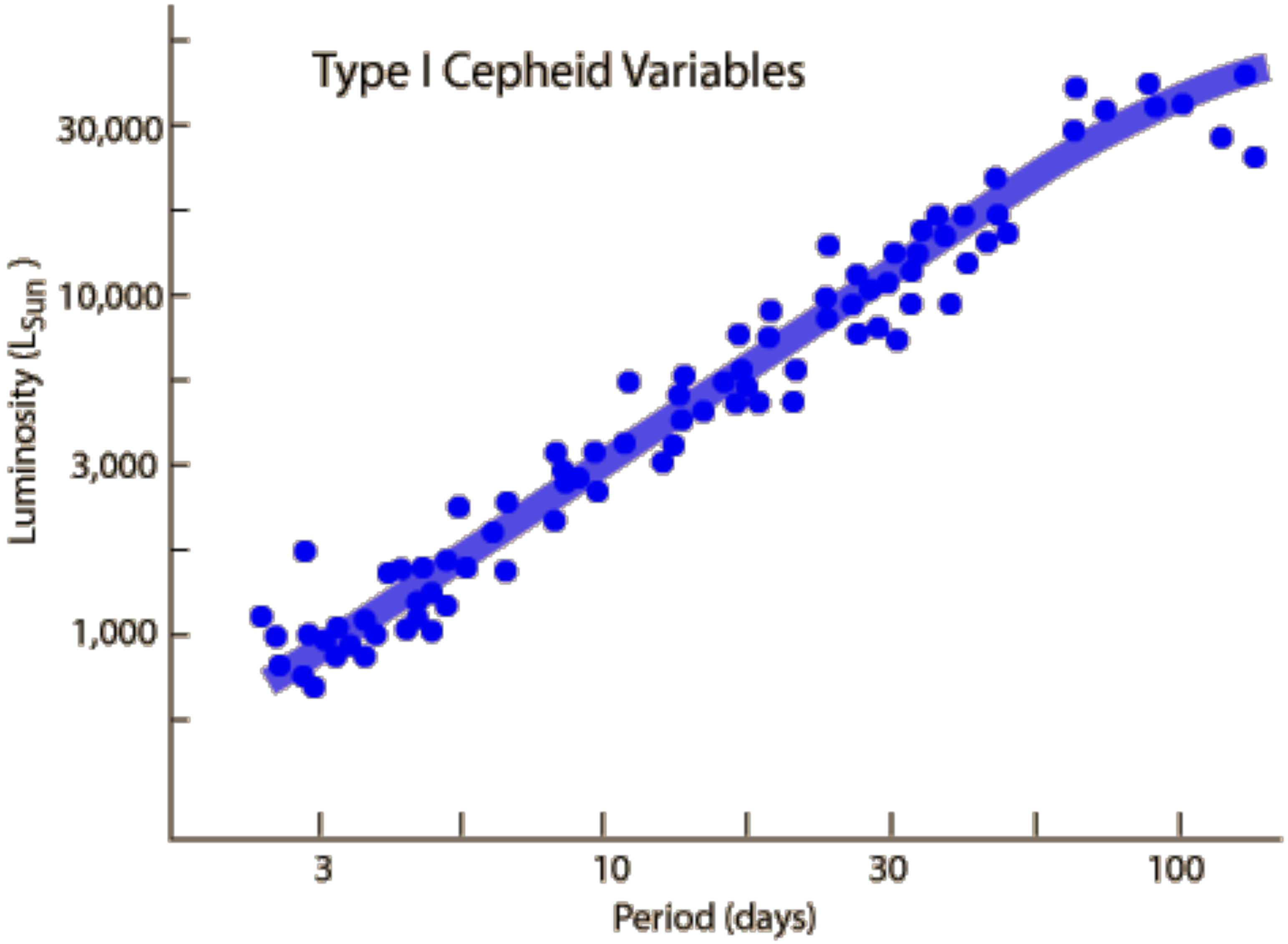


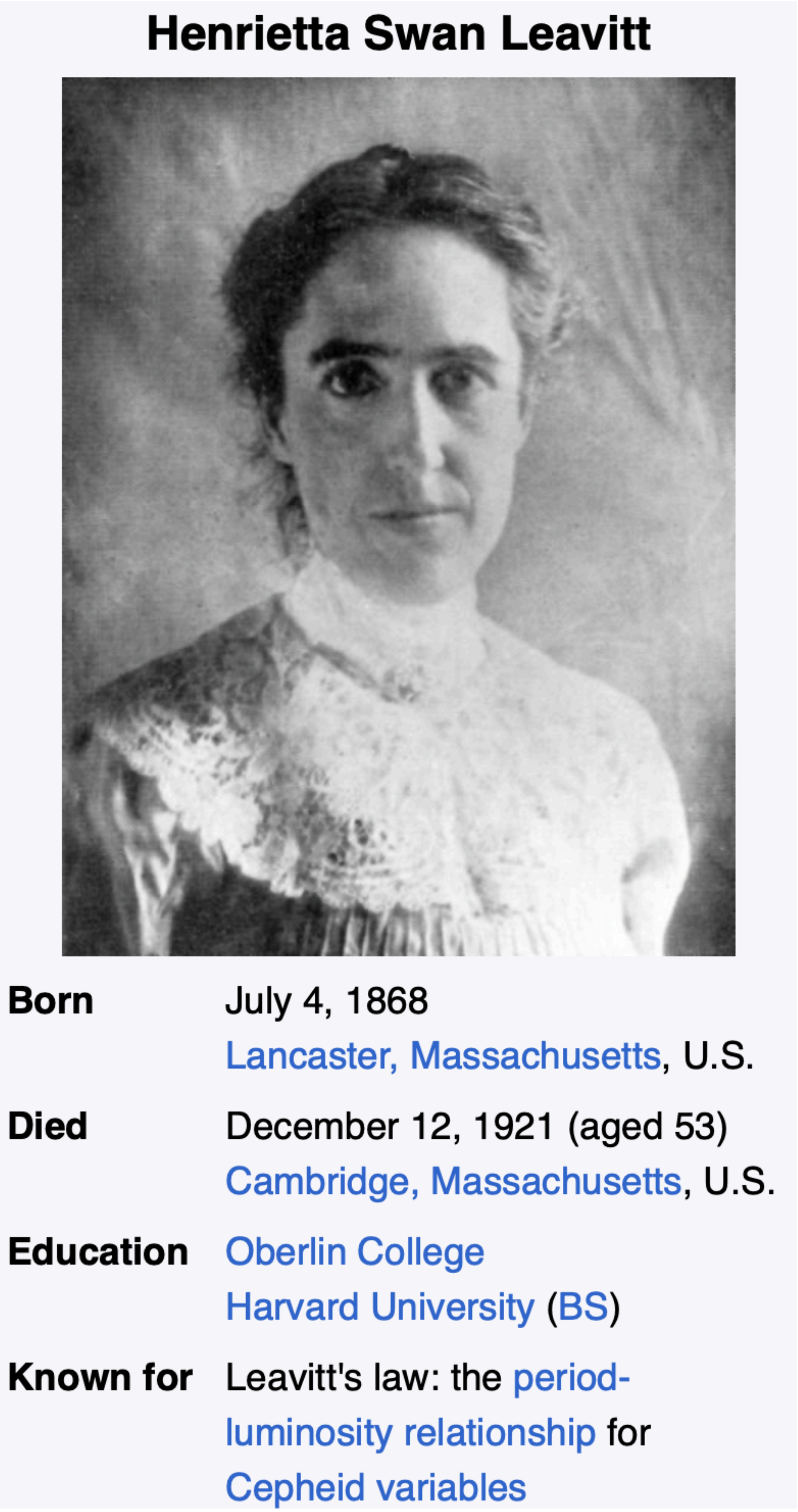


The Standard Candle — Cepheids' Period-Luminosity Relation

- $M = \alpha \log(P_{\text{day}}) + \beta$

  where *M* is the **absolute magnitude** (defined as the apparent magnitude at a distance of 10 parsec), and P the **period** of pulsation.





Henrietta Swan Leavitt

Born	July 4, 1868 Lancaster, Massachusetts, U.S.
Died	December 12, 1921 (aged 53) Cambridge, Massachusetts, U.S.
Education	Oberlin College Harvard University (BS)
Known for	Leavitt's law: the period-luminosity relationship for Cepheid variables

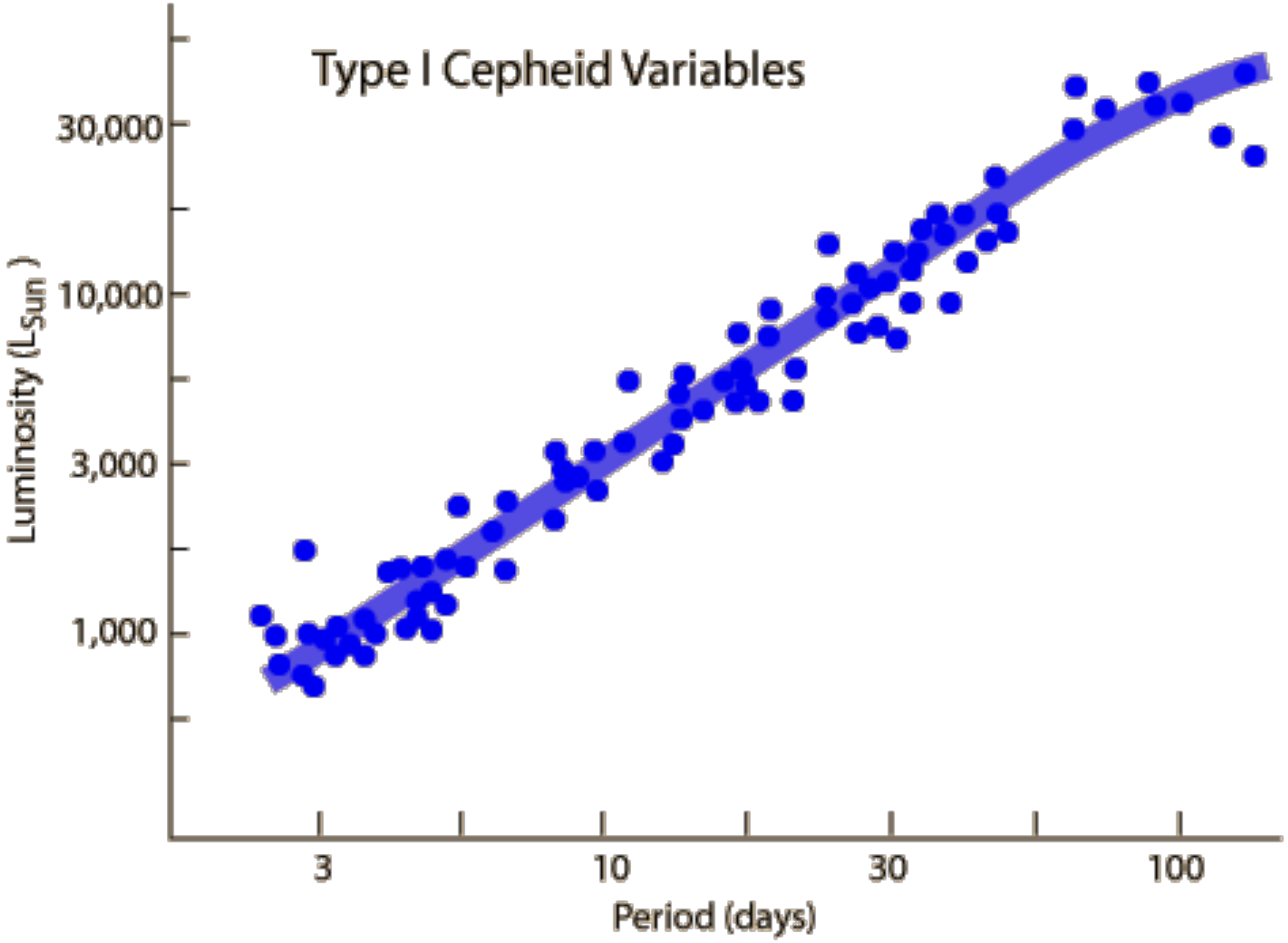
To use Cepheids to measure distances, their Period-Luminosity relation must be calibrated using Geometric Distances from parallaxes and other techniques

- Example calibrated P-L relation (Fritz et al. 2007):

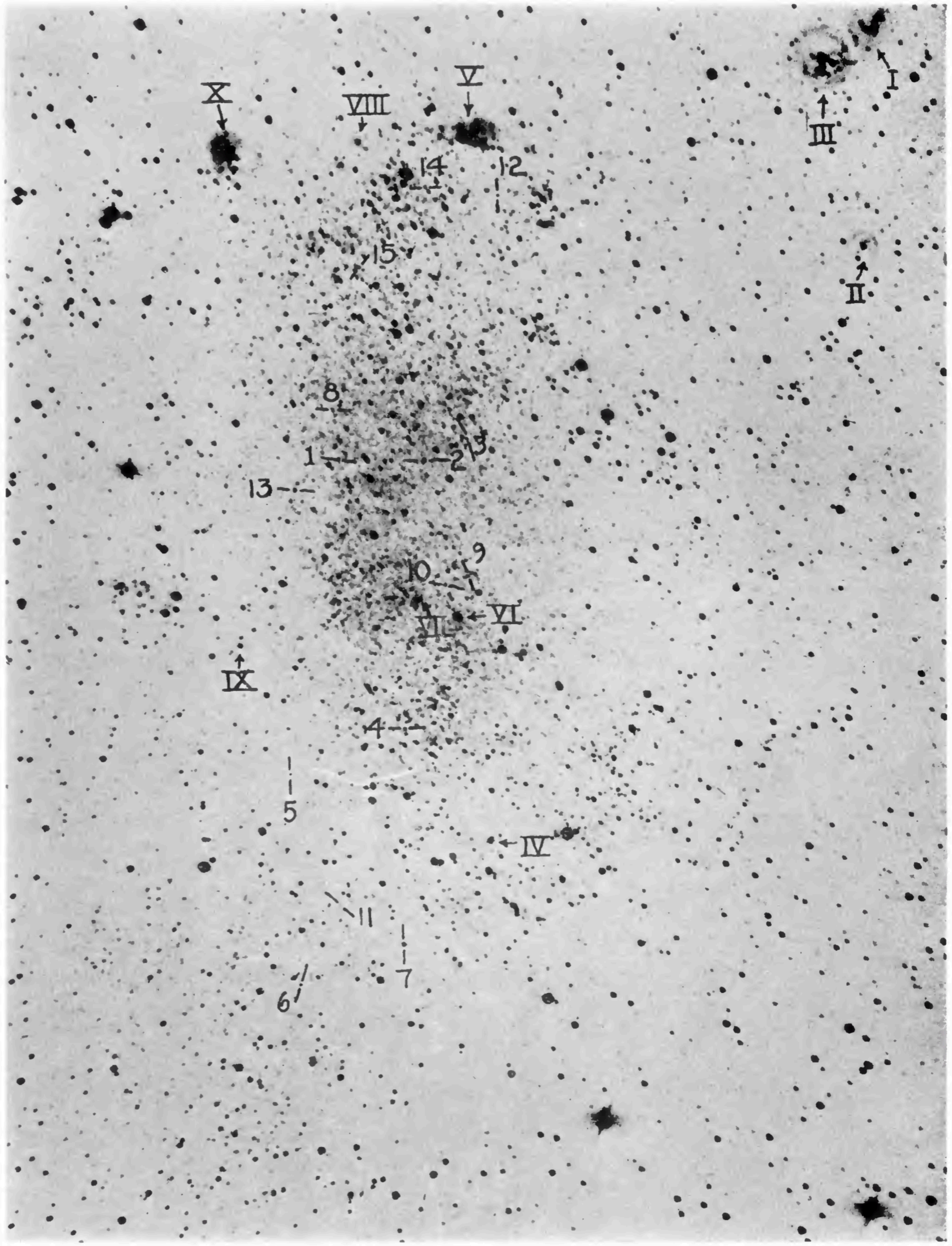
$$M_V = -2.43\log(P_{\rm day}) - 1.62\ (-6 \lesssim M_V \lesssim -3,\ M_{\odot,V} = 4.83)$$

- Calibration critical for determining distances to other galaxies*:*

$$5\log d_{\rm pc} - 5 = m_V - M_V(P)$$



The debate was resolved by Hubble's observations of NGC 6822



1925ApJ....62..409H

N.G.C. 6822

Negative print of Plate XIV. Variable stars are designated by Arabic figures; nebulae involved in or superposed on the cluster by Roman numerals.

Hubble (1925)

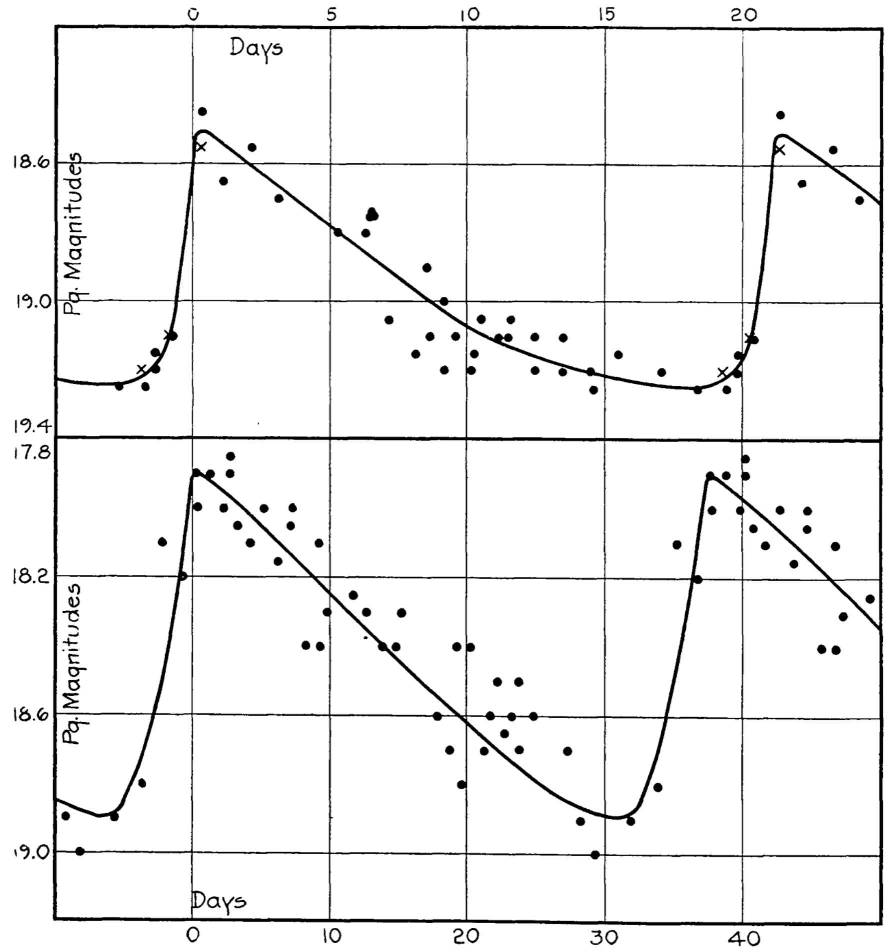


FIG. 1.—Light curves for two Cepheids in N.G.C. 6822. Upper curve, variable No. 6. Period 21.06 days; range 18.5–19.25. Lower curve, variable No. 2. Period 37.45 days; range 17.9–18.9. The three crosses on the rising slope of the upper curve represent observations on successive days and illustrate the rapid brightening of the variables.

$$m - M = 21.65 .$$

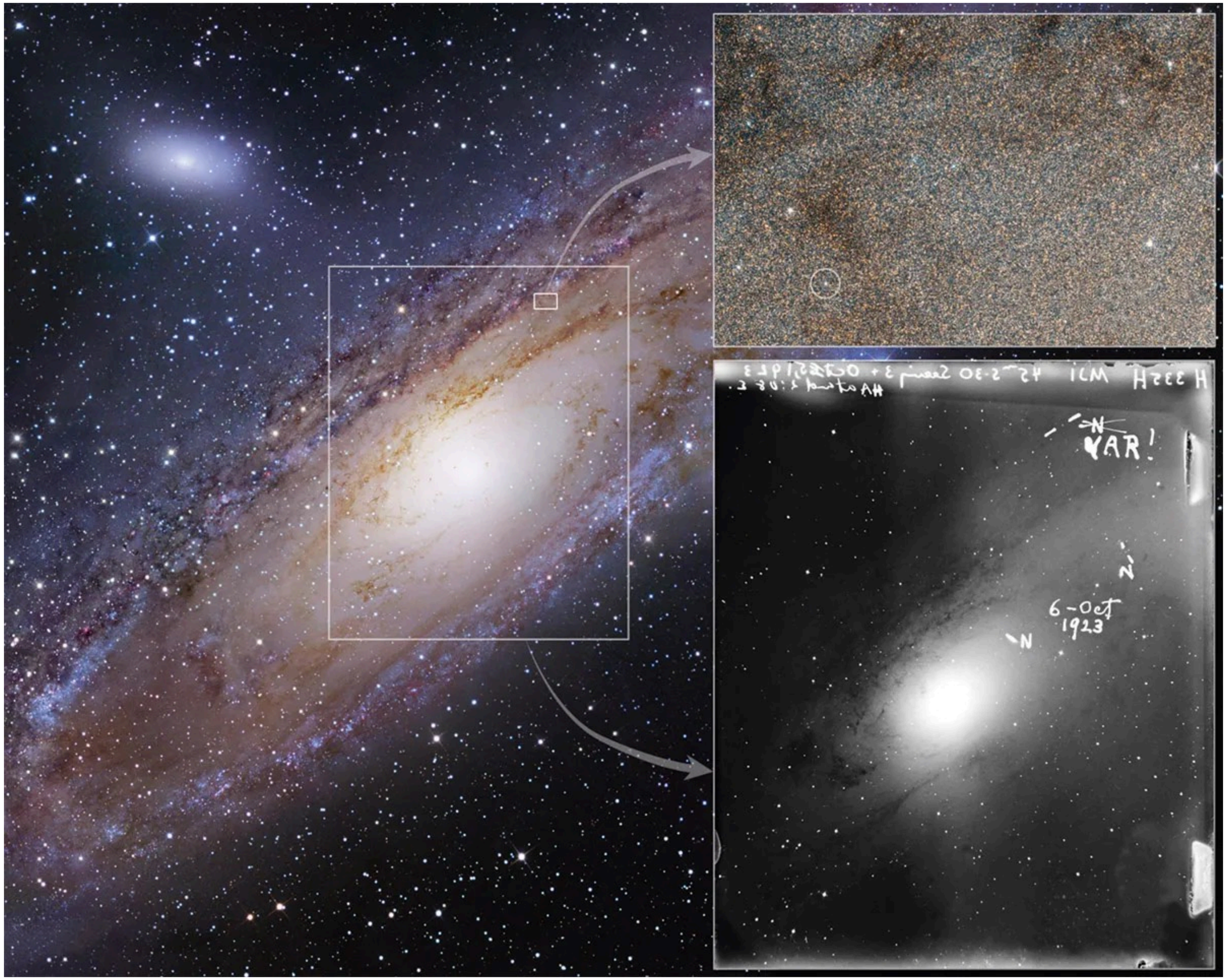
$$\pi = 0\overset{''}{.}00000468$$

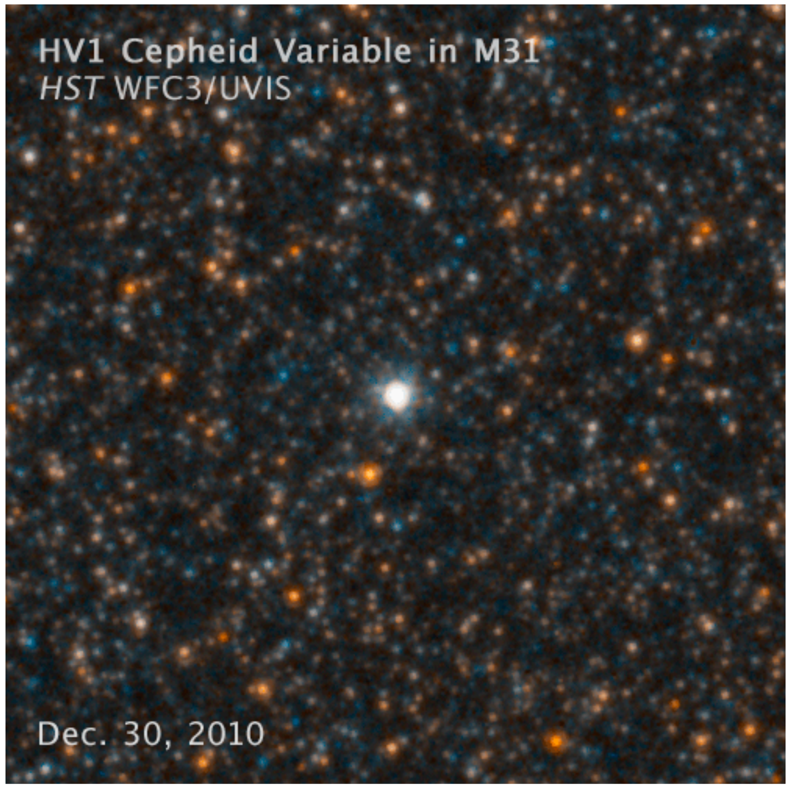
Distance = 214,000 parsecs

= 700,000 light-years

Hubble Discovered Cepheids in the Andromeda Galaxy

- **Hubble (1929)**: *"A Spiral Nebula as a Stellar System, Messier 31"*
- Hubble discovered **Cepheid variables** inside of the Andromeda (**M31**) and estimated the distance to M31 that is much greater than the size of the Milky Way, concluding "**spiral nebulae" are galaxies outside of the Galaxy.**

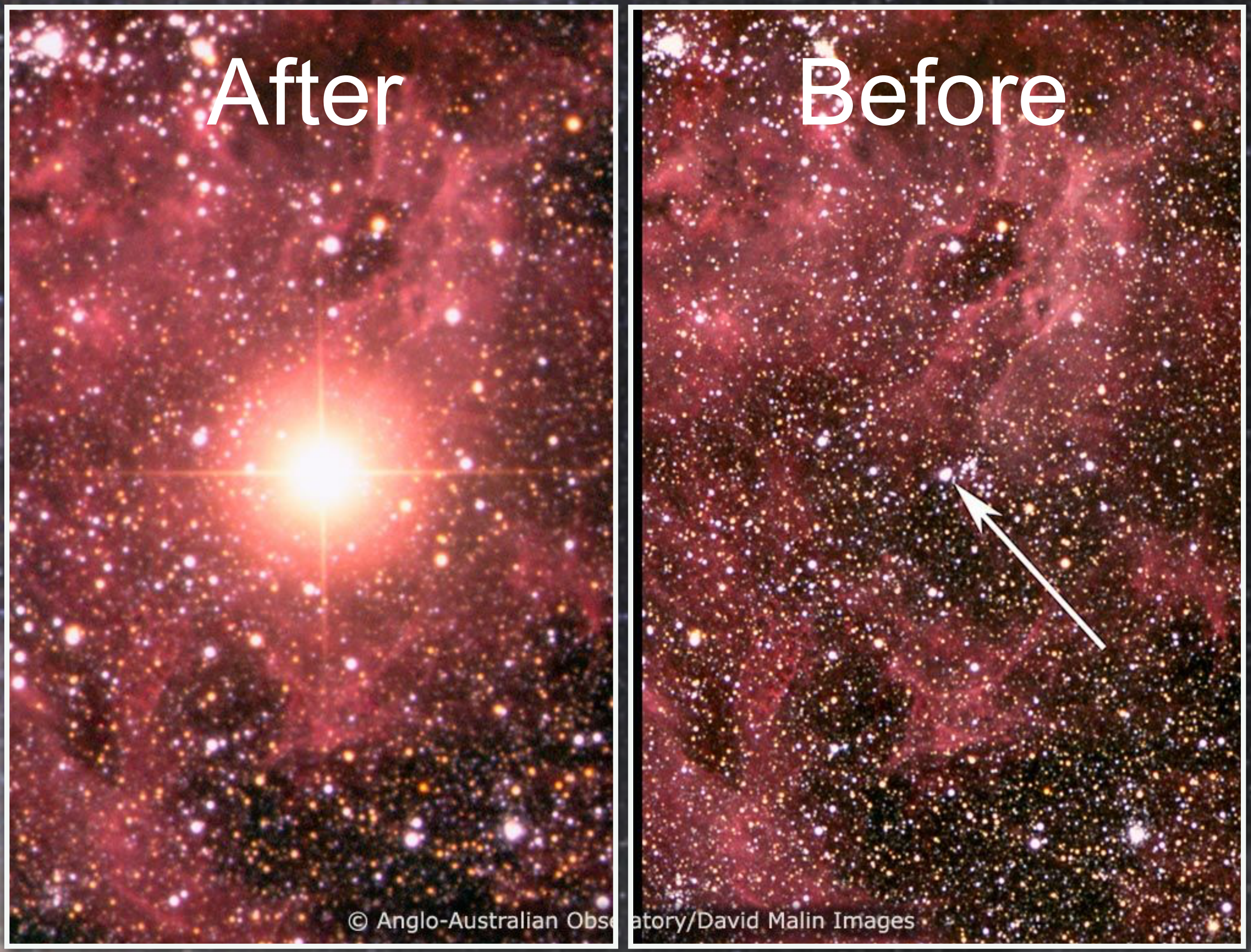




The Distance Ladder

from parallax to supernovae

SN 1987A



© Anglo-Australian Obs atory/David Malin Images

LMC

d = 50 kpc

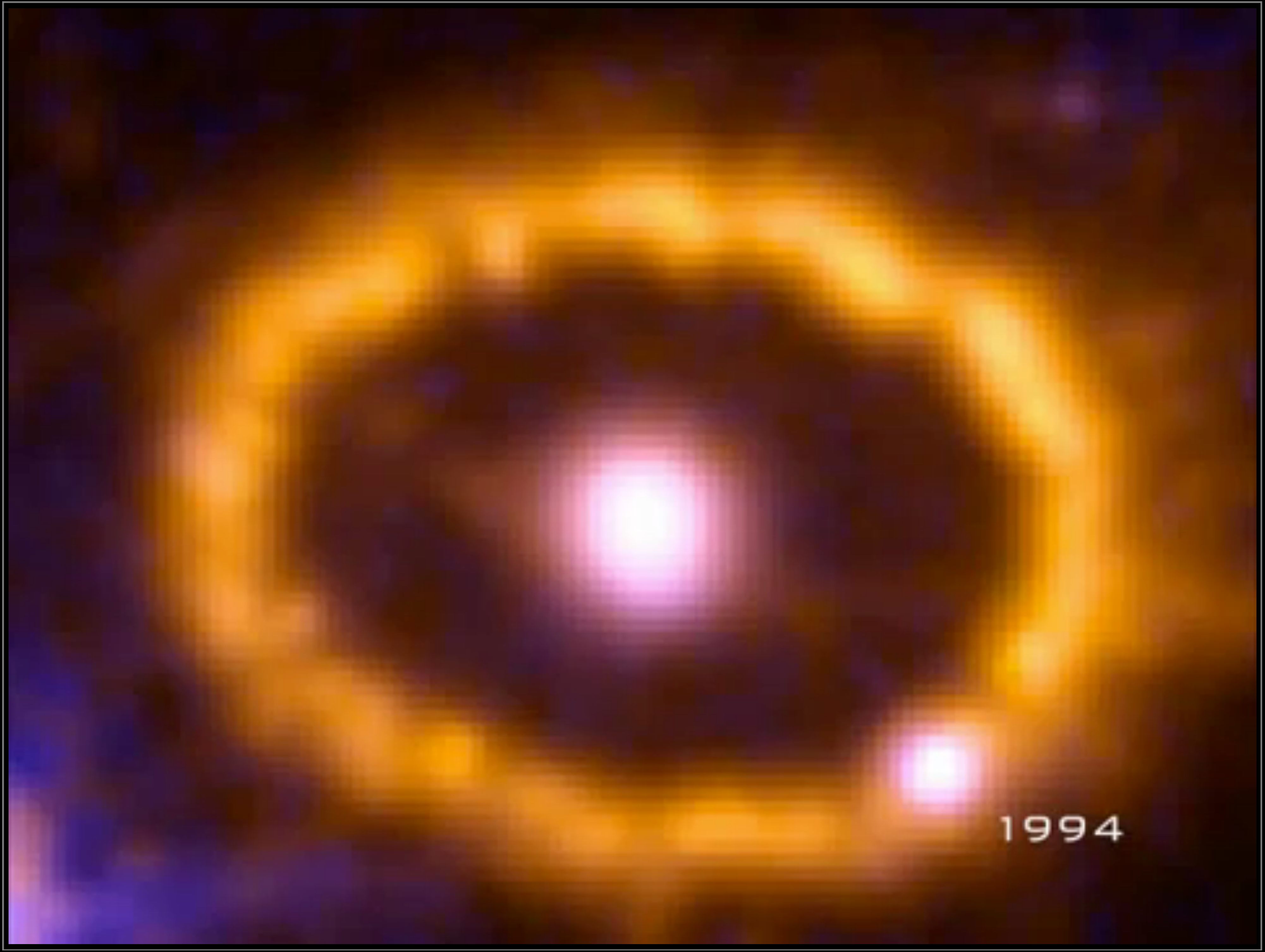
Dec = -70d

$M^* = 1e10\ M_{sun}$

Zoom in onto SN 1987A

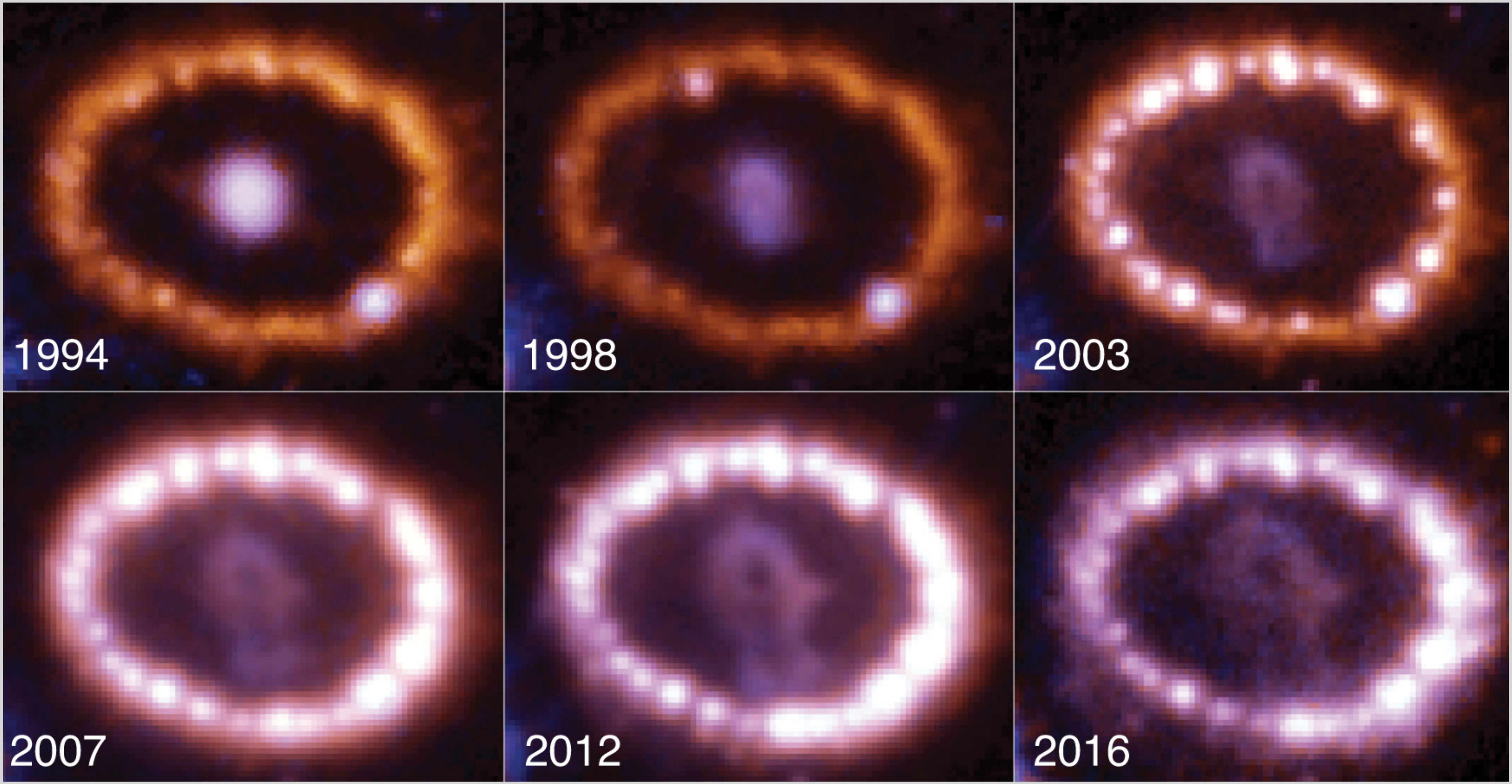
The firework after the explosion

SN ejecta catching up with the mass loss from stellar winds



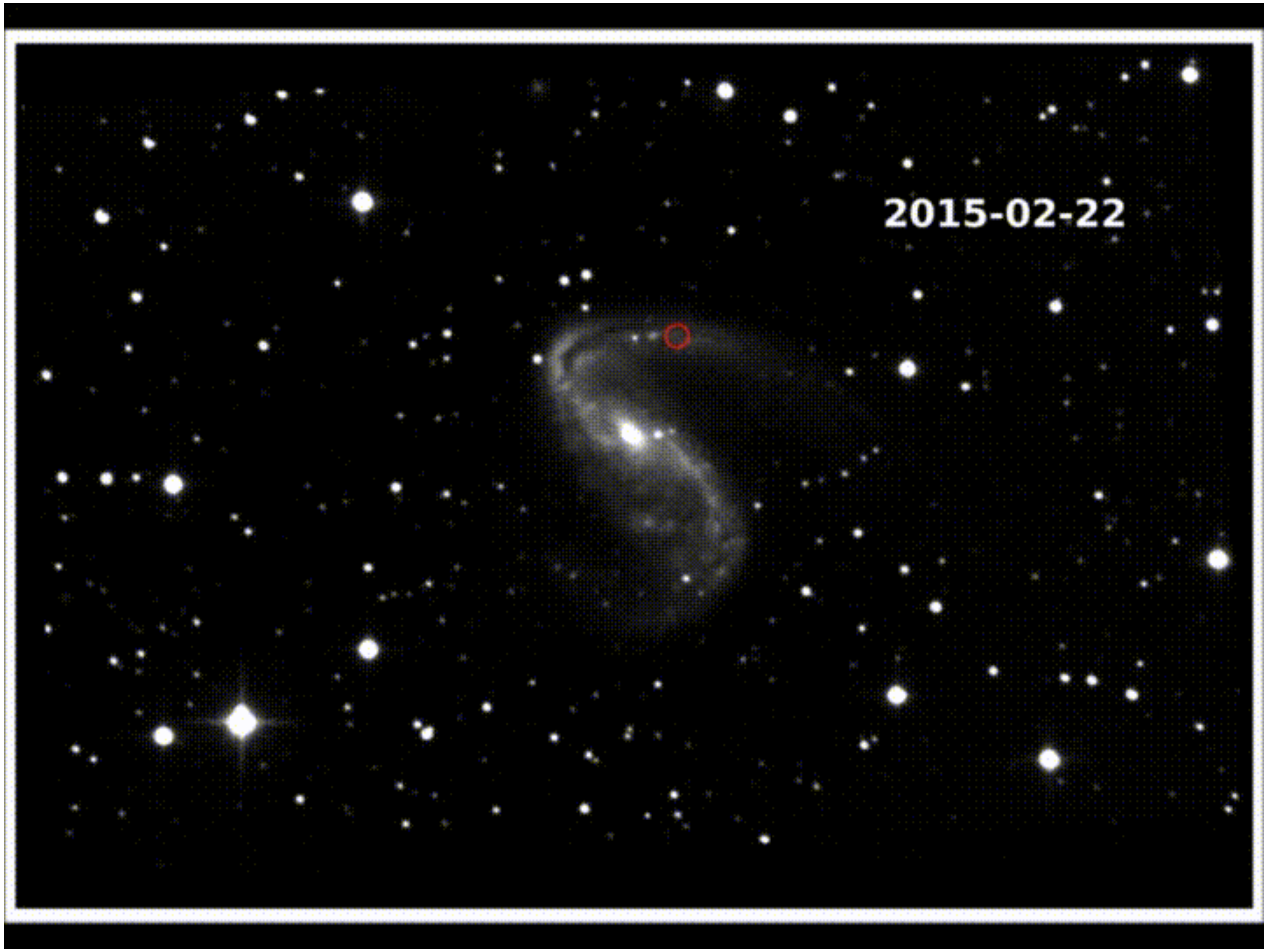
The firework after the explosion

SN ejecta catching up with the mass loss from stellar winds



The Standard Candle Method 2 — Type Ia SNe

- **Type Ia supernovae (SNe Ia)** have been used as standard candles to measure cosmological distances to other galaxies.
- They work as standard candles because presumably the white dwarfs have to reach **1.44 solar mass (the Chandrasekhar mass)** to trigger the thermonuclear explosion, reaching a peak absolute magnitude of **M_V = -19**.



Practice: The Standard Candle Method of Distance Measurement

- Type Ia supernovae (SNe) have been used as standard candles to measure cosmological distances to other galaxies.
- They work as standard candles because presumably the white dwarfs have to reach 1.44 solar mass (the Chandrasekhar mass) to trigger the thermonuclear explosion
- At its peak, the absolute magnitude in V-band (550 nm) is ***M_V = -19***, and you measured a peak apparent magnitude of ***m_V = 10***, what's the distance in parsec?

$$m - M = 5\ [\log d(\text{parsec}) - 1]$$

$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$

10 parsec * 10^(0.2*(10-(-19))) = 6.3 Mpc

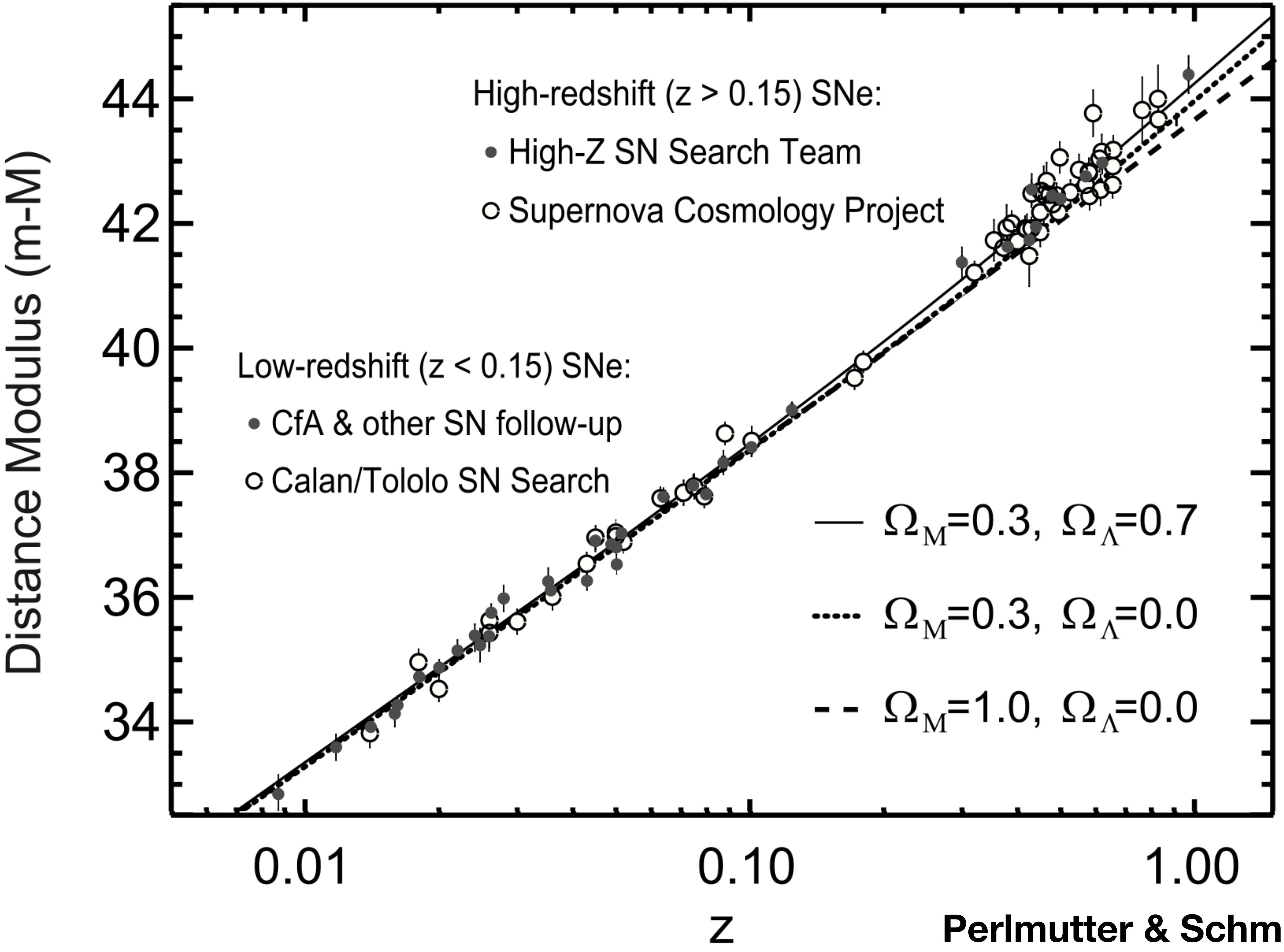
The Distance Ladder to Galaxies

- **Parallax** uses geometry to measure the distances to stars.
- **Standard candles** are objects with a luminosity inferred from other properties, so that their brightness and luminosity can be combined to calculate a distance.

- Finding distances to galaxies requires the use of the **distance ladder** in which short-distance methods are used to calibrate long-distance methods.

- **Geometric anchors** from parallax and other methods tied to our knowledge of the astronomical unit (AU)
    - **Cepheid variables** uses the luminosity-period relation of pulsating variable stars (calibrated by geometric distances)
        - **Type Ia supernovae** uses the luminosity-fading-rate relation of exploding white dwarfs (calibrated by Cepheids)

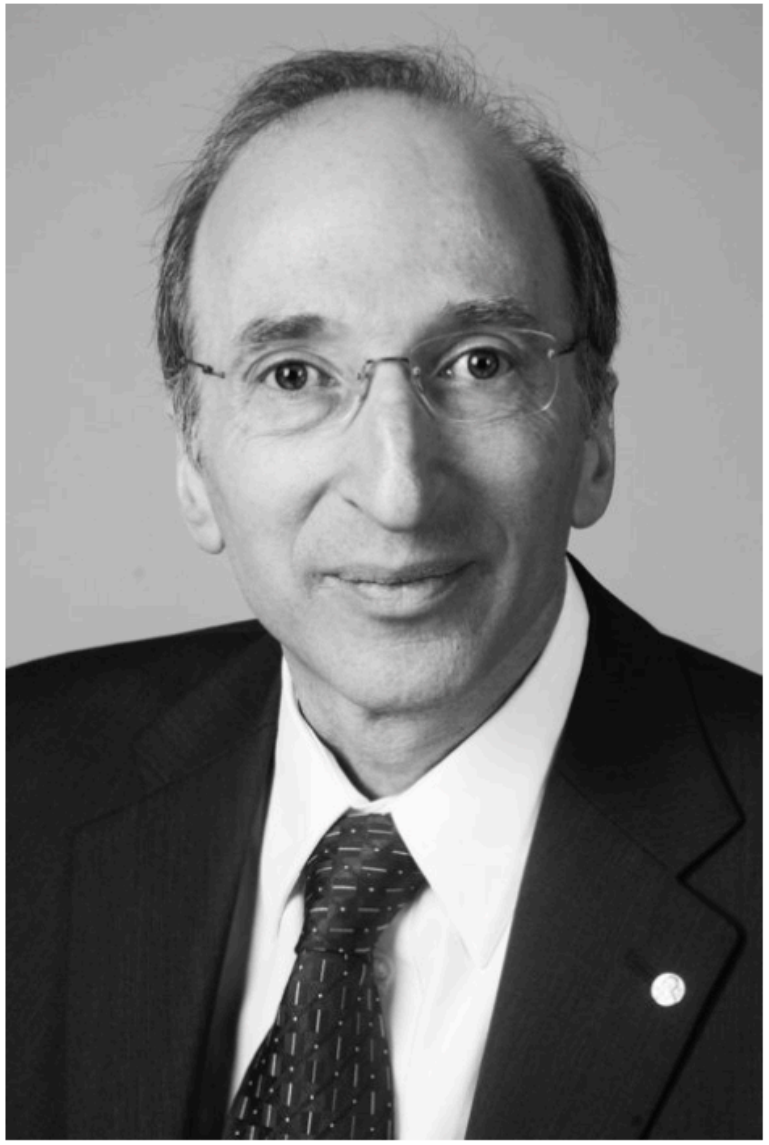
Distance Modulus vs. Cosmological Redshifts (Hubble Diagram)



Perlmutter & Schmidt 2003

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

The Nobel Prize in Physics 2011



© The Nobel Foundation. Photo: U. Montan

Saul Perlmutter

© The Nobel Foundation. Photo: U. Montan

Brian P. Schmidt

© The Nobel Foundation. Photo: U. Montan

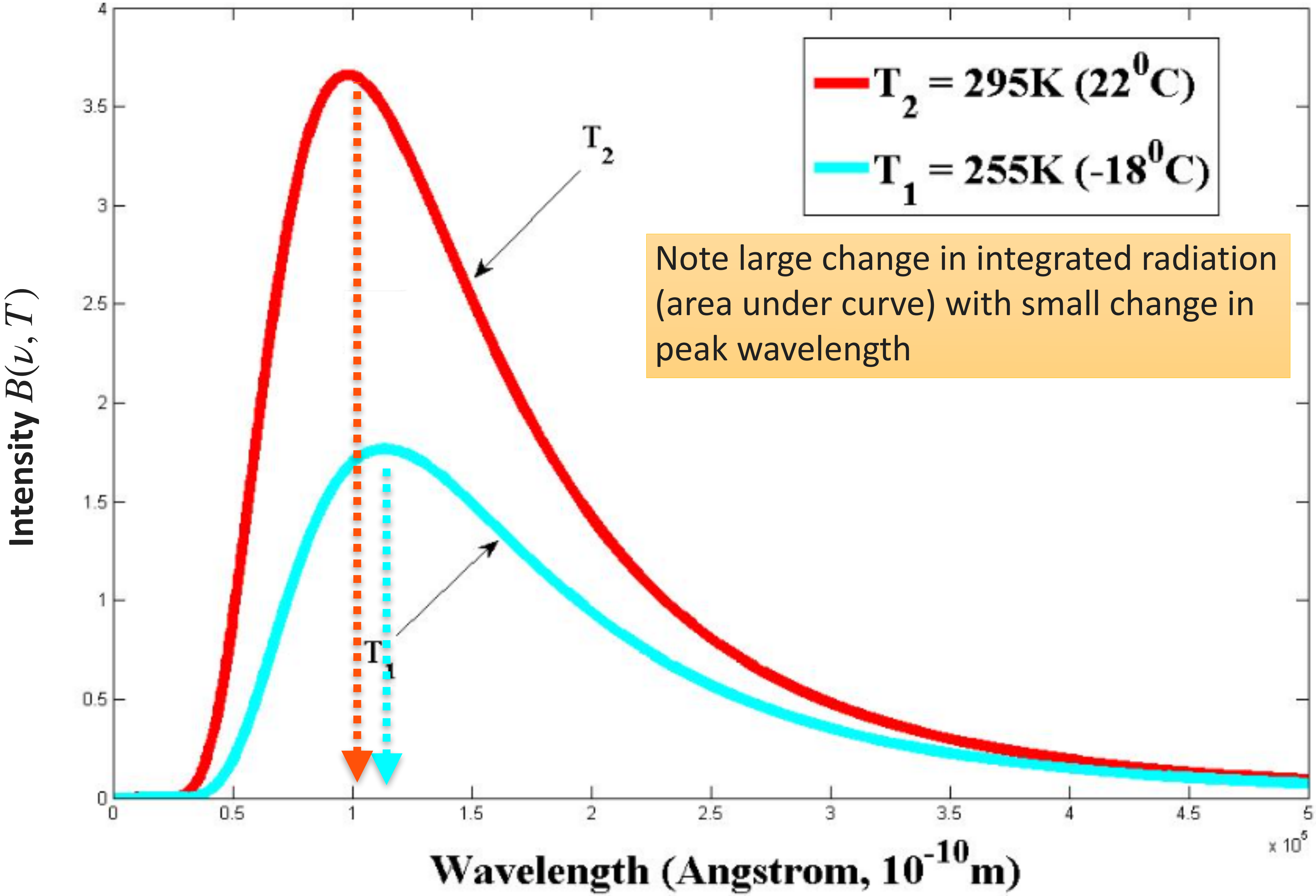
Adam G. Riess

Blackbody Emission

Surface Flux & Luminosity

Planck Function gives **radiative intensity of a blackbody emitter**: power emitted per **unit projected surface area** per **unit solid angle** per **unit frequency**

$$I_\nu = B_\nu(T) \equiv \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$$

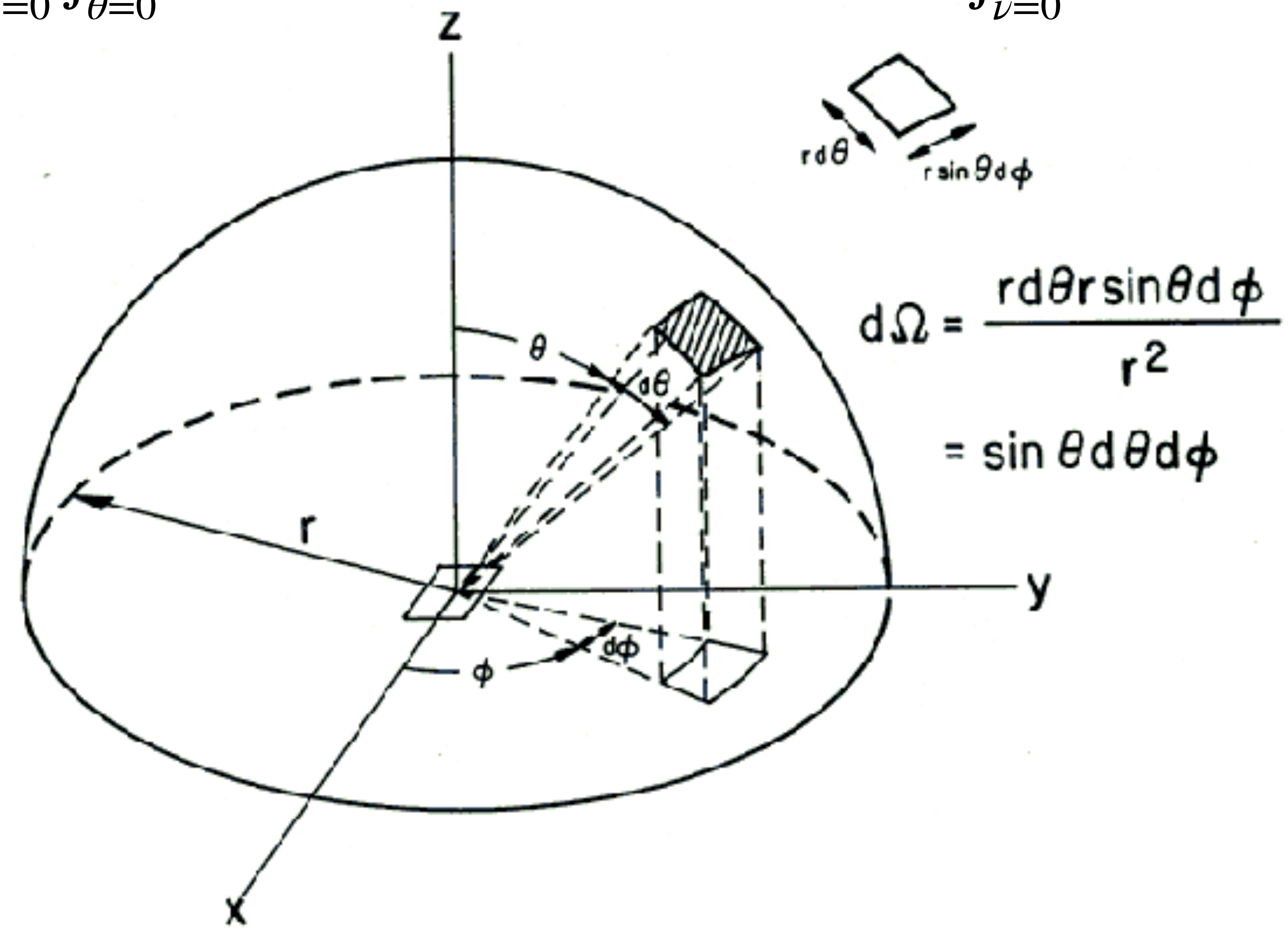


Note large change in integrated radiation (area under curve) with small change in peak wavelength

Stefan-Boltzmann Law - Derived based on definition of intensity

- **Differential solid angle in polar coordinates (see diagram below):**
  $d\Omega = \sin\theta d\theta d\phi$
- **Projected area towards direction θ:** $A\cos\theta$
- **Power radiated into $d\Omega$ from** A **between $\nu, \nu + d\nu$:** $dP = B(\nu, T)A\cos\theta d\Omega d\nu$
  given that **intensity** $B(\nu, T)$ is the power (dP) emitted per <u>unit projected surface area</u> ($A\cos\theta$) per <u>unit solid angle ($d\Omega$)</u> per <u>unit frequency ($d\nu$)</u>
- **Bolometric surface flux to the upper half hemisphere:**

$$F_S = \frac{P}{A} = \int_{\nu=0}^{\infty}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} B(\nu, T)\ \cos\theta\ \sin\theta\ d\theta\ d\phi\ d\nu = \pi\int_{\nu=0}^{\infty} B(\nu, T)d\nu$$



Stefan-Boltzmann Law - How to Integrate the Planck Function

Intensity: power emitted per unit projected surface area per unit solid angle per unit frequency

$$I_\nu = B(\nu, T) \equiv \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Surface Flux: power emitted per unit surface area

$$F_S = \int_{\nu=0}^{\infty}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} B(\nu, T)\ \cos\theta\ \sin\theta\ d\theta\ d\phi\ d\nu = \pi\int_{\nu=0}^{\infty} B(\nu, T)d\nu$$

To evaluate this integral, do a substitution,

$$u = \frac{h\nu}{kT}$$

$$du = \frac{h}{kT}\,d\nu$$

$$F = \frac{2\pi h}{c^2}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{u^3}{e^u - 1}\,du.$$

Surface Flux – Stefan-Boltzmann Law

- **Surface Flux** (F_S) is the total amount of energy emitted per square meter every second (the luminosity per area).

$$F_S = \sigma_{\mathrm{SB}} T^4$$

$$T(K) = T(C) + 273.15$$

$$\sigma_{\mathrm{SB}} = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$$

where *T* is the temperature in K, *F* is the flux, and σ (sigma) is called the **Stefan-Boltzmann constant.**

- Hotter objects emit *much* more energy (per square meter per second) than cool objects: ***surface flux ~ T^4***

The `L-R-T` Relation of Spherical Blackbody Radiators

- The Stefan-Boltzmann law gives us the **surface flux** from thermal emission of a blackbody emitter, can we use it to calculate the **luminosity** of the thermal emission?

$$F_S = \sigma_{\mathrm{SB}} T^4$$

$$\sigma_{\mathrm{SB}} = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$$

- Assuming spherical shape for the thermal emitter, its total surface area is $4\pi R^2$, where R is its radius (note that this is not distance)
- The luminosity is then the product of the surface flux and surface area:

$$L = 4\pi R^2 F_S = 4\pi \sigma_{\mathrm{SB}} R^2 T^4$$

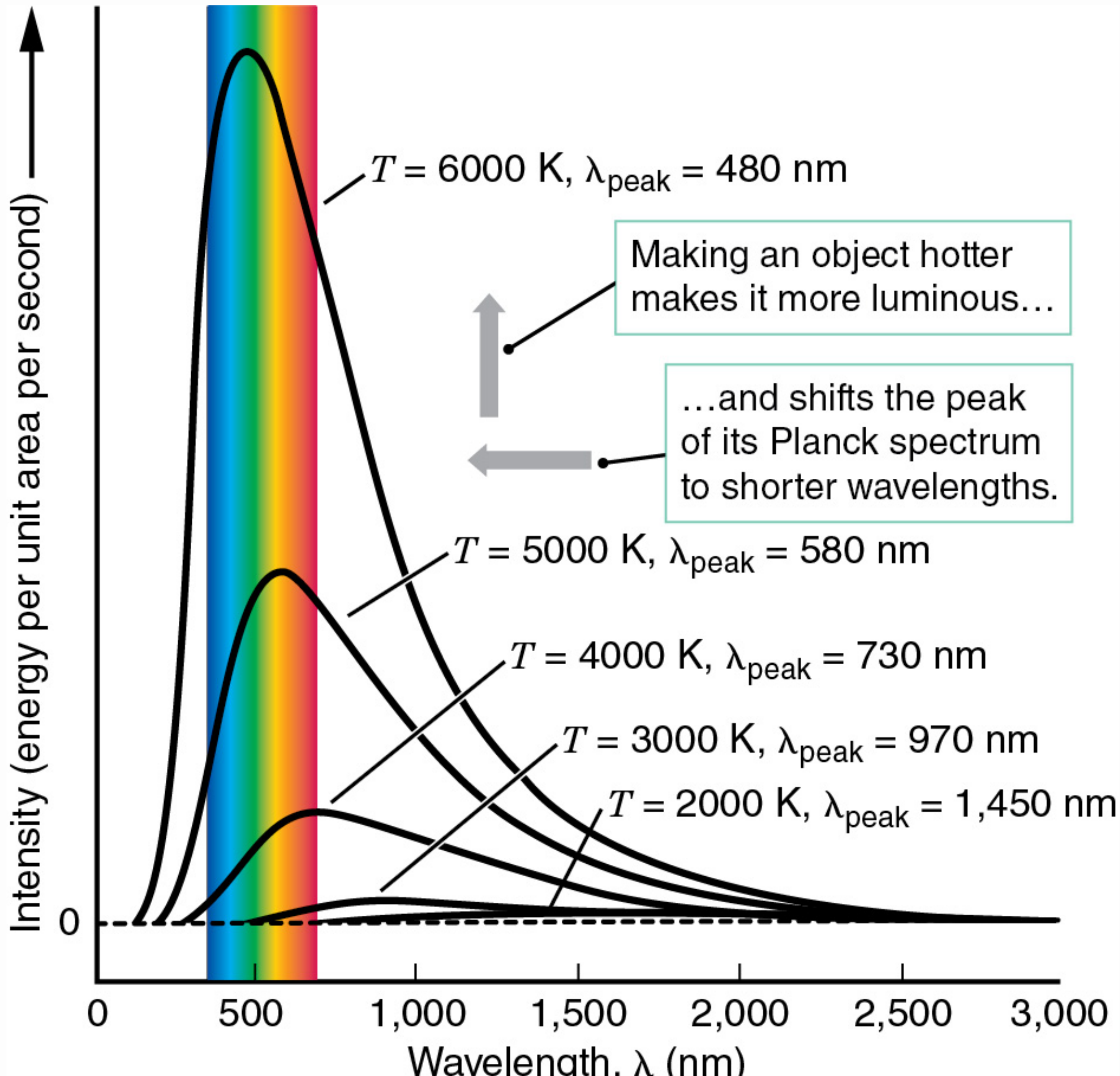
Distinction: Luminosity Density vs. Bolometric Luminosity

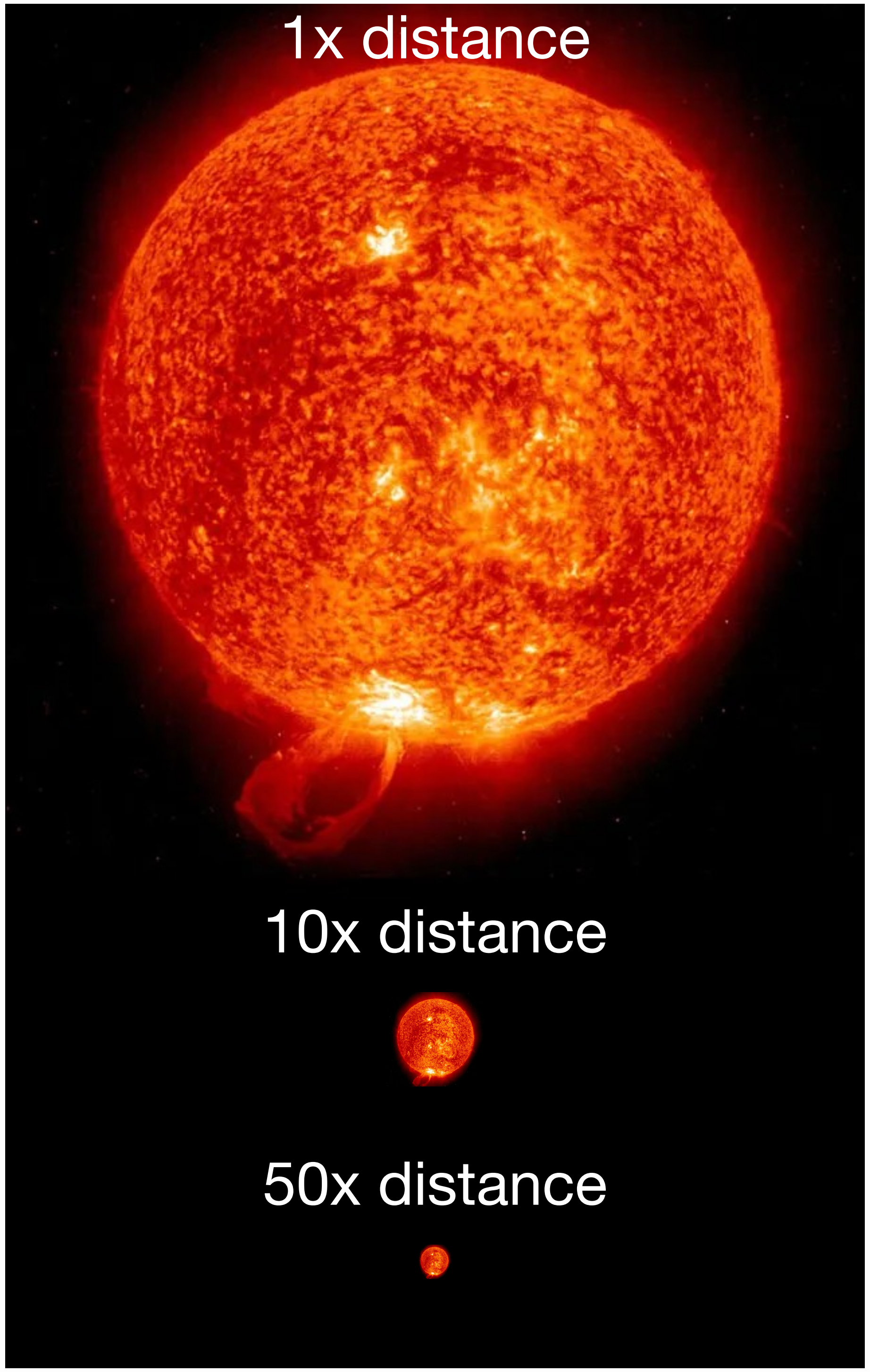
- **Luminosity Density** is the luminosity measured at a given wavelength:
  $L_\lambda = dL/d\lambda$
- For spherical blackbody radiators of uniform surface temperature, we have:
  $L_\lambda = 4\pi R^2 \times \pi B_\lambda(T)$,
  where
  $$B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}} - 1}$$ is the **Planck function**

- **Bolometric Luminosity** is the luminosity density integrated over all wavelengths:
  $$L_{\rm bol} = \int_0^\infty L_\lambda d\lambda$$
- For spherical blackbody radiators, we have:
  $L_\lambda = 4\pi R^2 \times \pi B_\lambda(T)$
  as a result,
  $$L_{\rm bol} = 4\pi R^2 \int \pi B_\lambda(T) d\lambda$$
  $$= 4\pi R^2 \sigma_{\rm SB} T^4$$

So … What $B_\lambda(T)$ actually is to an observer?

- **Flux density** can be derived from **luminosity density** using the **inverse distance square law**: $F_\lambda = L_\lambda/4\pi d^2 = (4\pi R^2 \cdot \pi B_\lambda(T))/(4\pi d^2) = B_\lambda(T) \cdot \pi R^2/d^2$
- Since $\pi R^2/d^2$ is the **angular area** of the source, hence **Planck function** $B_\lambda(T)$ gives the **surface brightness** of the source *(at λ), which is **distance invariant.***



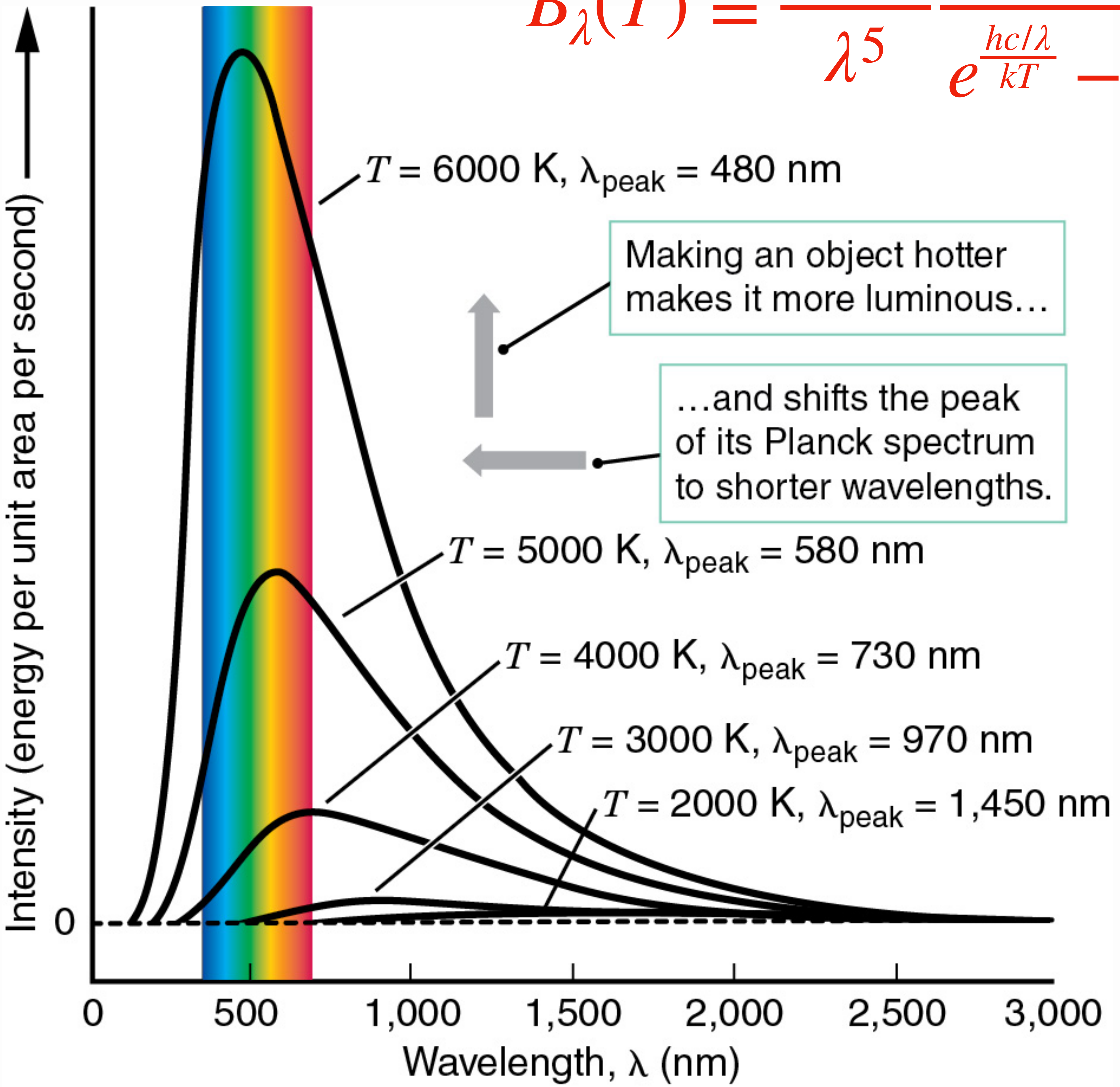


Surface Temperatures of Stars

spectroscopic methods: Wien’s law and spectral classification

Planck Curves at Various T

$$B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}}-1}$$



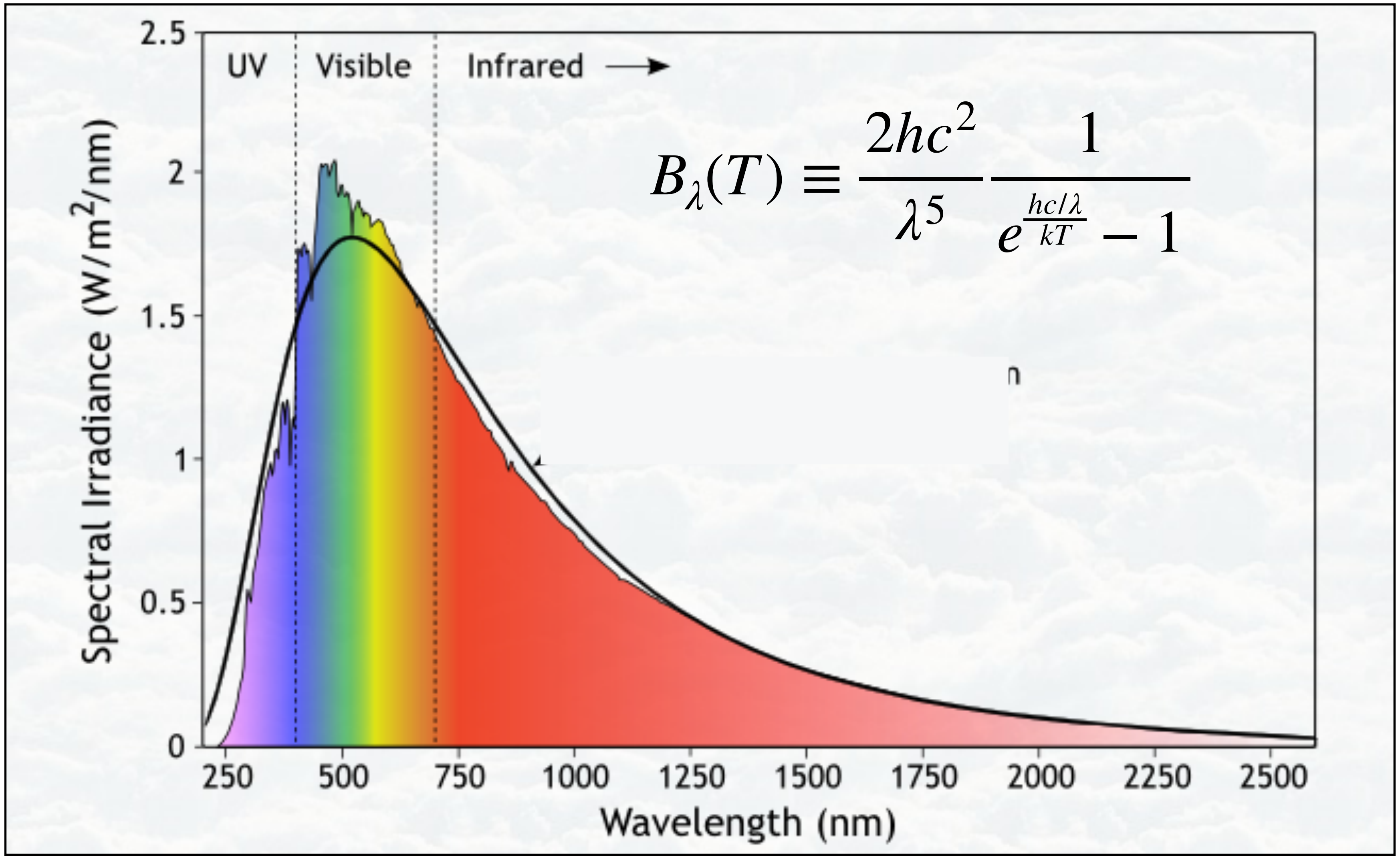
Making an object hotter makes it more luminous…

…and shifts the peak of its Planck spectrum to shorter wavelengths.

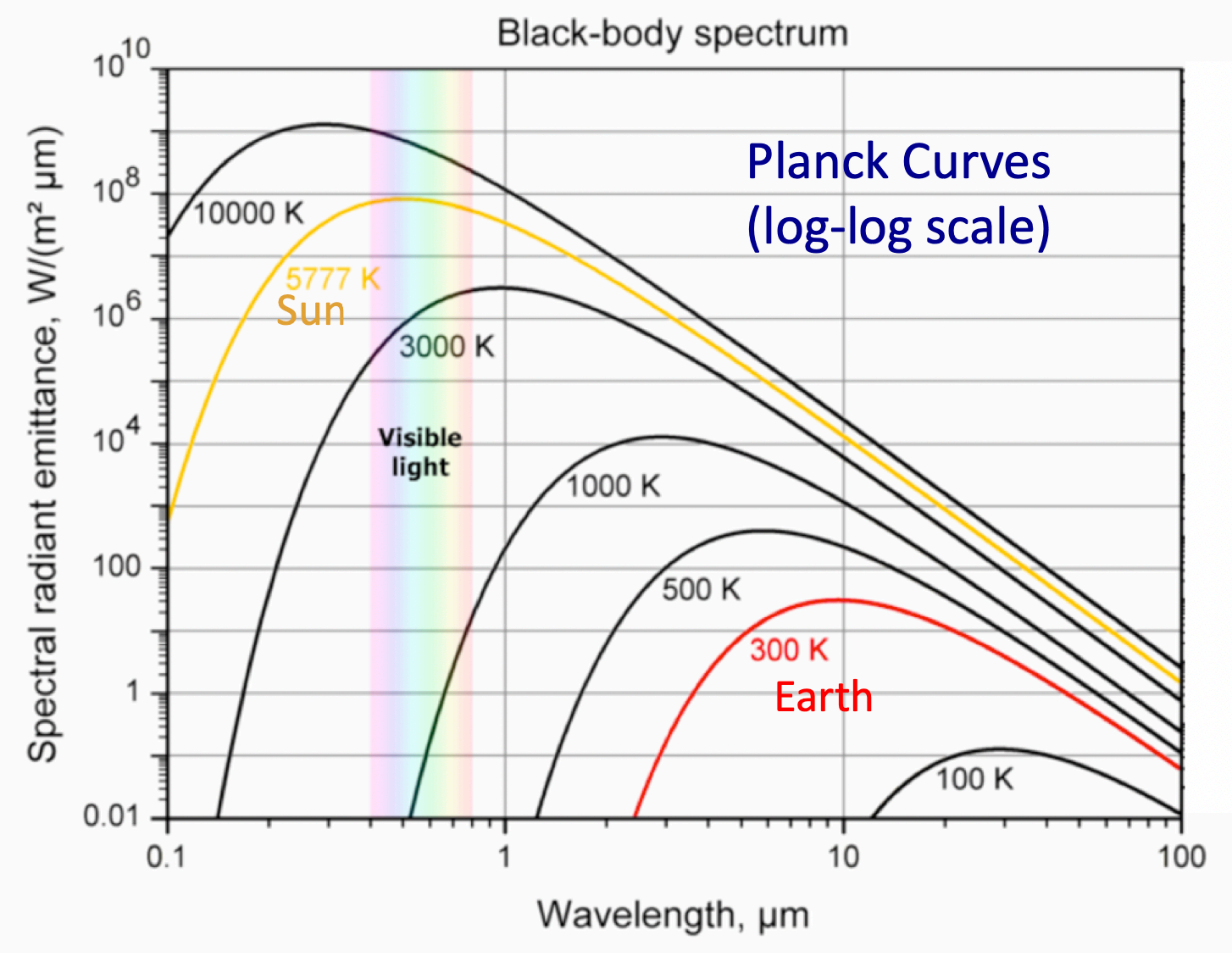
Temperature from Wien's Displacement Law

$$\lambda_{\text{peak}} = \frac{2.9\ \text{mm K}}{T} \Rightarrow T = \frac{2.9\ \text{mm K}}{\lambda_{\text{peak}}}$$

- Given a temperature, calculate the wavelength at which the BB emission's flux density peaks; Or given a peak wavelength, calculate the temperature.



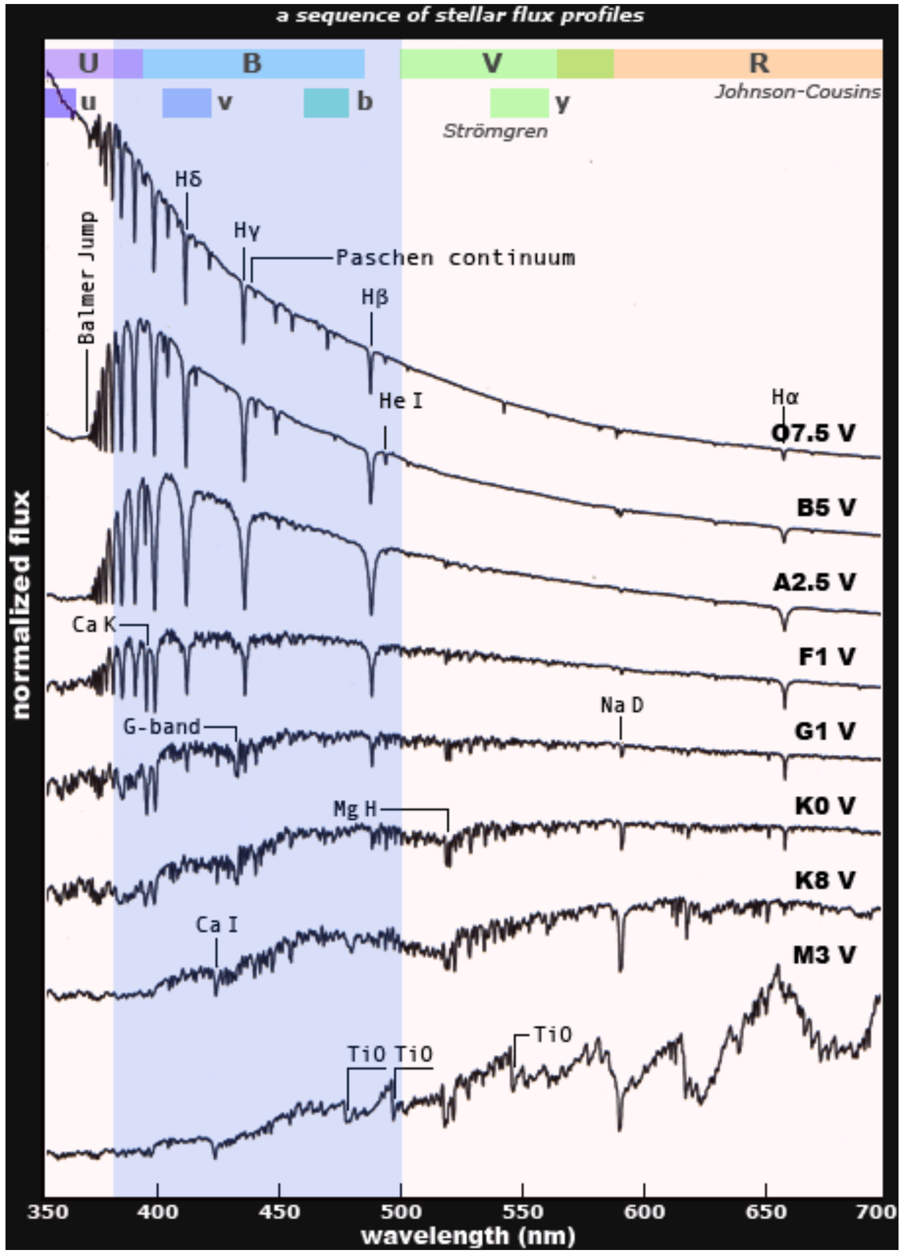
What to do when the peak shifts outside of the visible light window?
e.g. when T > 9000 K or T < 3000 K

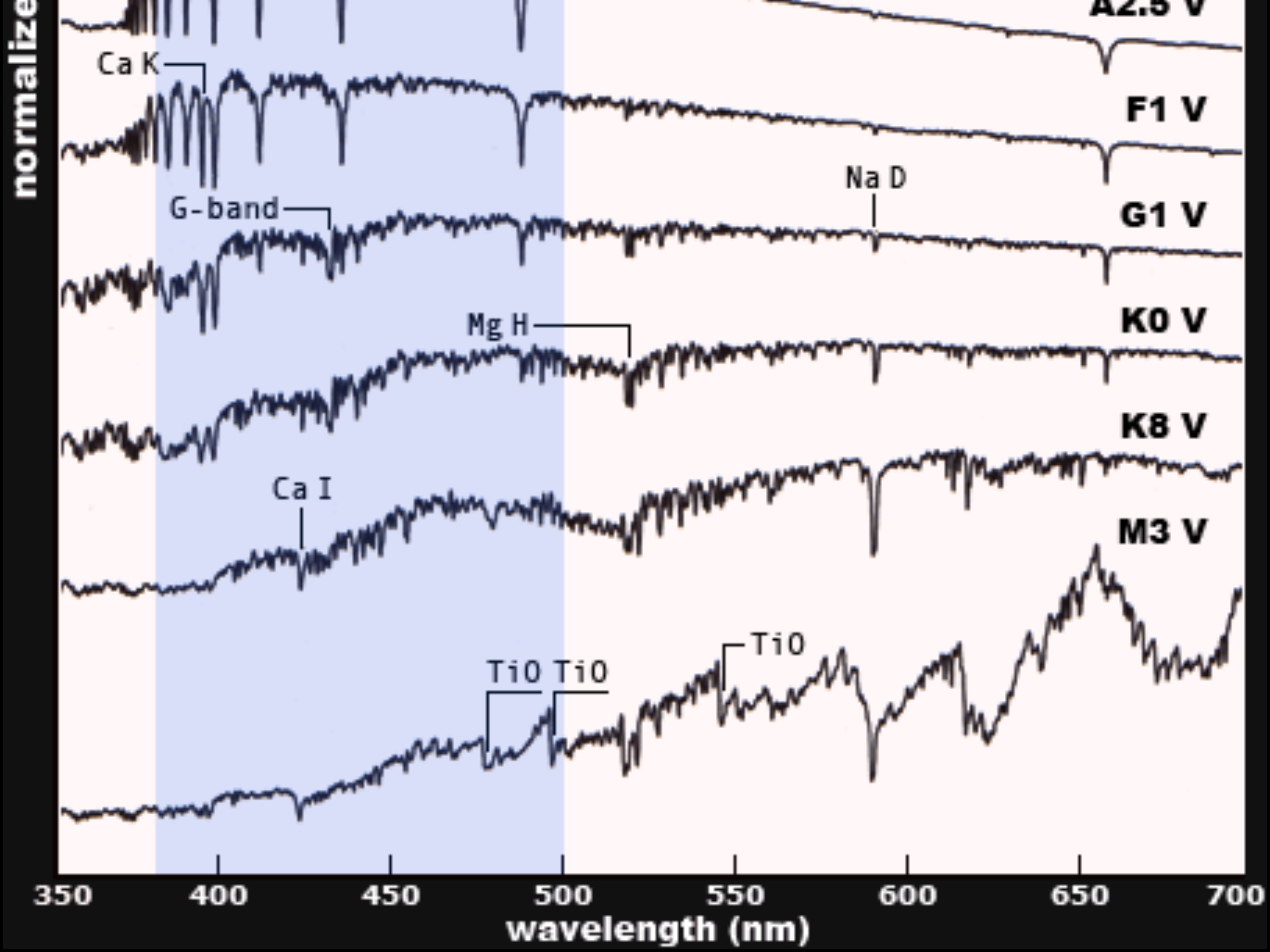


Planck Curves (log-log scale)

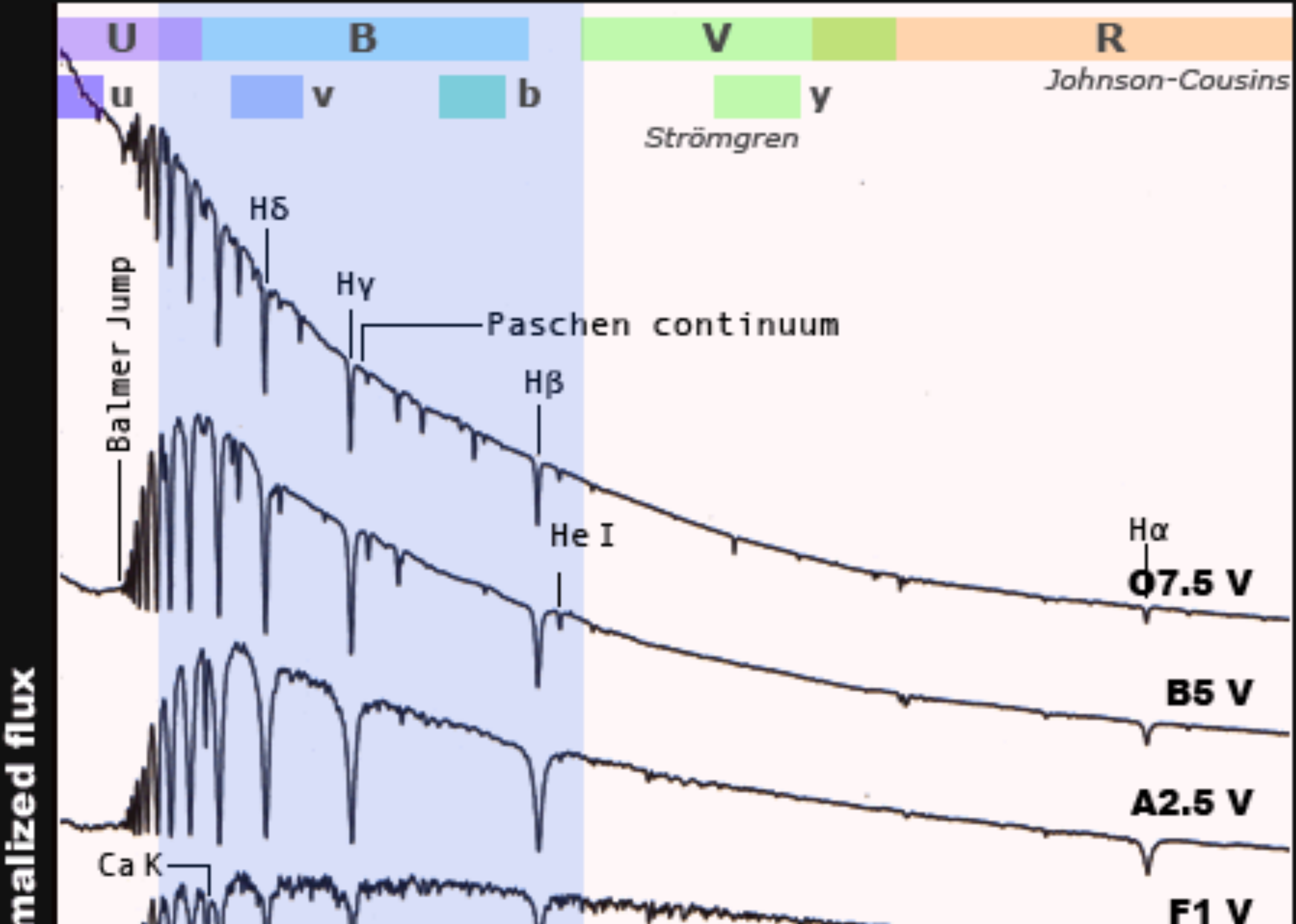
Optical spectral classification of stars

- **The strength of absorption lines from different elements depend mainly on the temperature (because of ionization equilibrium).**
- The current classification scheme was **re-ordered** and **simplified** by **Annie Jump Cannon** (1863–1941) at Harvard College Observatory.
- The full sequence is **O B A F G K M**, which are further subdivided by adding numbers to the letter. The Sun is a **G2** spectral-type star.

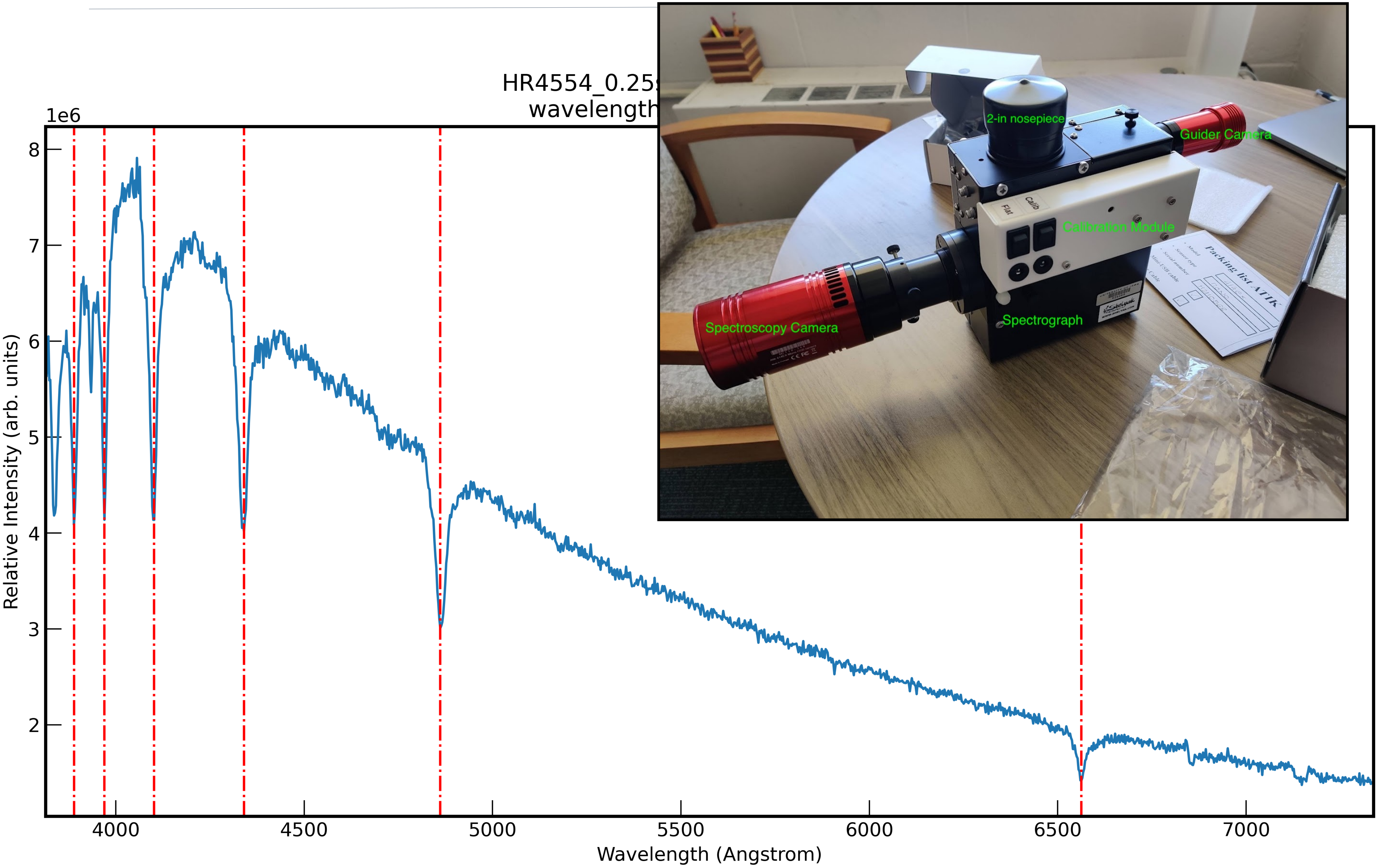



normalize
A2.5 V
Ca K
F1 V
Na D
G-band
G1 V
Mg H
K0 V
K8 V
Ca I
M3 V
TiO
TiO
TiO
350
400
450
500
550
600
650
700
wavelength (nm)

a sequence of stellar flux profiles

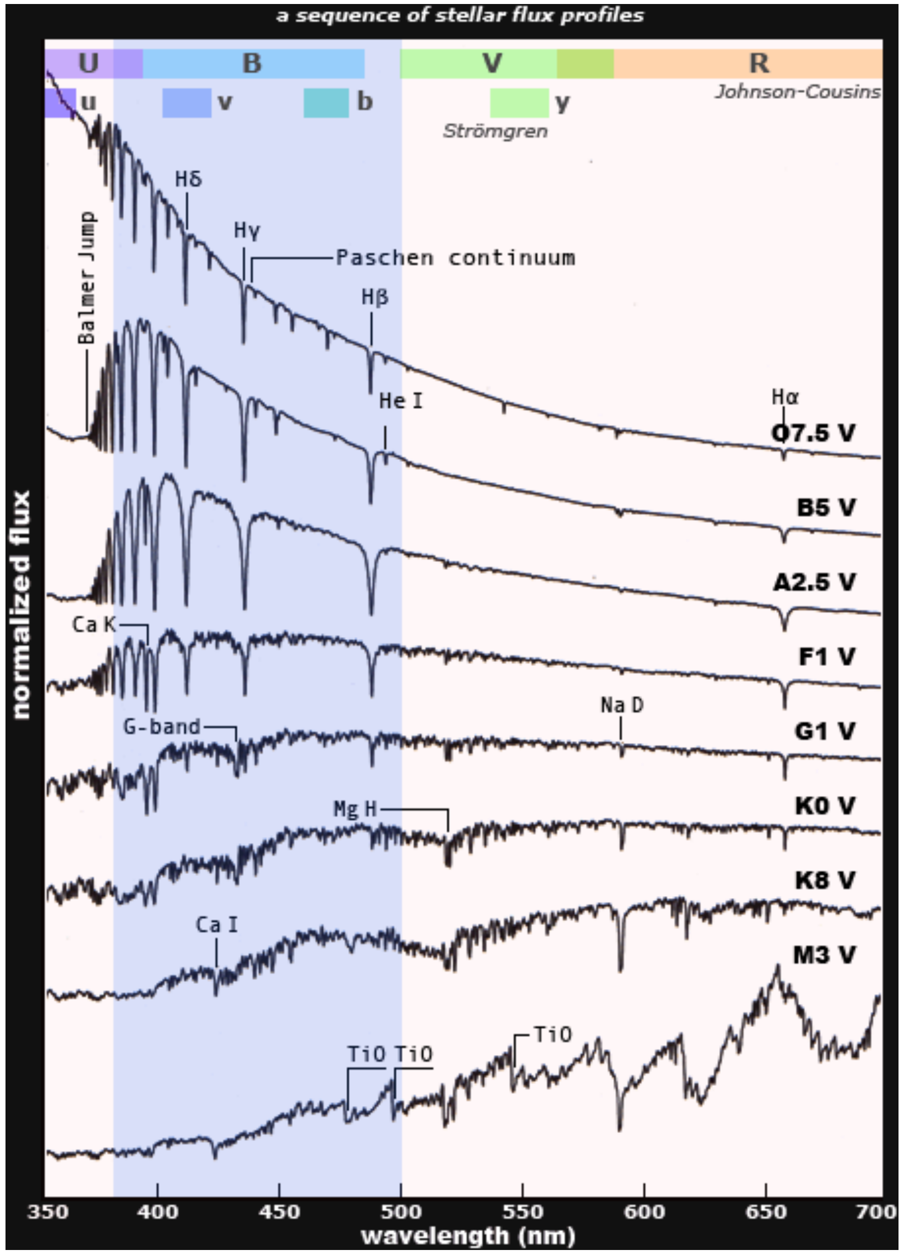


An A-type star’s spectrum taken by the Van Allen Observatory

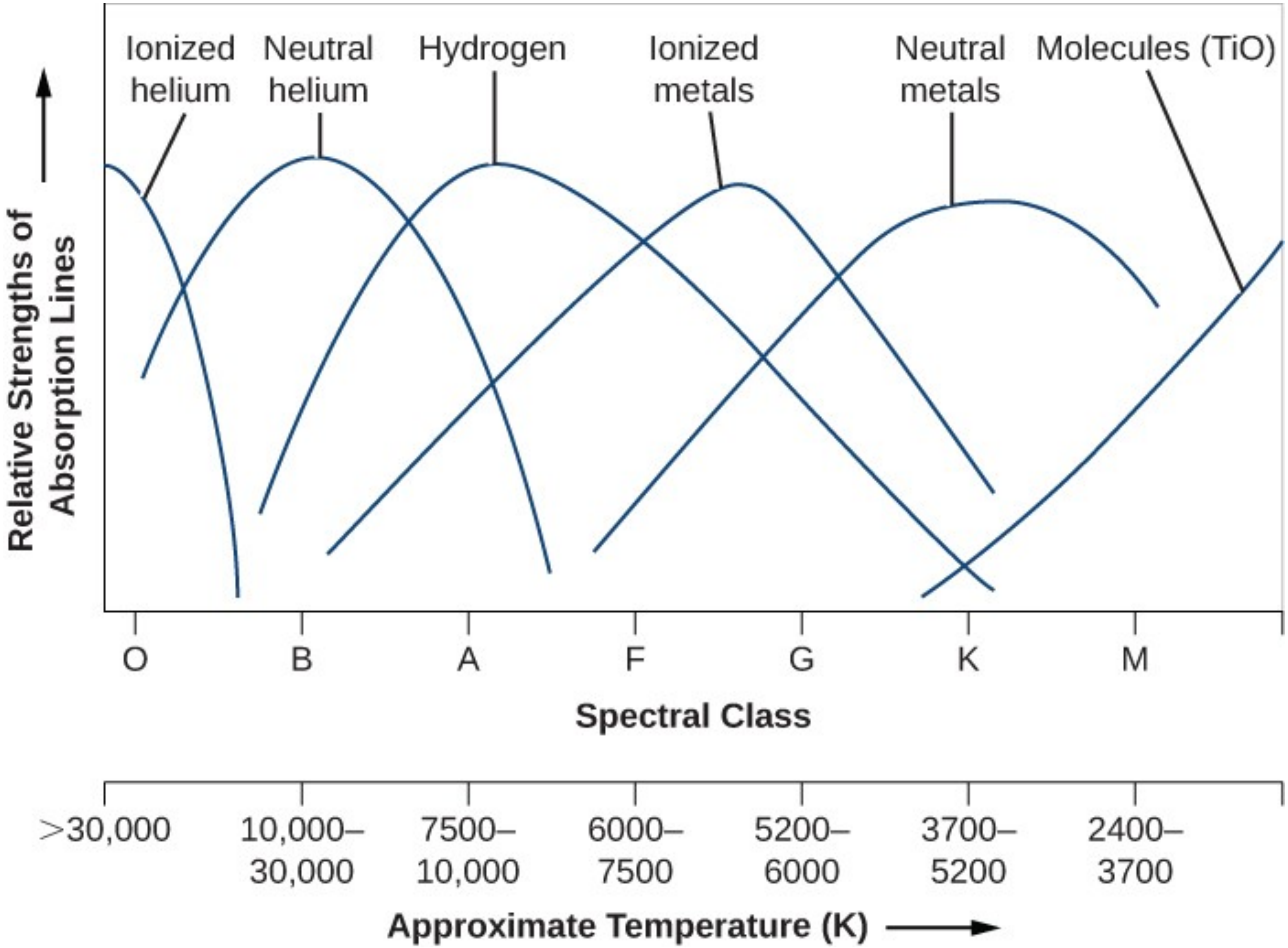


Optical spectral classification of stars

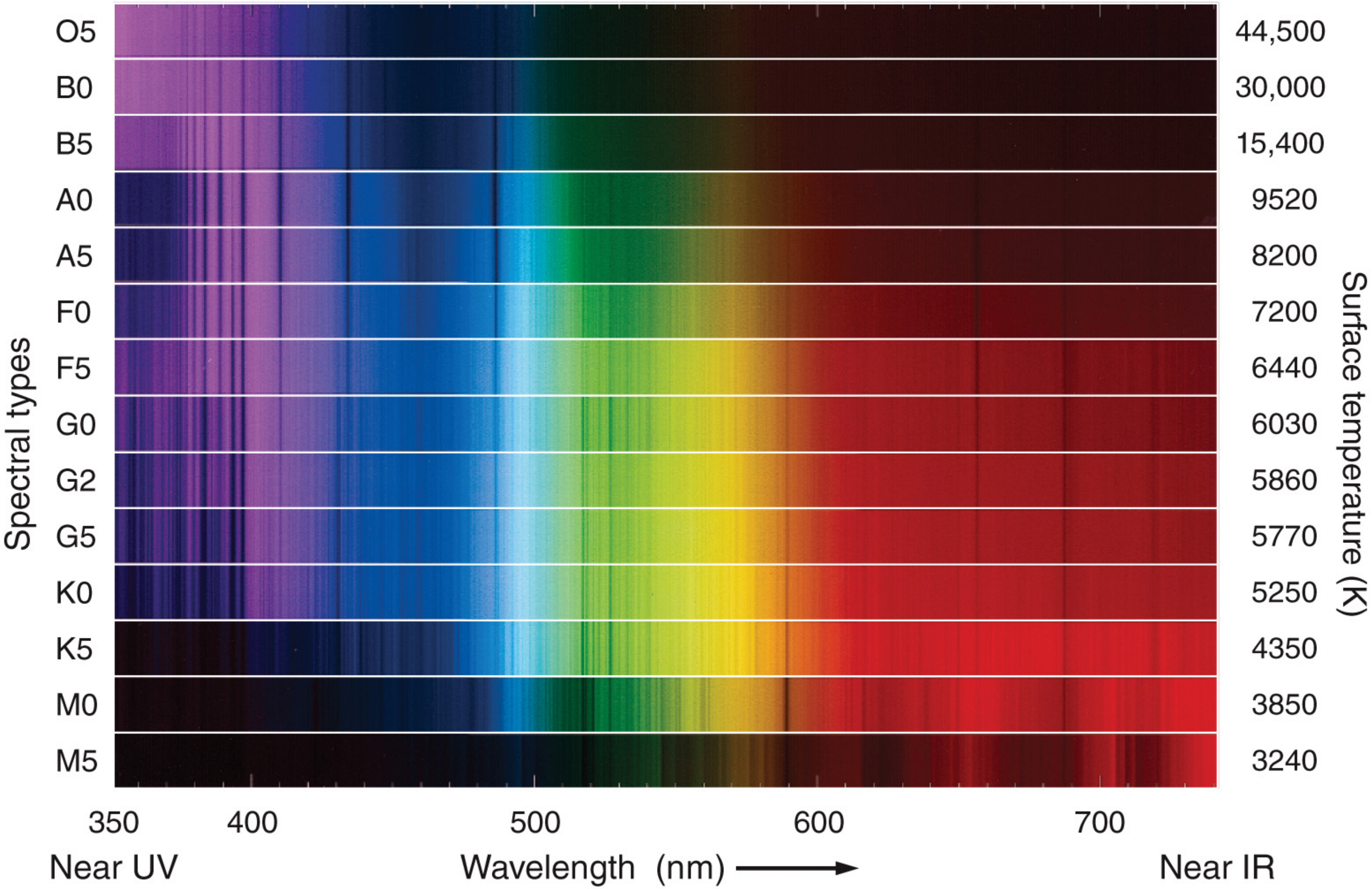
- **The strength of absorption lines from different elements depend mainly on the temperature (because of ionization equilibrium).**
- The current classification scheme was **re-ordered** and **simplified** by **Annie Jump Cannon** (1863–1941) at Harvard College Observatory.
- The full sequence is **O B A F G K M**, which are further subdivided by adding numbers to the letter. The Sun is a **G2** spectral-type star.



Relative Strengths of Absorption Lines vs. Atmospheric Temperature

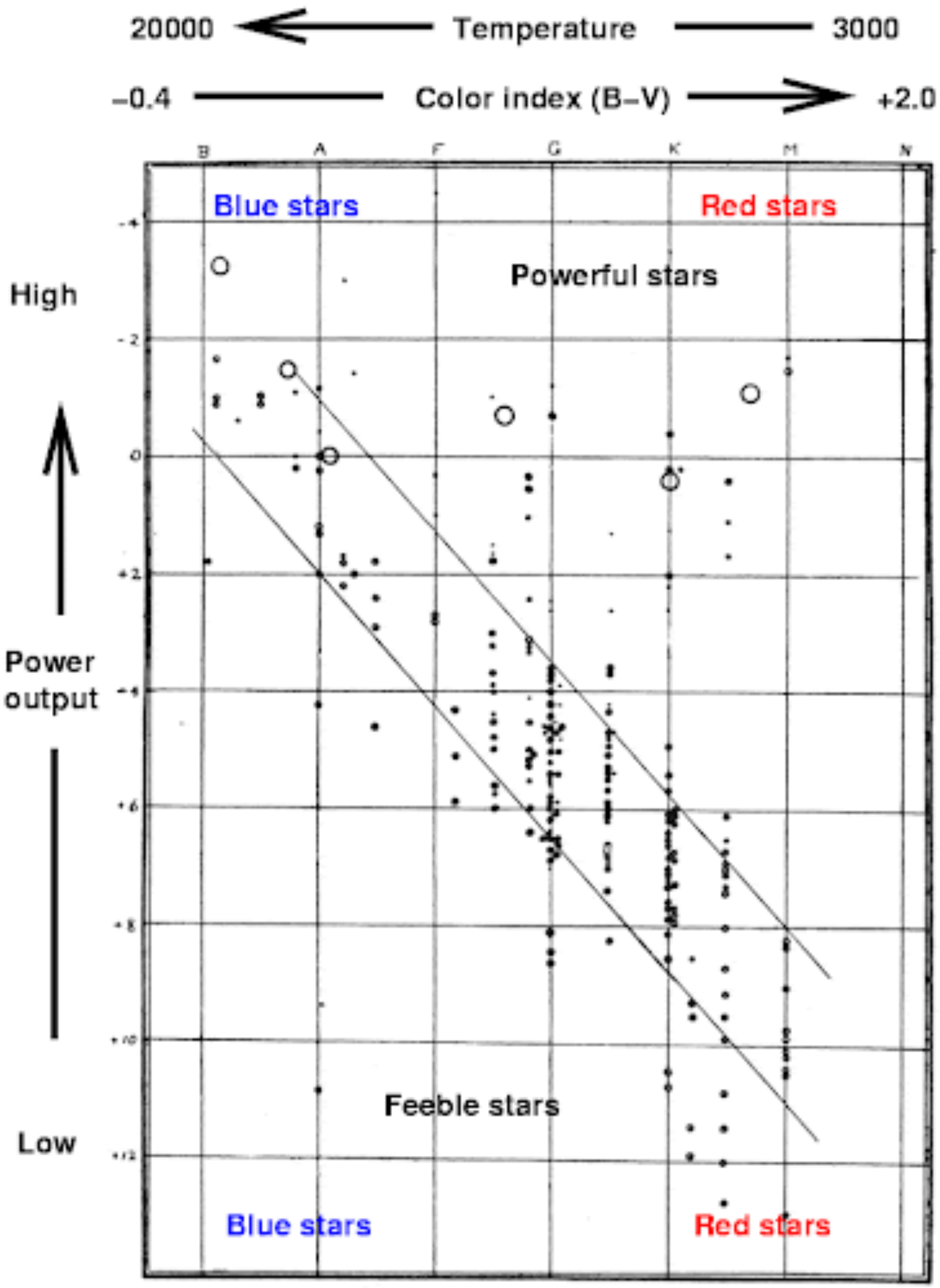


Temperature from Spectral Classes



The Hertzsprung-Russell Diagram (with Spectral Types)

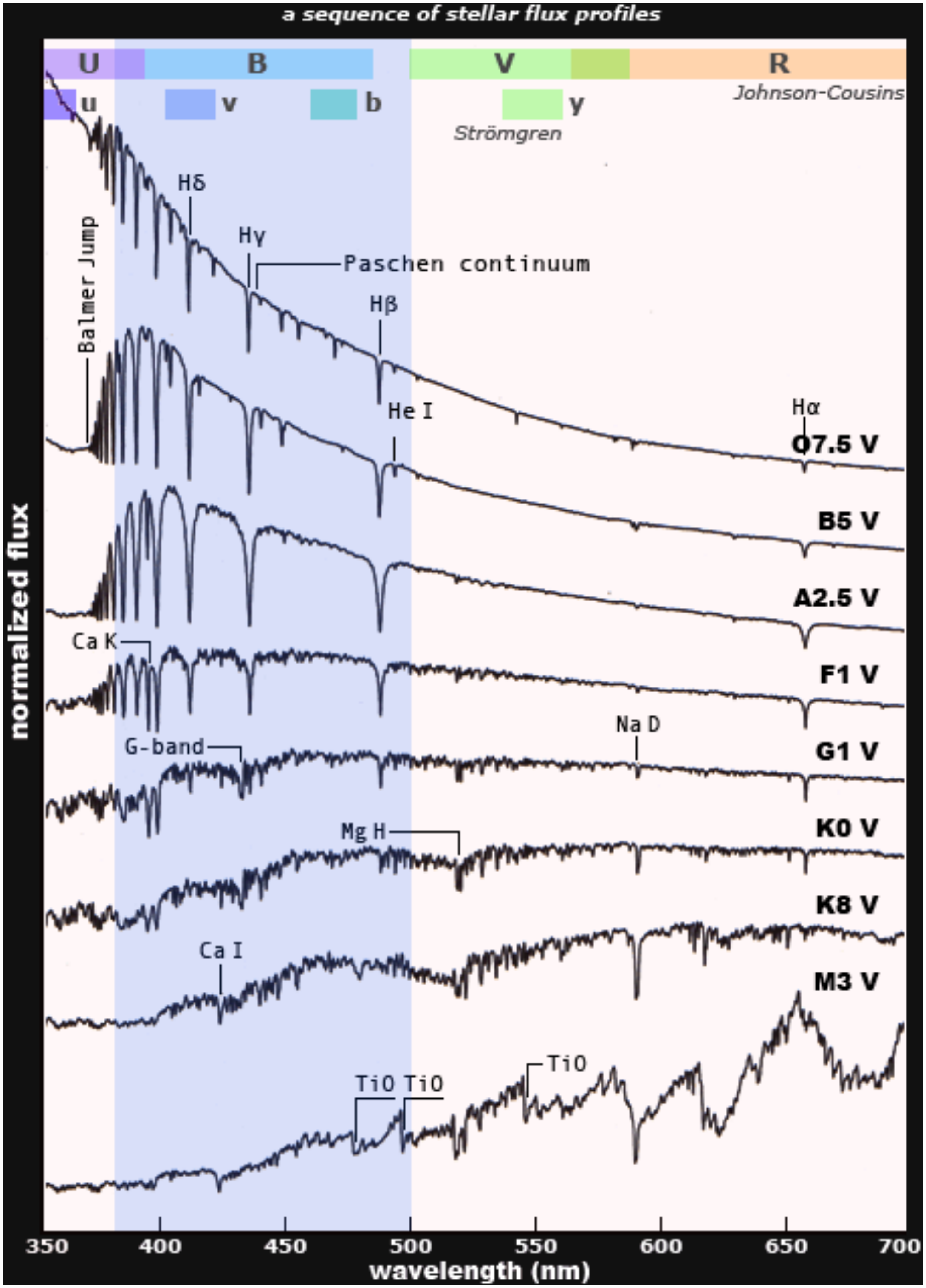
- In **1905, Hertzsprung** first published the measurements in a Table.
- In **1914, Russell** published his independent measurements in the format of a Figure
- He plotted absolute magnitude of nearby stars against their spectral types (sorted in temperature, as OBAFGKM, with 4-5 subclasses in each type)



Temperature

photometric method: color index

Spectroscopy takes longer time to acquire, because each star would require its own spectroscopic observations with a traditional longslit spectrograph



The Colors of Stars

From Hottest to Coldest

These are the Apparent Colors to Your Eyes

hottest

BLUE
Rigel
25,000 K

BLUE-WHITE
Sirius
10,000 K

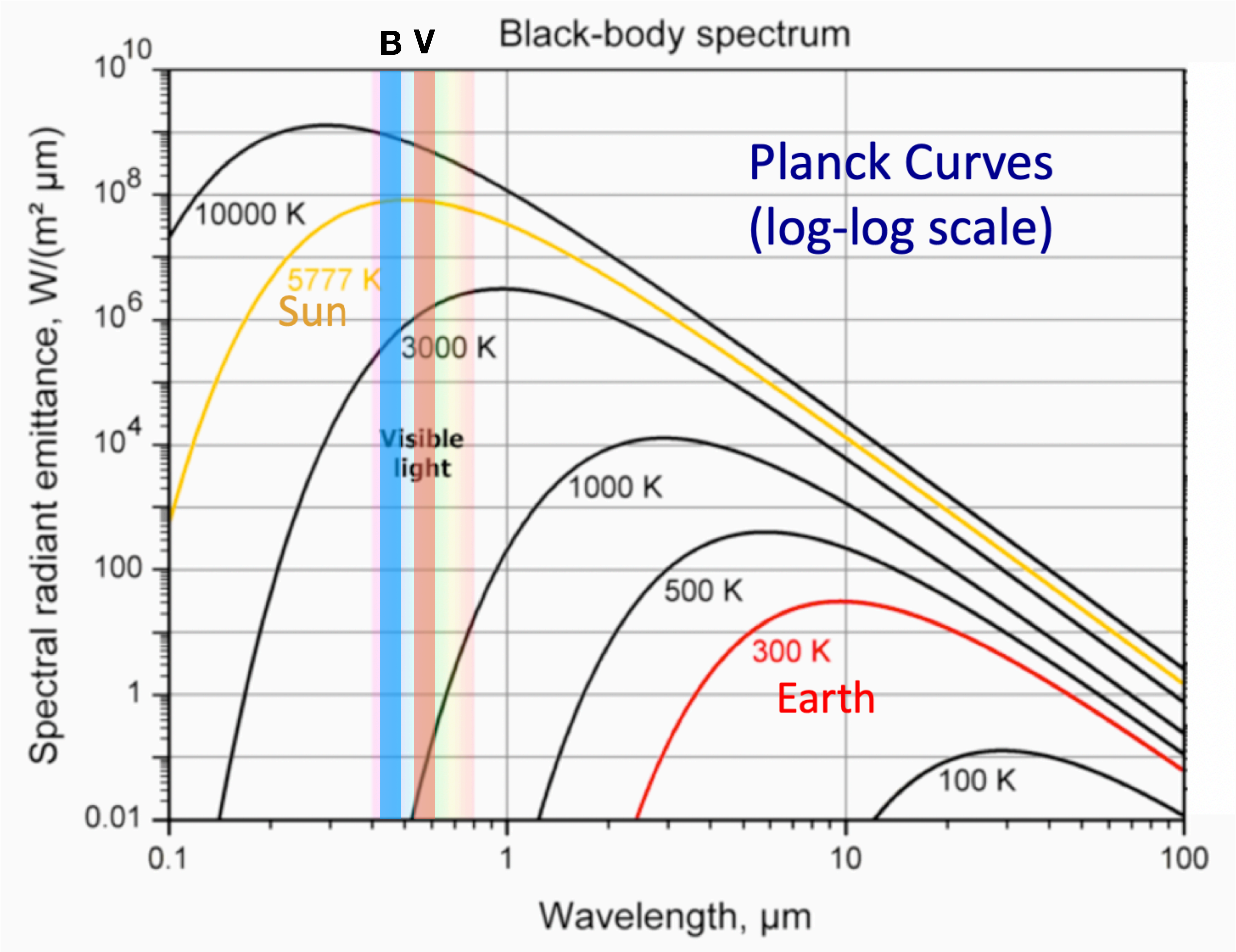
YELLOW
Sun
6,000 K

ORANGE
Aldebaran
4,000 K

RED
Antares
3,000 K

coldest

Two-band photometry offers a much simpler way to estimate temperature



Color Index: Blue Magnitude - Red Magnitude

- **Color index** is defined as the **magnitude difference** of the same object at two different wavelengths.

- According to **Pogson's magnitude definition**, the magnitude difference corresponds to a flux ratio at two different wavelengths:

$$m_B - m_V = -2.5[\log(f_B/f_{B,0}) - \log(f_V/f_{V,0})]$$

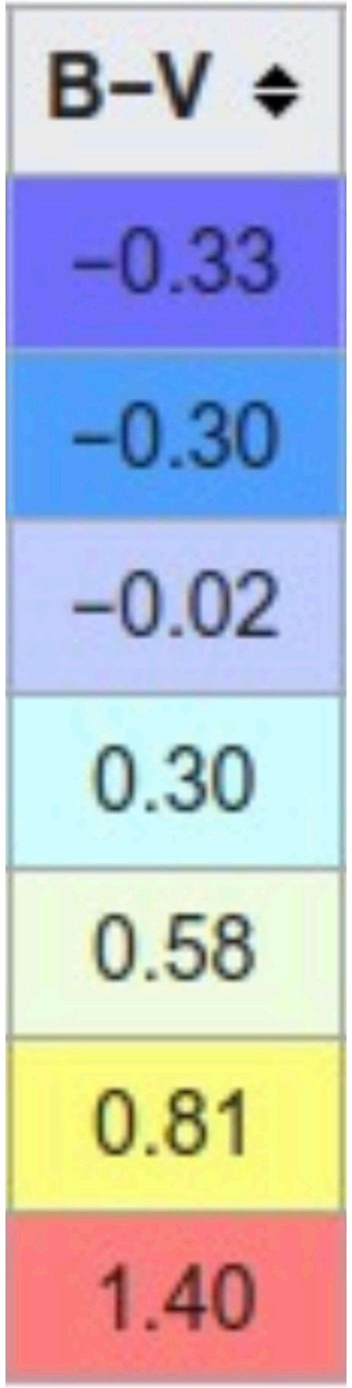
or simply (after rearranging):

$$B - V = -2.5[\log(f_B/f_V) - \log(f_{B,0}/f_{V,0})]$$

- Typically, we subtract a bluer magnitude (e.g., B) to a redder magnitude (e.g., V), so that **the higher the value** of the color index, **the redder the object** appears (i.e., the object appears much fainter in B-band than in V-band)

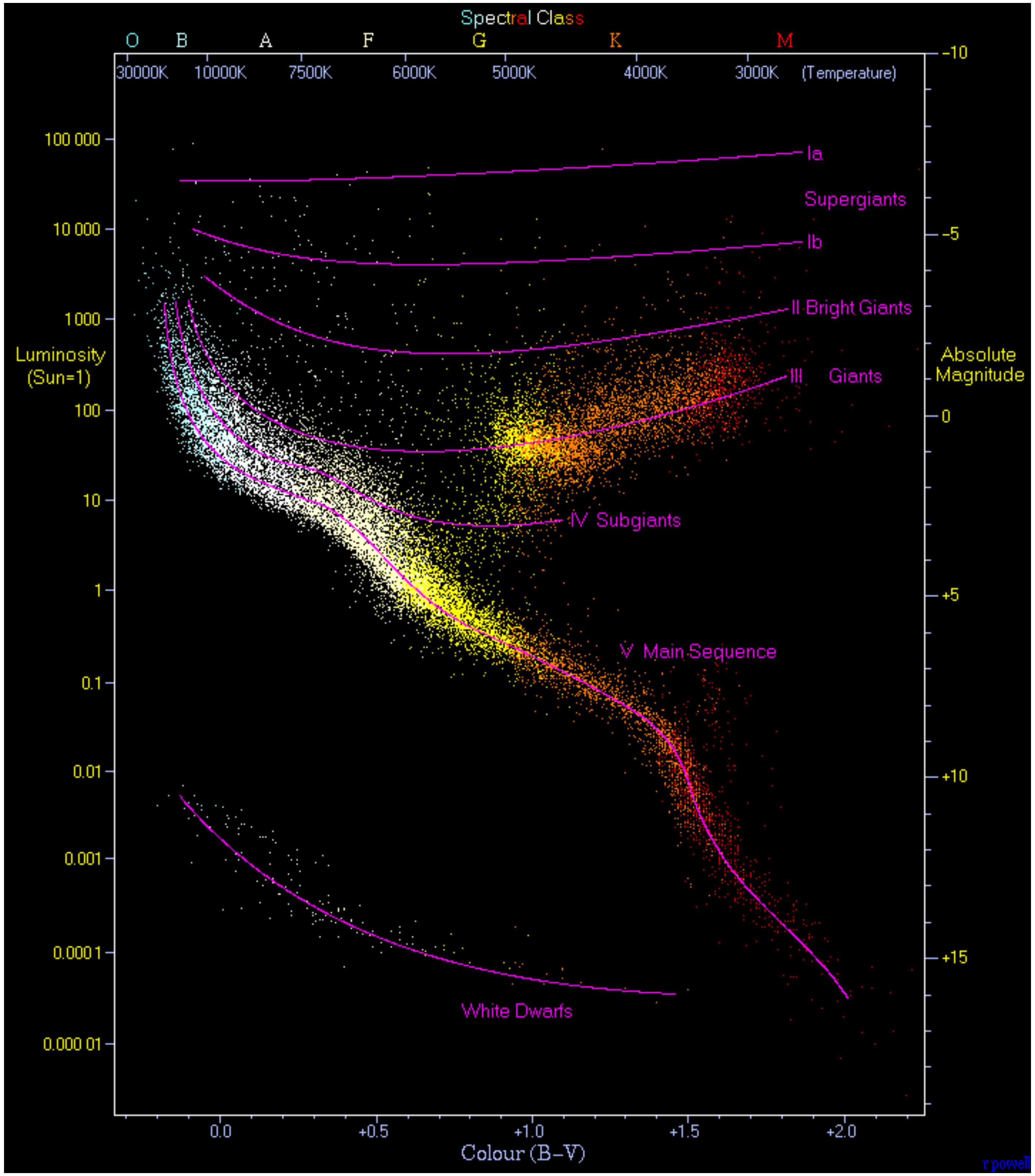
Temperature vs. Color Index vs. Apparent Color

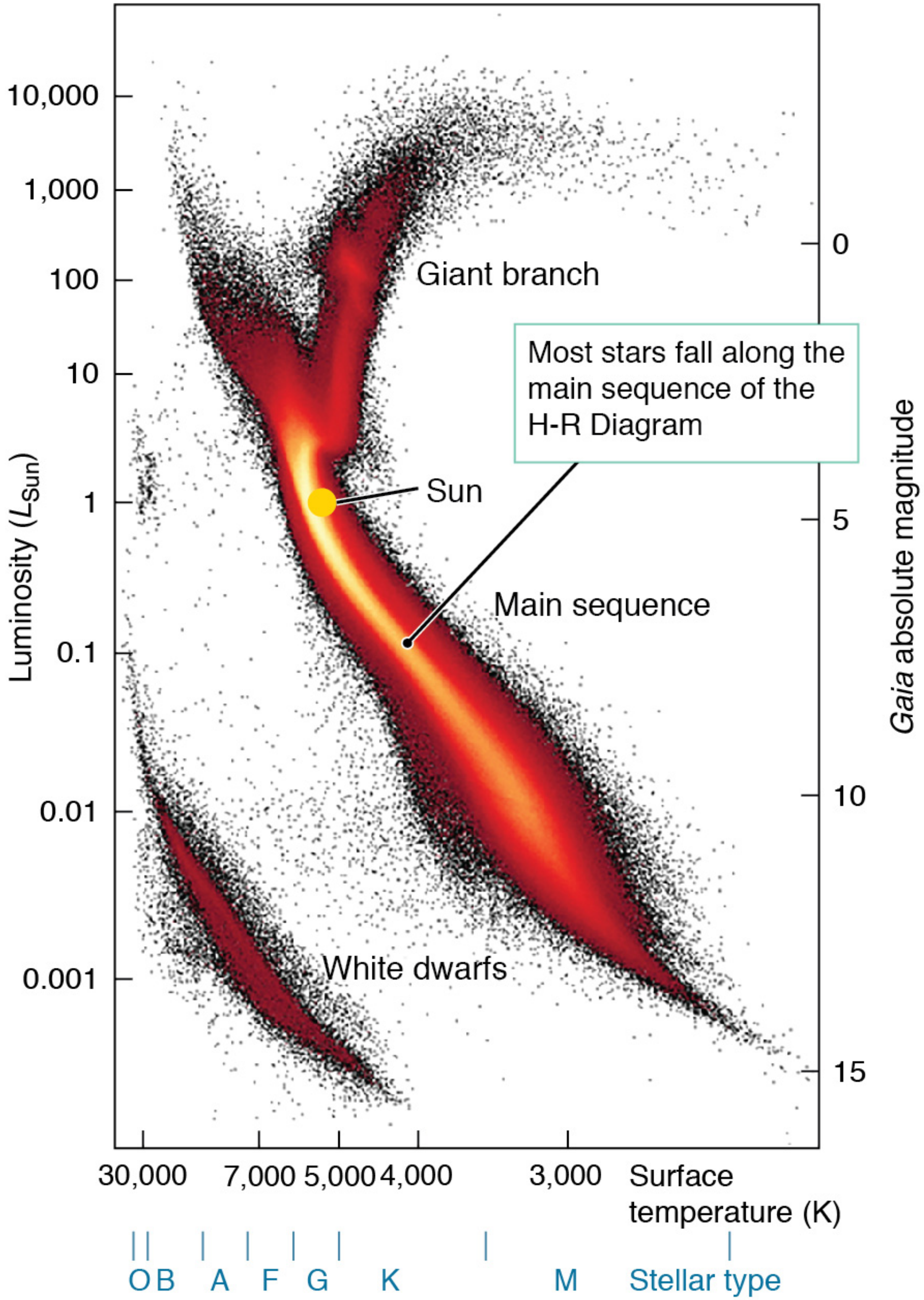
$$B - V = -2.5[\log(f_B/f_V) - \log(f_{B,0}/f_{V,0})]$$


Spec Type	Surface Temperature	B–V ⬍	Apparent Color
O	≥ 33,000 K	−0.33	blue
B	10,000–30,000 K	−0.30	blue white
A	7,500–10,000 K	−0.02	white to blue white
F	6,000–7,500 K	0.30	white
G	5,200–6,000 K	0.58	yellowish white
K	3,700–5,200 K	0.81	yellow orange
M	≤ 3,700 K	1.40	orange red

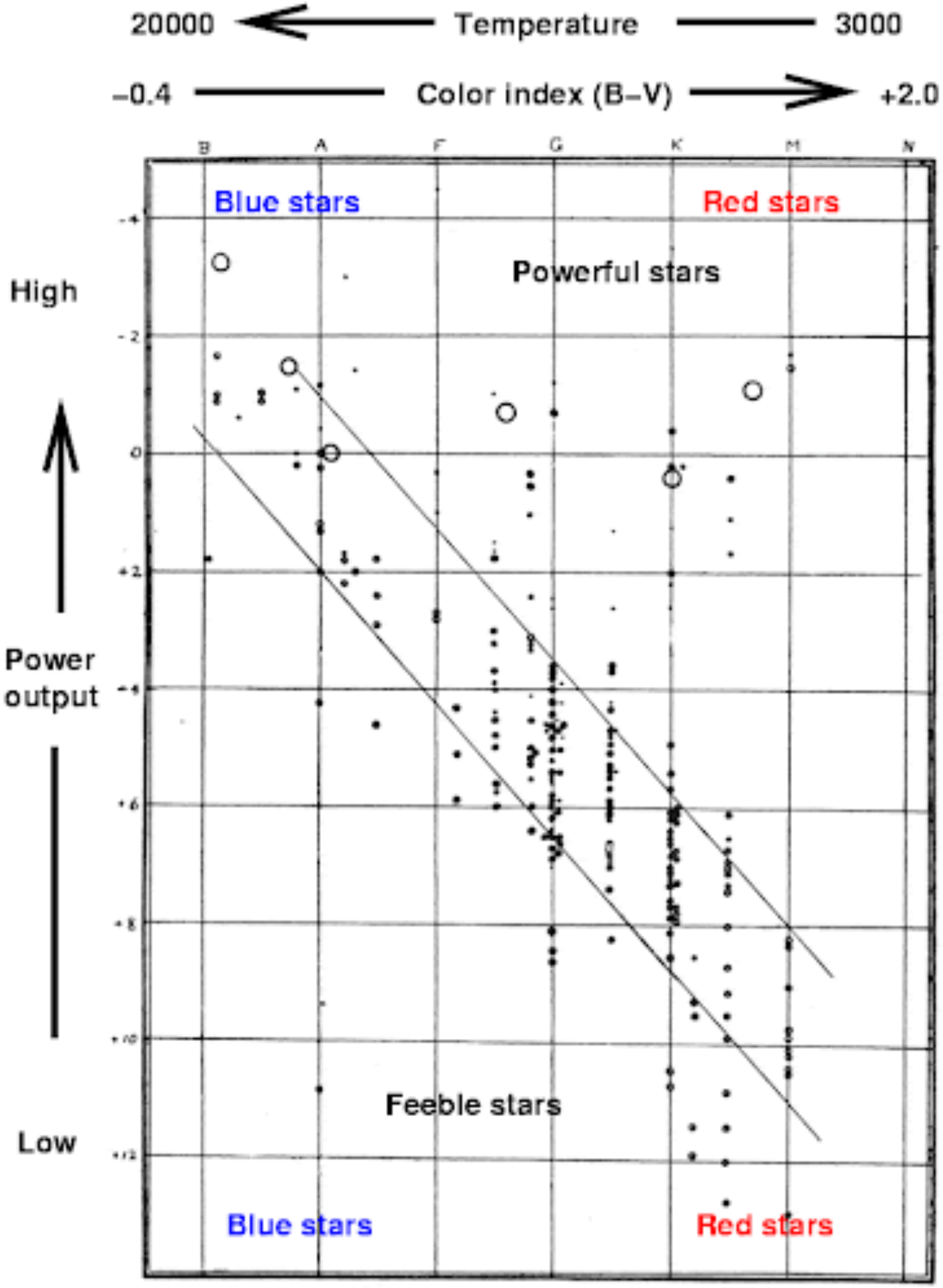
The Hertzsprung-Russell Diagram (with Color Indices)

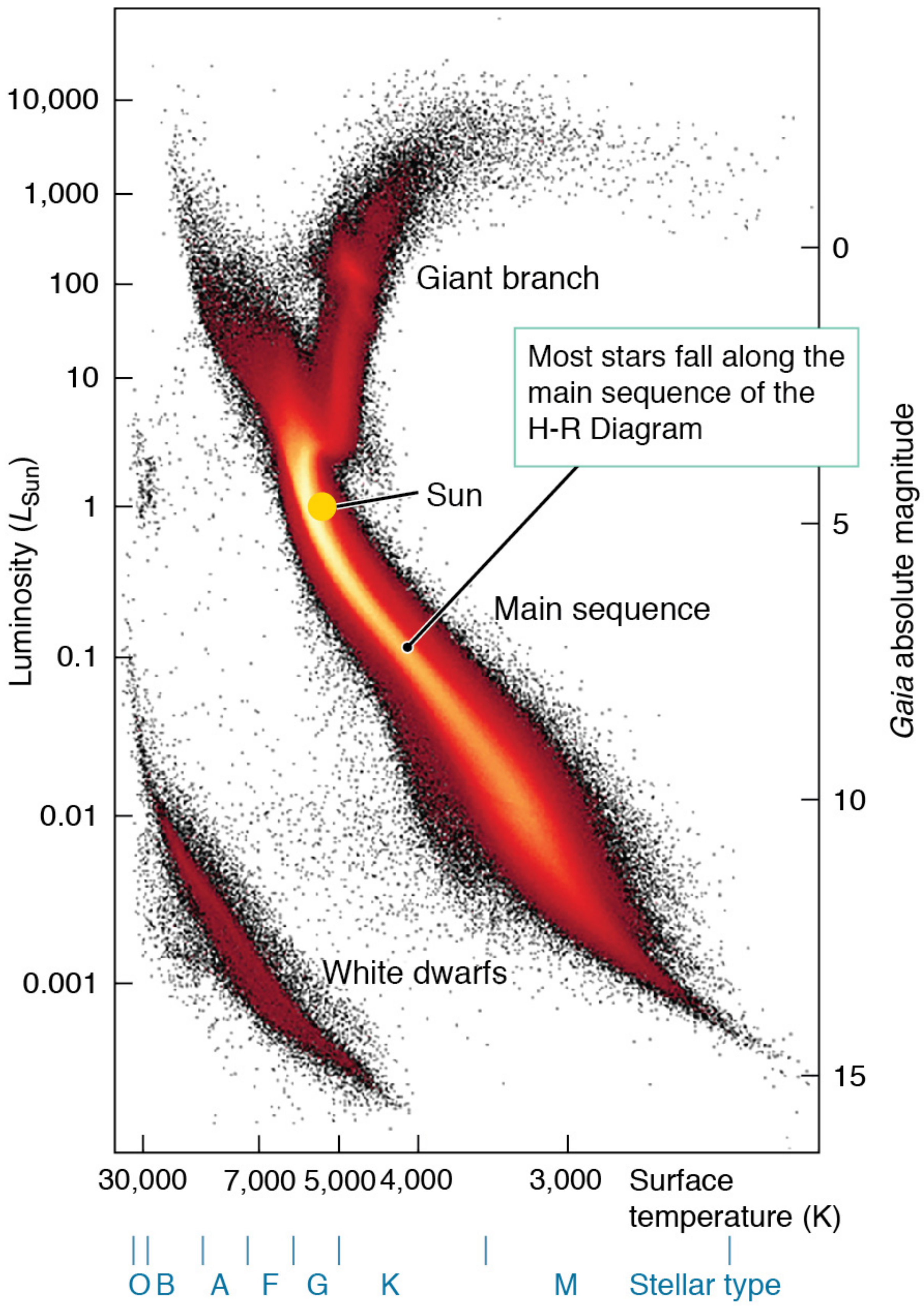




Most stars fall along the main sequence of the H-R Diagram

The Hertzsprung-Russell Diagram (1914 vs. 2020s)





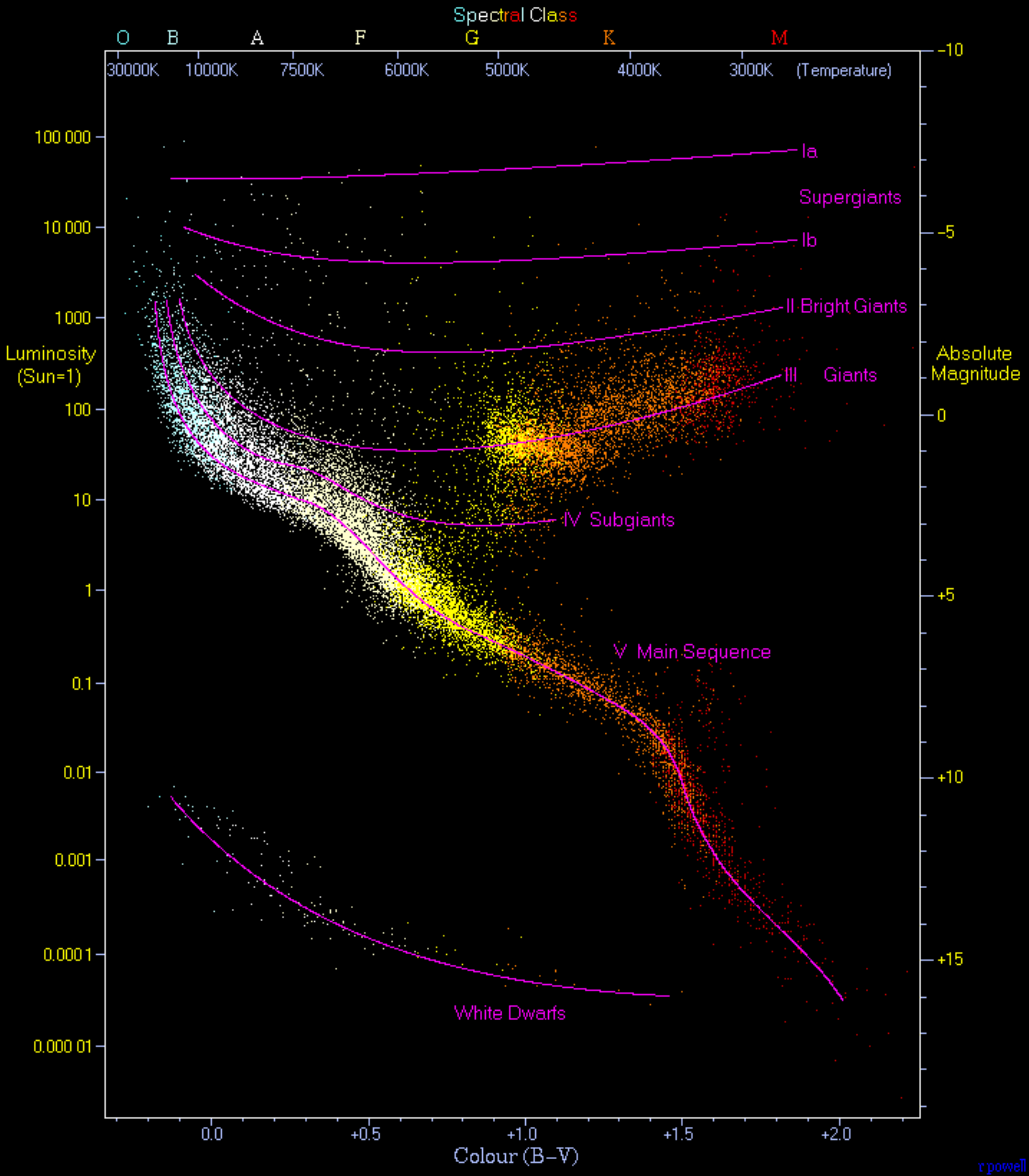
Most stars fall along the main sequence of the H-R Diagram

The Hertzsprung-Russell Diagram:

Implications

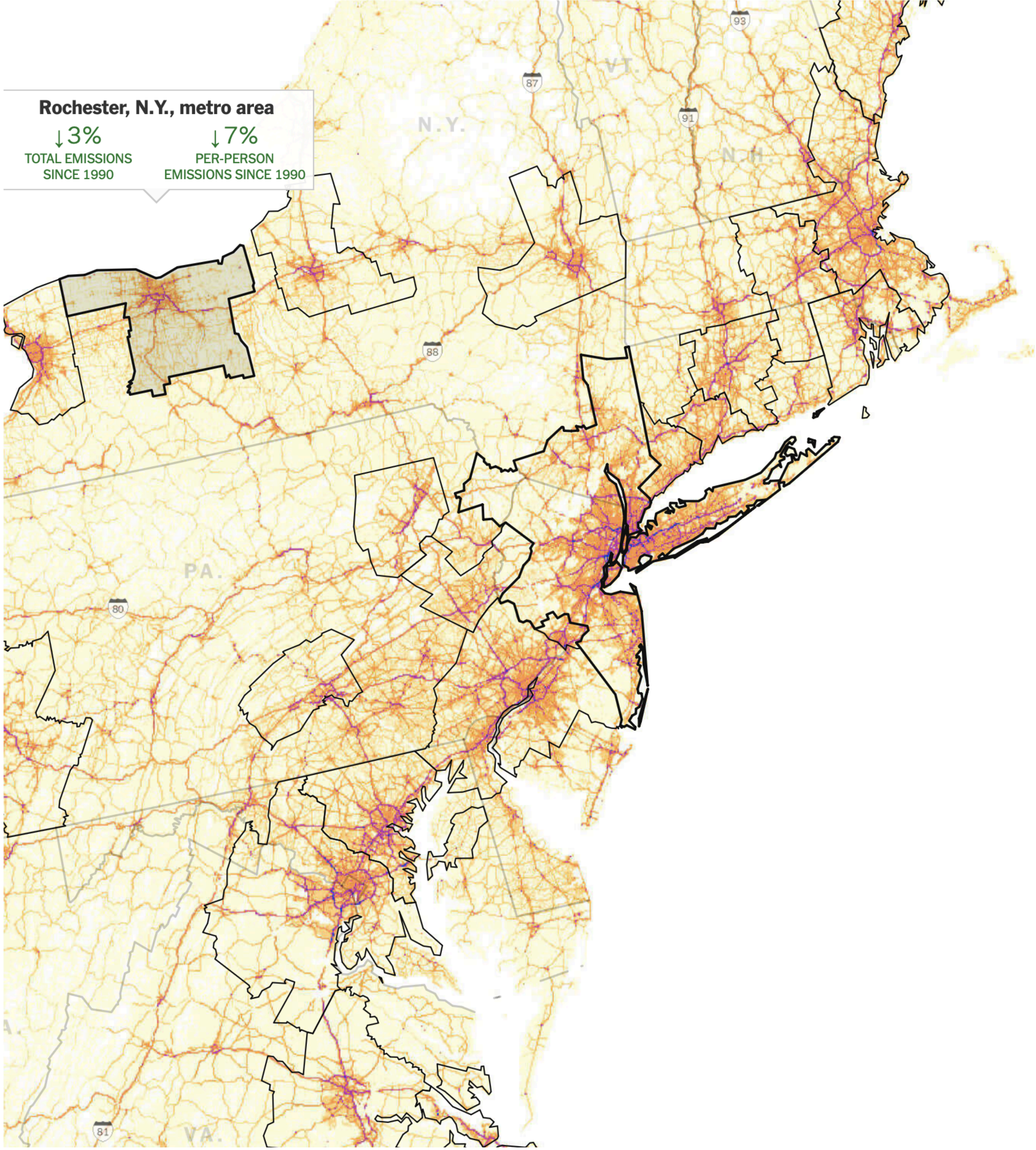
HR Diagram

- The luminosity-temperature diagram is the most important graph in stellar astronomy and is the key to unraveling stellar evolution.
- Two questions to discuss today:
  - What does the concentration of stars in certain areas imply?
  - What does the location of any star on the HRD tell us about its physical properties?



I - Why stars concentrate in certain areas on the HRD

- On the right is the most detailed auto emission map of the US east coast
- It’s made by the New York Times in 2019.
- High emission areas are high concentration areas of internal combustion engines
- Conclusions:
  - People spend much more time in cities than in-between.
  - People spend a lot of time commuting along the DC-NY line



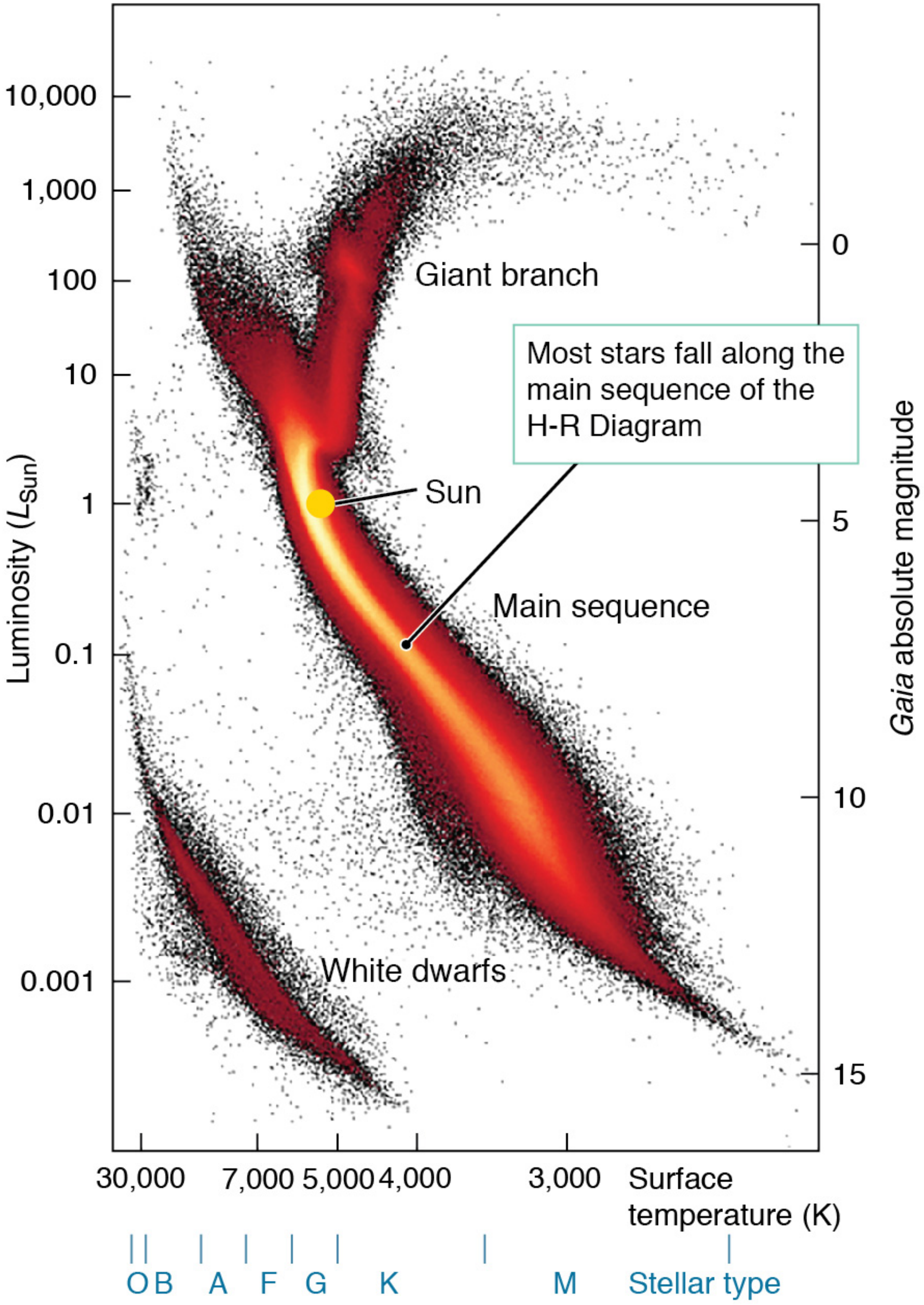
Rochester, N.Y., metro area

↓3% TOTAL EMISSIONS SINCE 1990

↓7% PER-PERSON EMISSIONS SINCE 1990

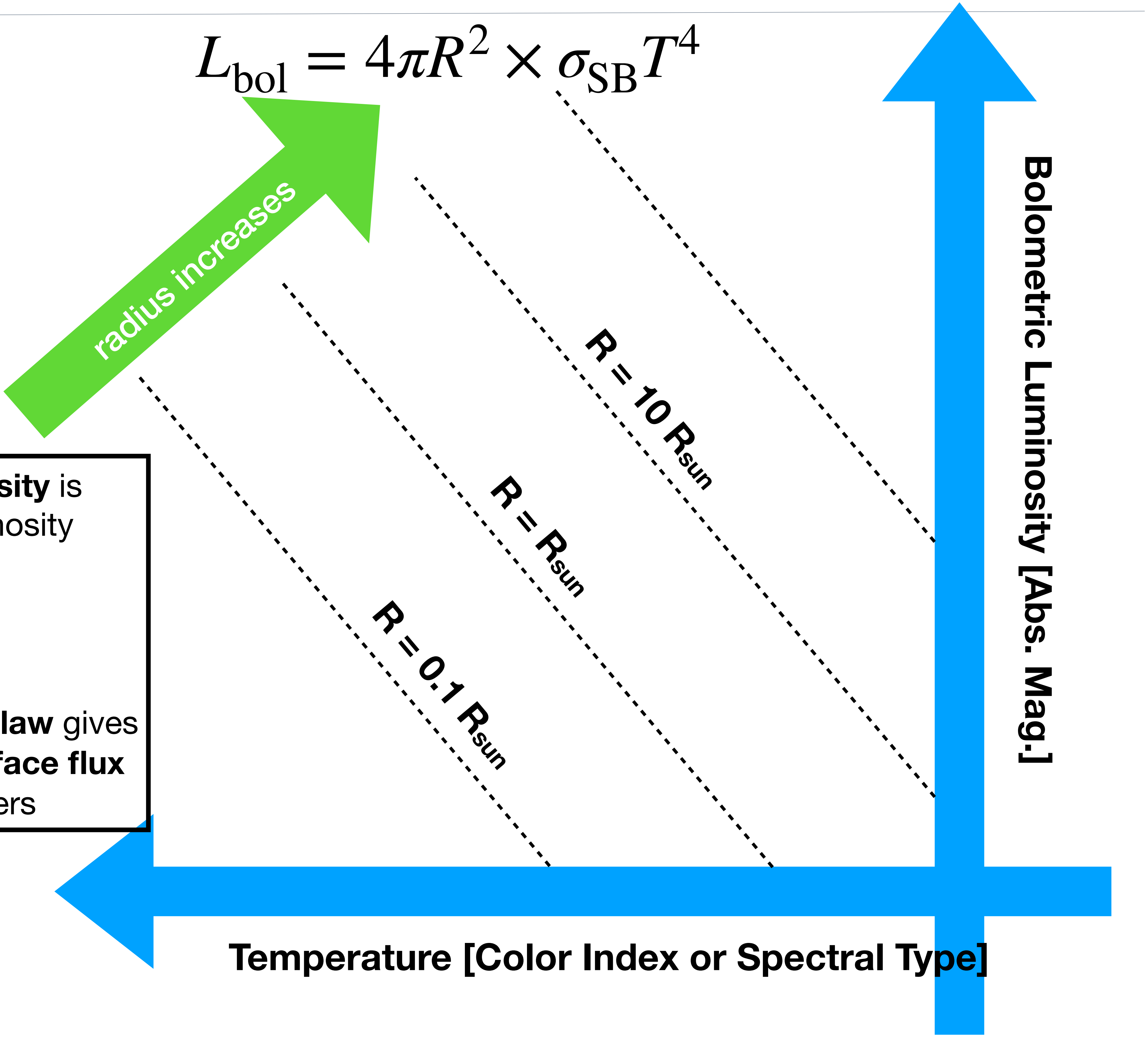
I - Why stars concentrate in certain areas on the HRD

- On the right is the HR diagram made using the Gaia data.
- Bright color indicates high concentration areas of stars:
  - Main sequence
  - Giant branch
  - White dwarfs
- Conclusions:
  - Stars spend much more time in the main sequence, the giant branch, and the white dwarf branch



II - Reading Physical Properties of Stars from the HRD

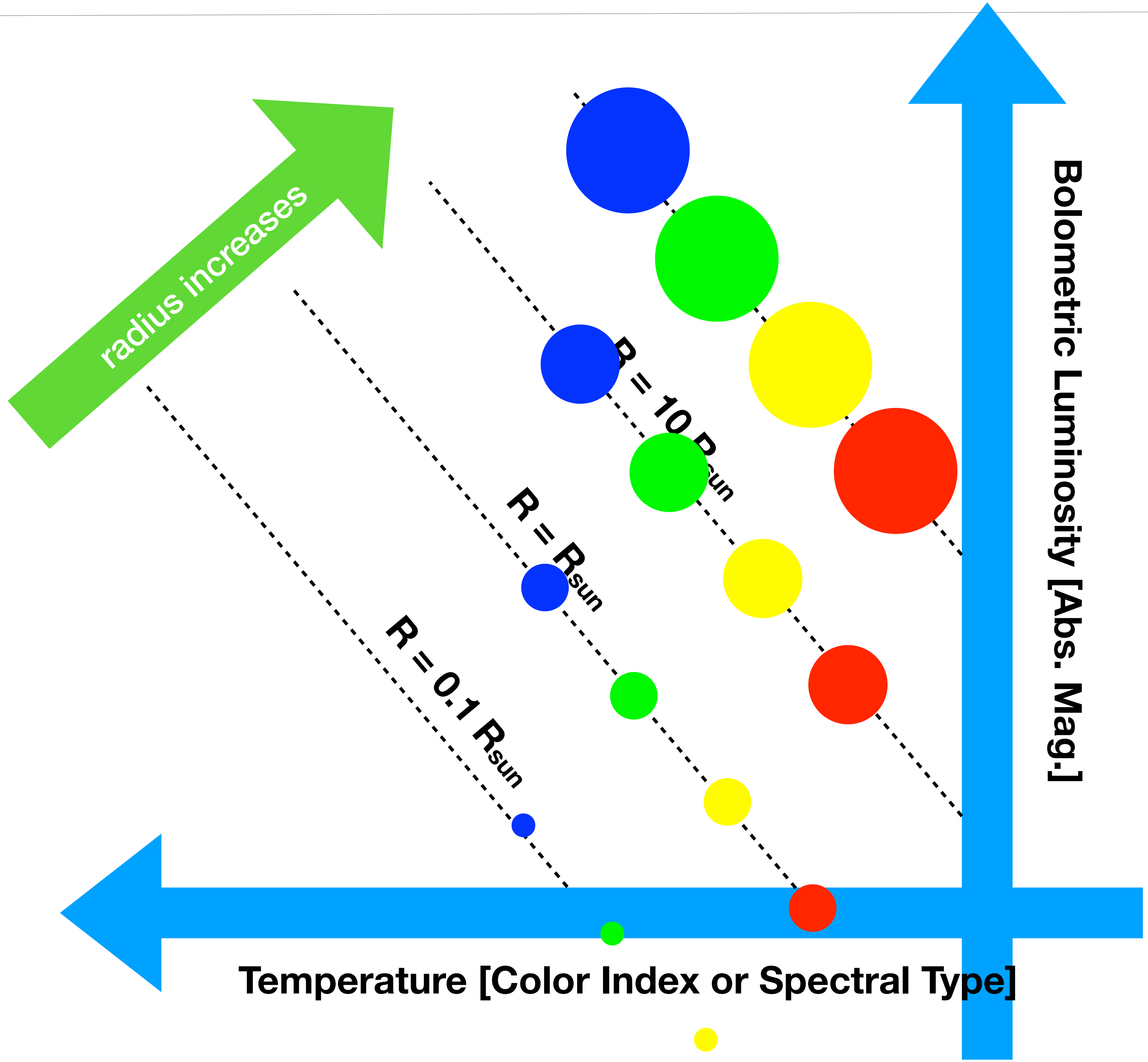
$$L_{\mathrm{bol}} = 4\pi R^2 \times \sigma_{\mathrm{SB}} T^4$$



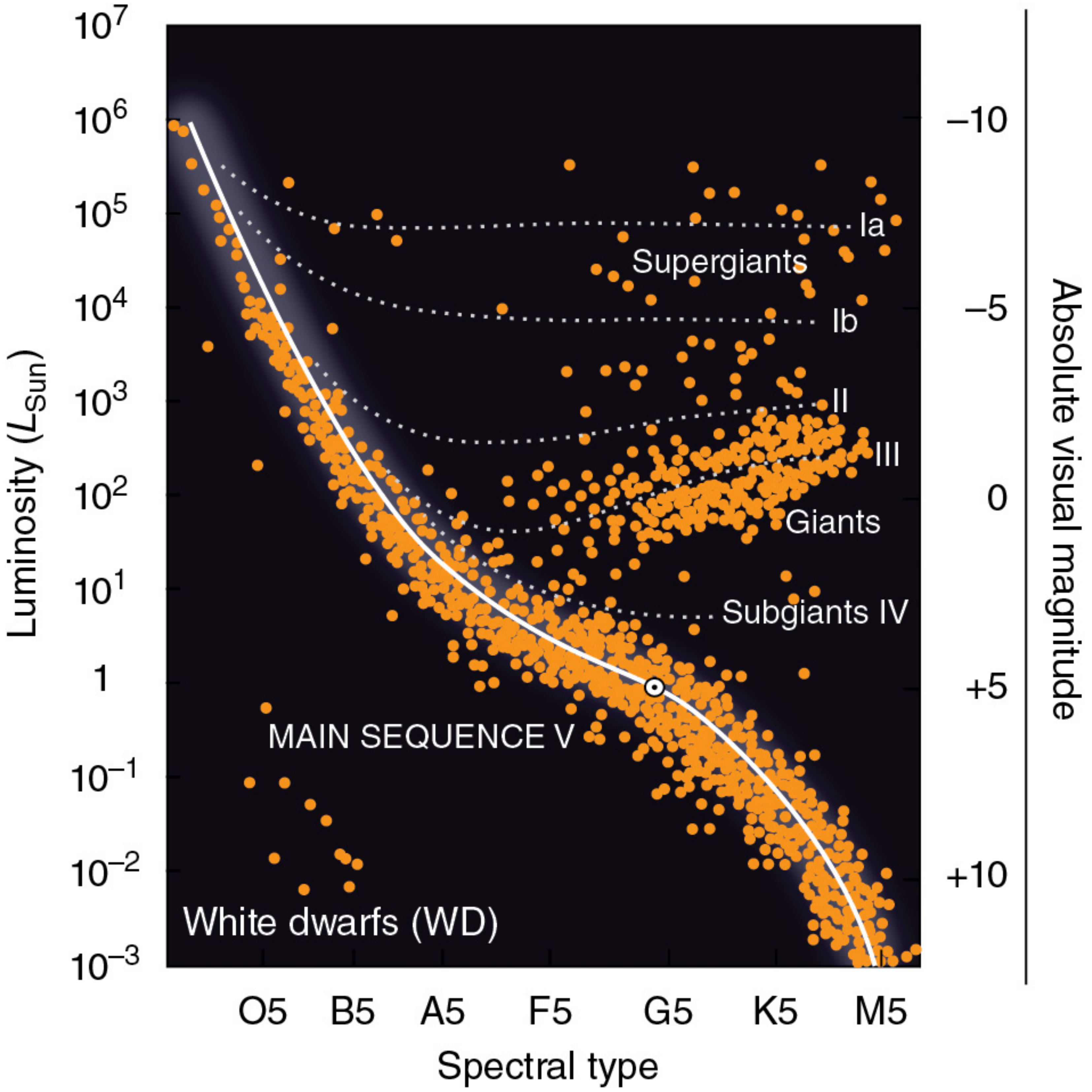
Bolometric luminosity is defined as the luminosity density of a source ***integrated over all wavelengths***.

Stefan-Boltzmann law gives the **bolometric surface flux** of **blackbody** emitters

Size Estimates using Stefan-Boltzmann Law



Luminosity Classes I-V: from Giant Stars and Dwarf Stars

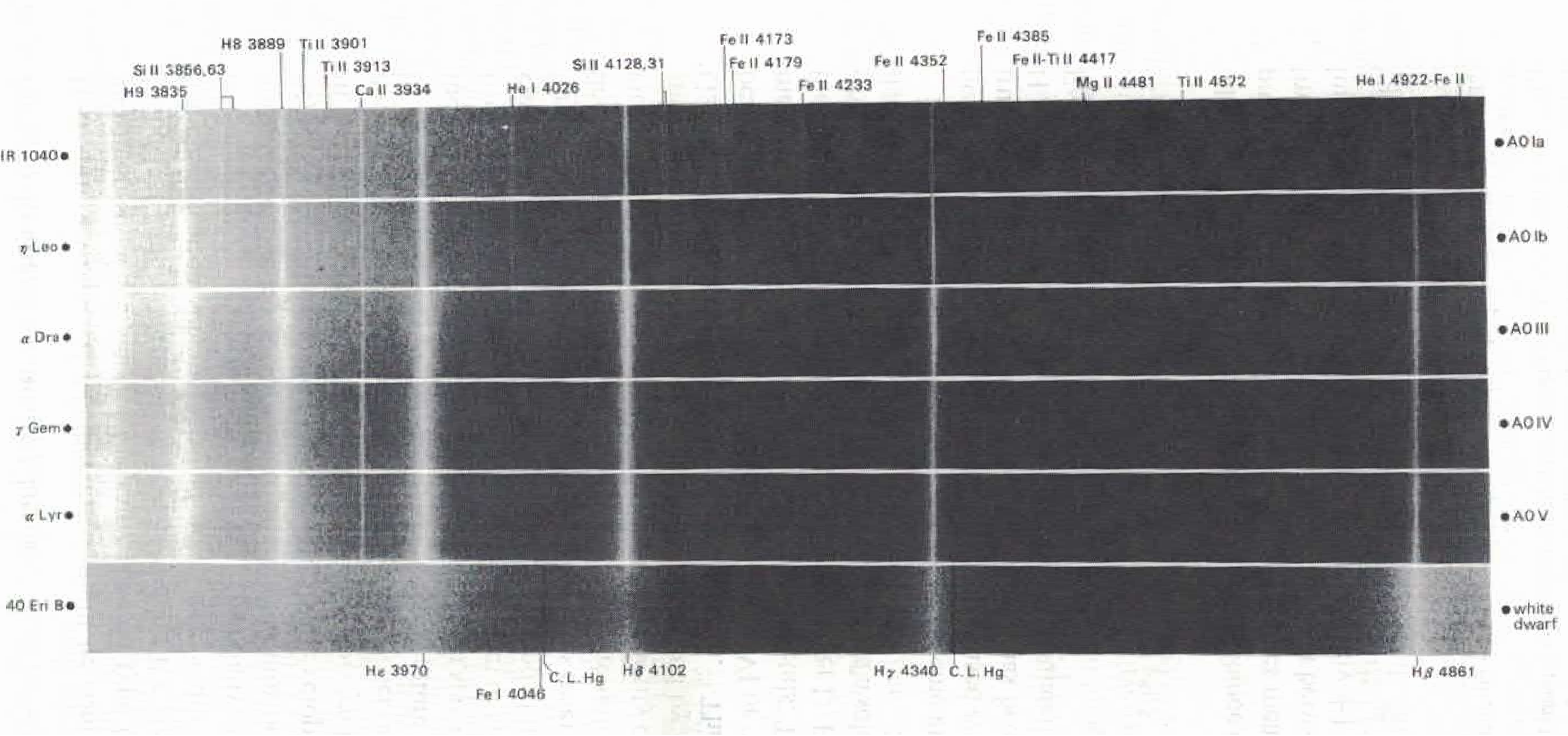


- *Broad* **luminosity classes** are defined roughly along the luminosity axis.
- This makes spectral classification of stars in a **two-dimensional** parameter space: T & L

- The Sun is a **G2V** star:
  G2 - spectral type
  V - luminosity class
- Betelgeuse is a **M1Ia**:
  M1 - spectral type
  Ia - luminosity class

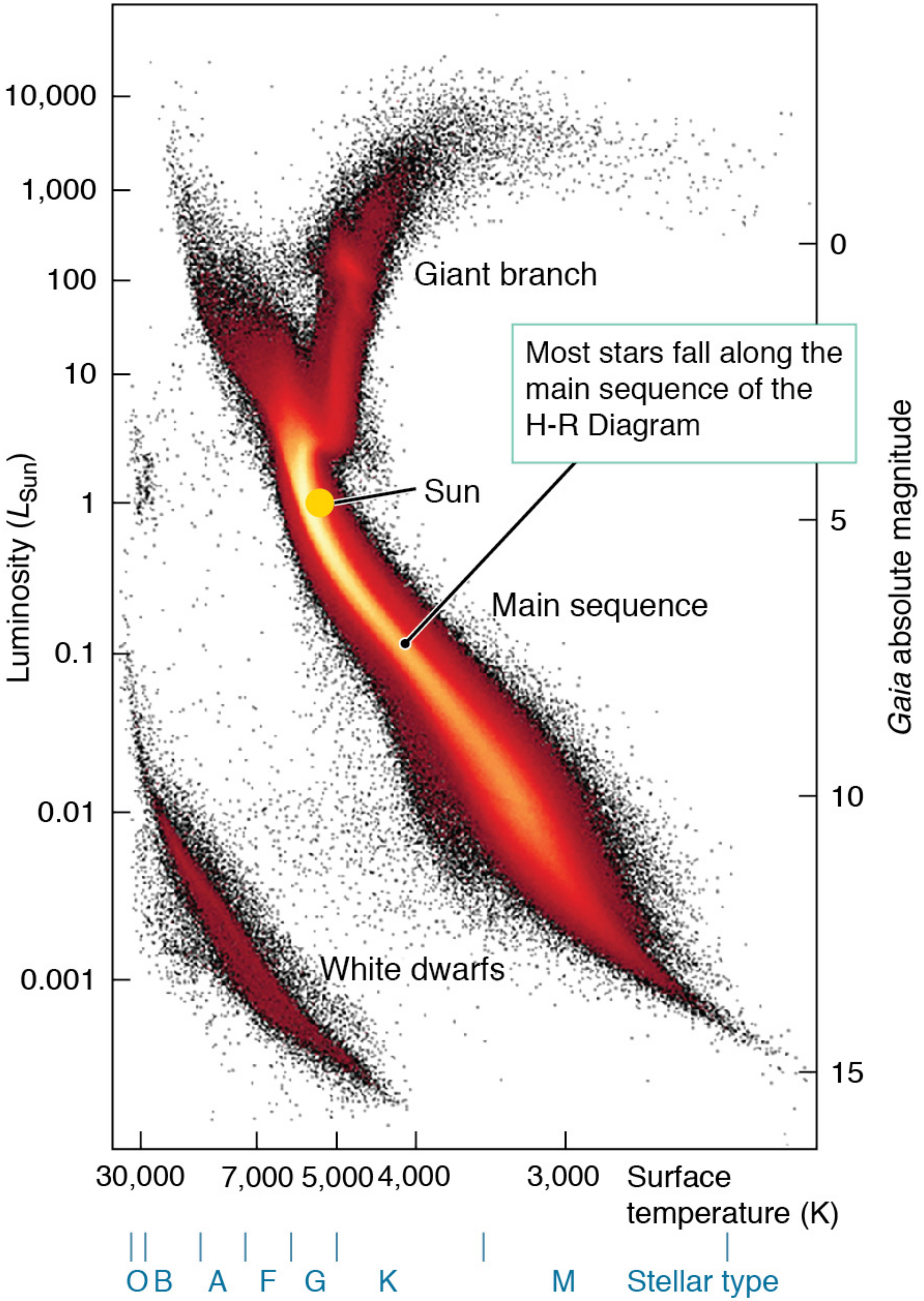
Luminosity Classes - Spectral signatures

- Stars in higher luminosity classes are denser, so they have larger surface gravity. As a result, the **absorption lines appear broader**.
- Below are examples of A0-type stars from class I to V, plus a white dwarf (WD).



Carroll & Ostlie, Fig 8.15

HR Diagram: Main Sequence, Giant Branch, White Dwarfs, & Luminosity Classes

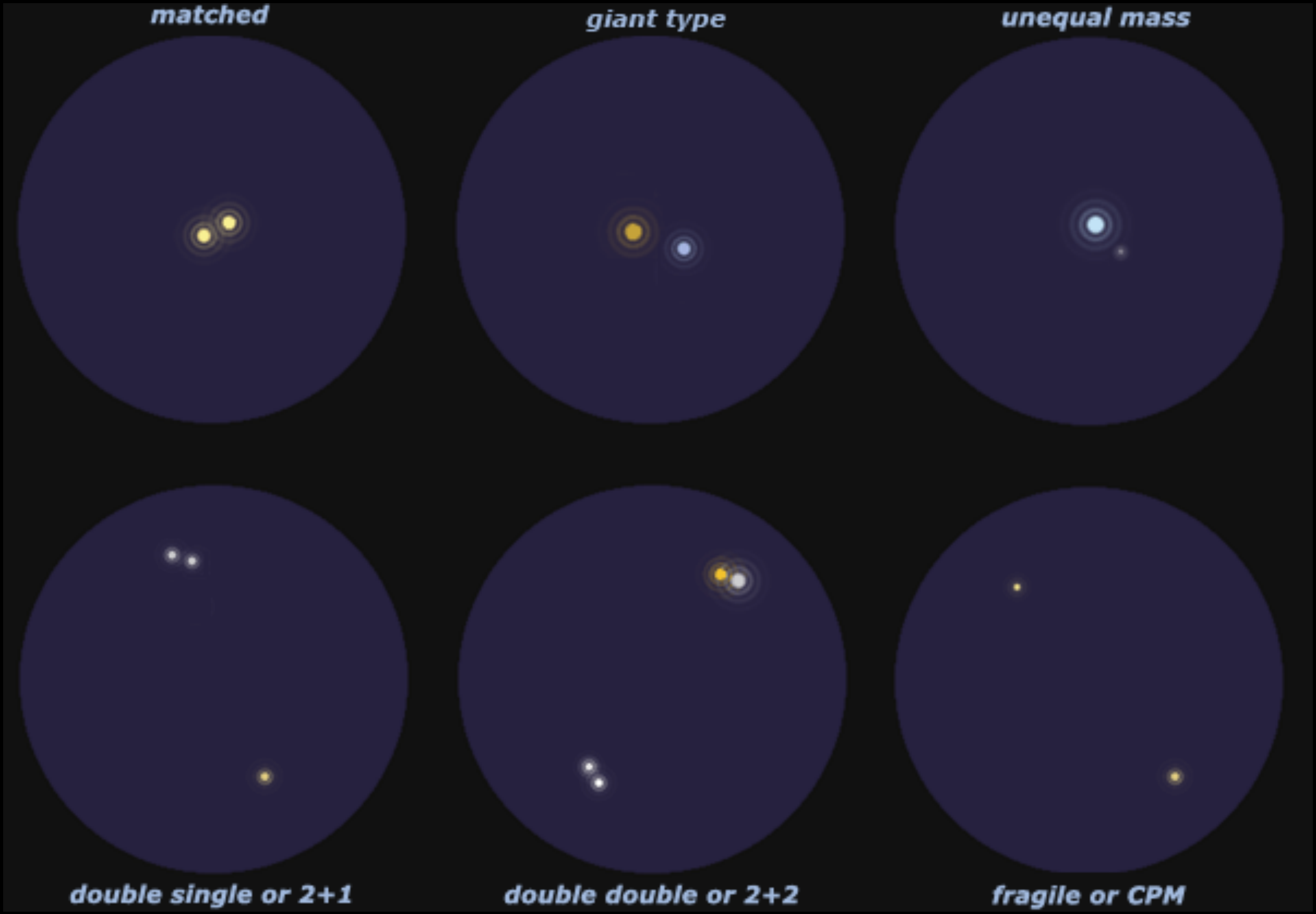


- Most stars, incl. the Sun, are found on the **main sequence,** which runs from luminous/hot stars in upper left corner to low-luminosity/cool stars in lower right corner
- It covers a temperature range of **~10**, and a radius range of **~100**, but a luminosity range of **~10^9**
- The **Giant branch** is connected to the main sequence but branches off to the lower temperature side. That is where the **red giant stars** live
- A separate branch parallel to the main sequence to the lower left, these stars have low luminosities but hot temperatures; this is where the **White Dwarfs** live
- **Spectral classification is 2-dimensional**: (1) temperature (OBAFGKM), and (2) luminosity (Ia, Ib, II, III, IV, V)

How to measure stellar mass?

Binary stars and Kepler’s Laws

The various configurations of visual binaries and multiples



B. MacEvoy 2012

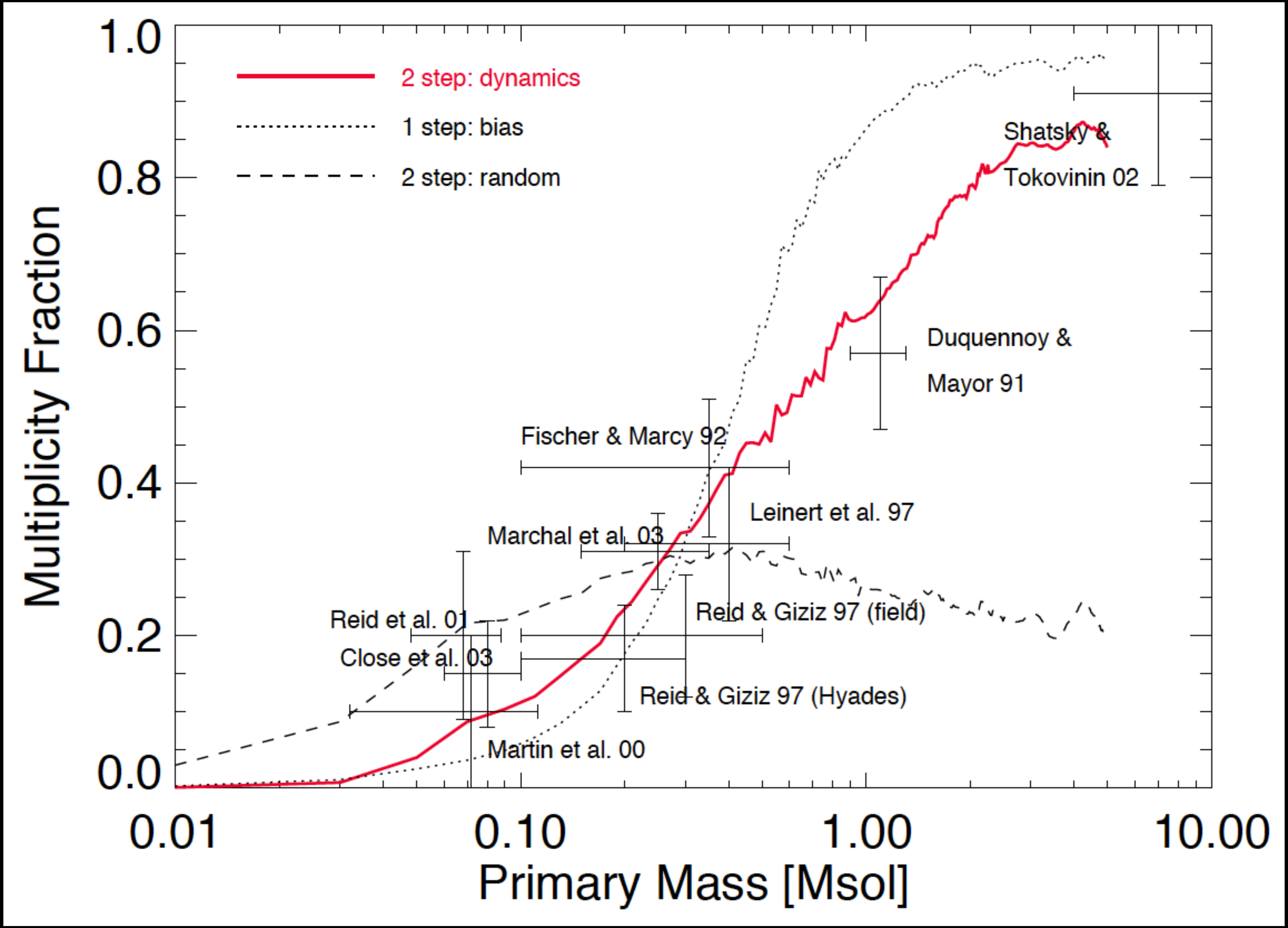
What fraction of stars are binaries?

About 60% of all stars are components of binary or multiple star systems for solar-type stars.

	Kuiper (1942)	*Heintz (1969)*	*Abt & Levy (1976)**	*Duquennoy & Mayor (1991)*	*Nordström et al. (2004)*	*Raghavan et al. (2010)*
Systems (N)	274	*n.a.*	123	164	16682	454
Single Star Systems	70%	30%	45%	**57%**	66%	**56%**
Binary	25%	47%	46%	38%	34%	33%
3	4%	16%	8%	4%	.	8%
4+	1%	7%	1%	1%	.	3%
Double Star Systems	30%	70%	55%	43%	34%	**44%**
Median R		50 *AU*		35 *AU*		40 *AU*
Stars in Doubles	52%	85%	73%	**62%**	51%	**65%**

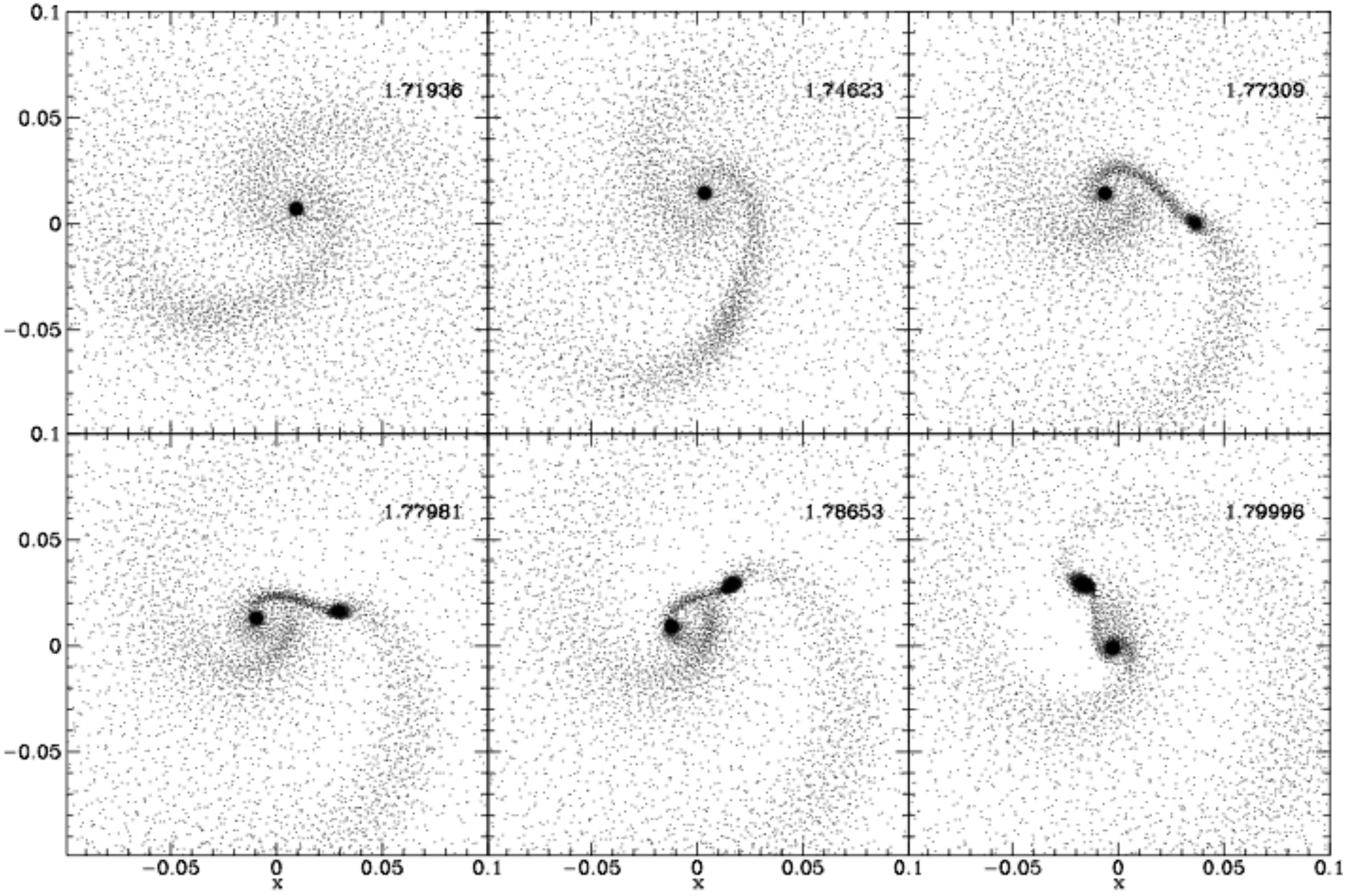
B. MacEvoy 2012

The multiplicity fraction increases with the mass of the primary



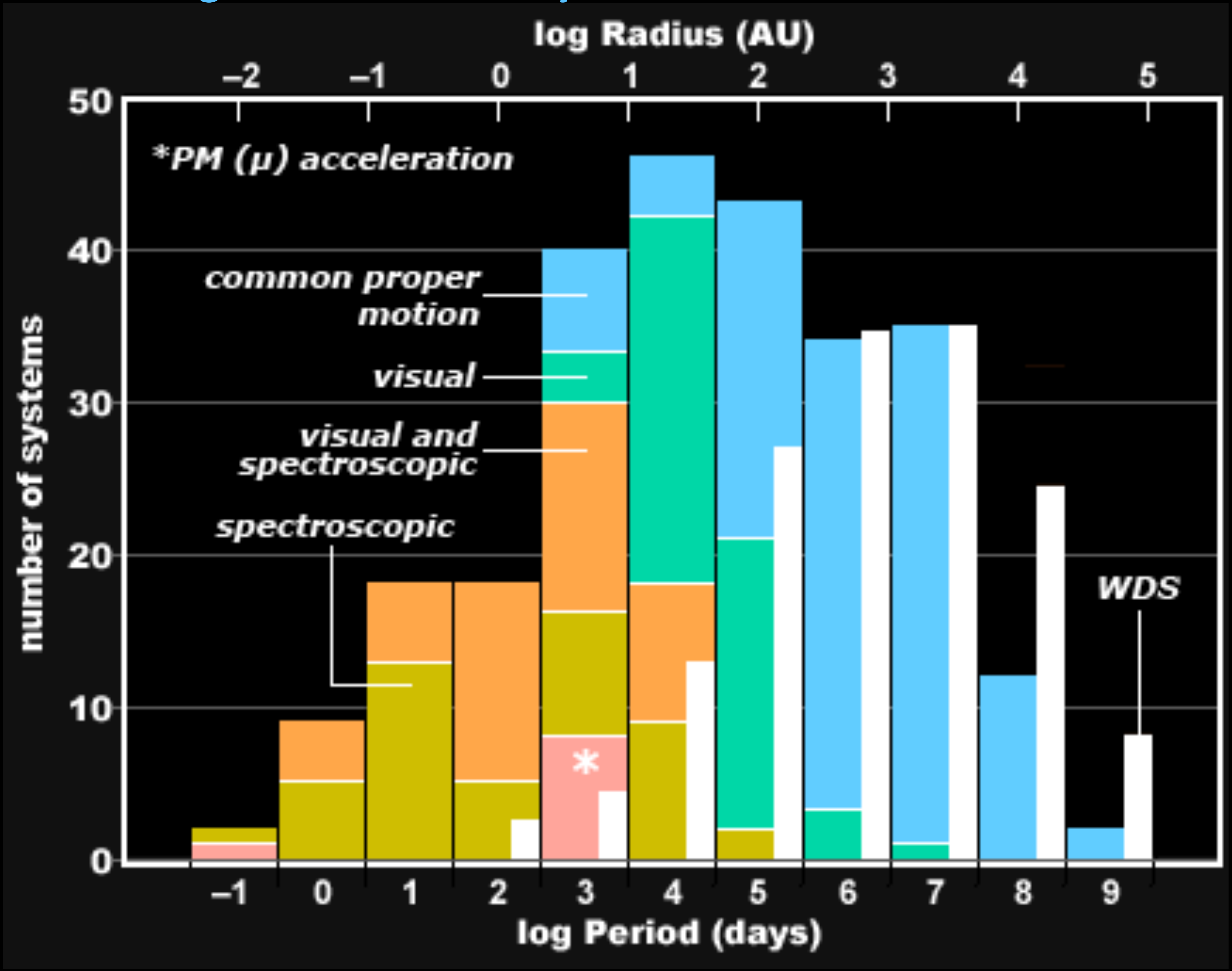
B. MacEvoy 2012

Binary Star Formation: Accretion Disk Fragmentation



Fragmentation of the protostar accretion disk is believed to be a frequent if not the most common path to binary formation at distances of around 40 AU … a massive spiral arm forces the protostar off the center of mass to produce a binary structure; the spiral arms draw more mass into the accretion disk while reducing the binary orbital momentum via gravitational (and possibly magnetic) torque (Source: Bonnell & Bate, 1994)

The logarithmic of Binary Periods follow a “Bell” curve



Binary stars tell us the masses of stars

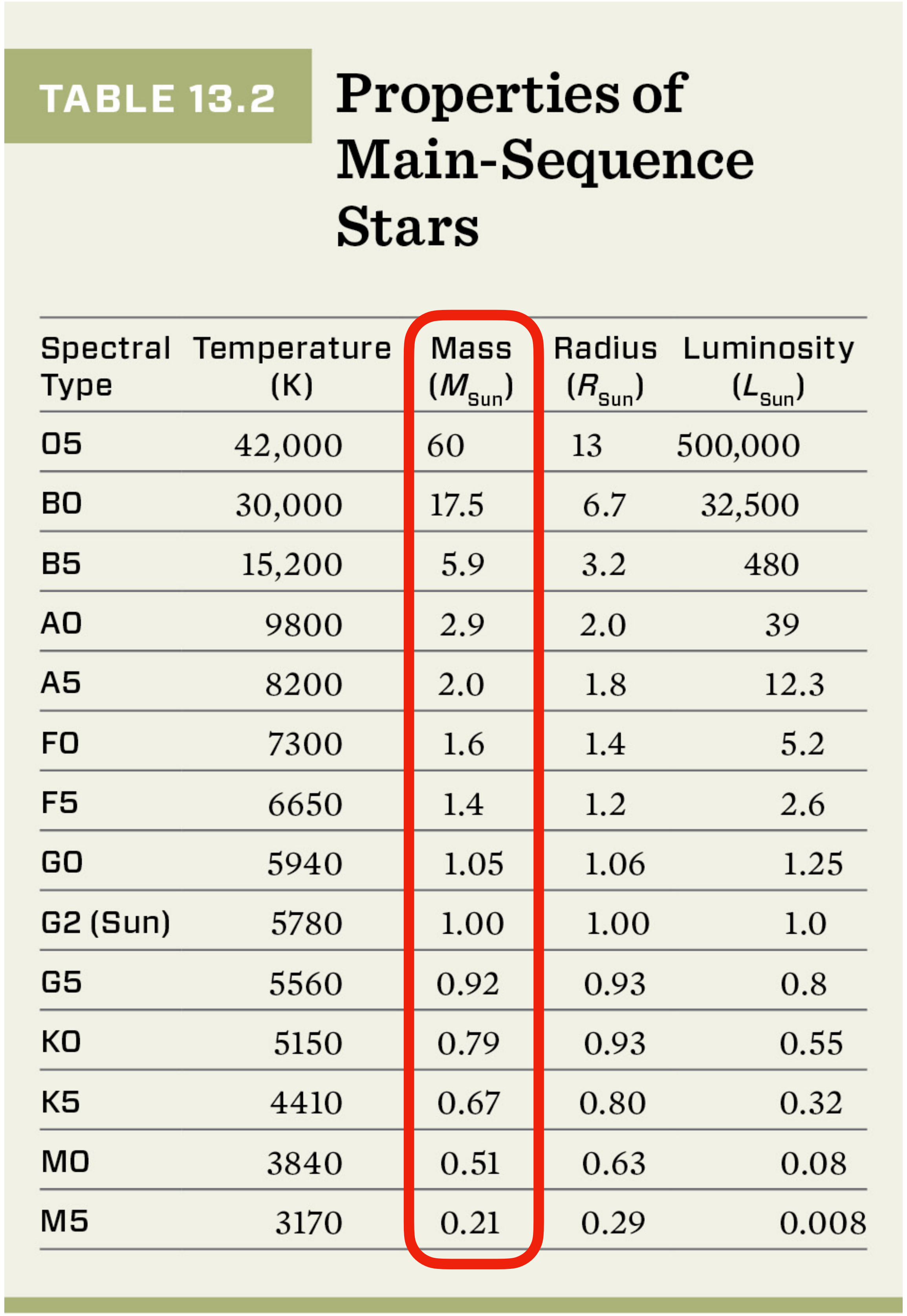
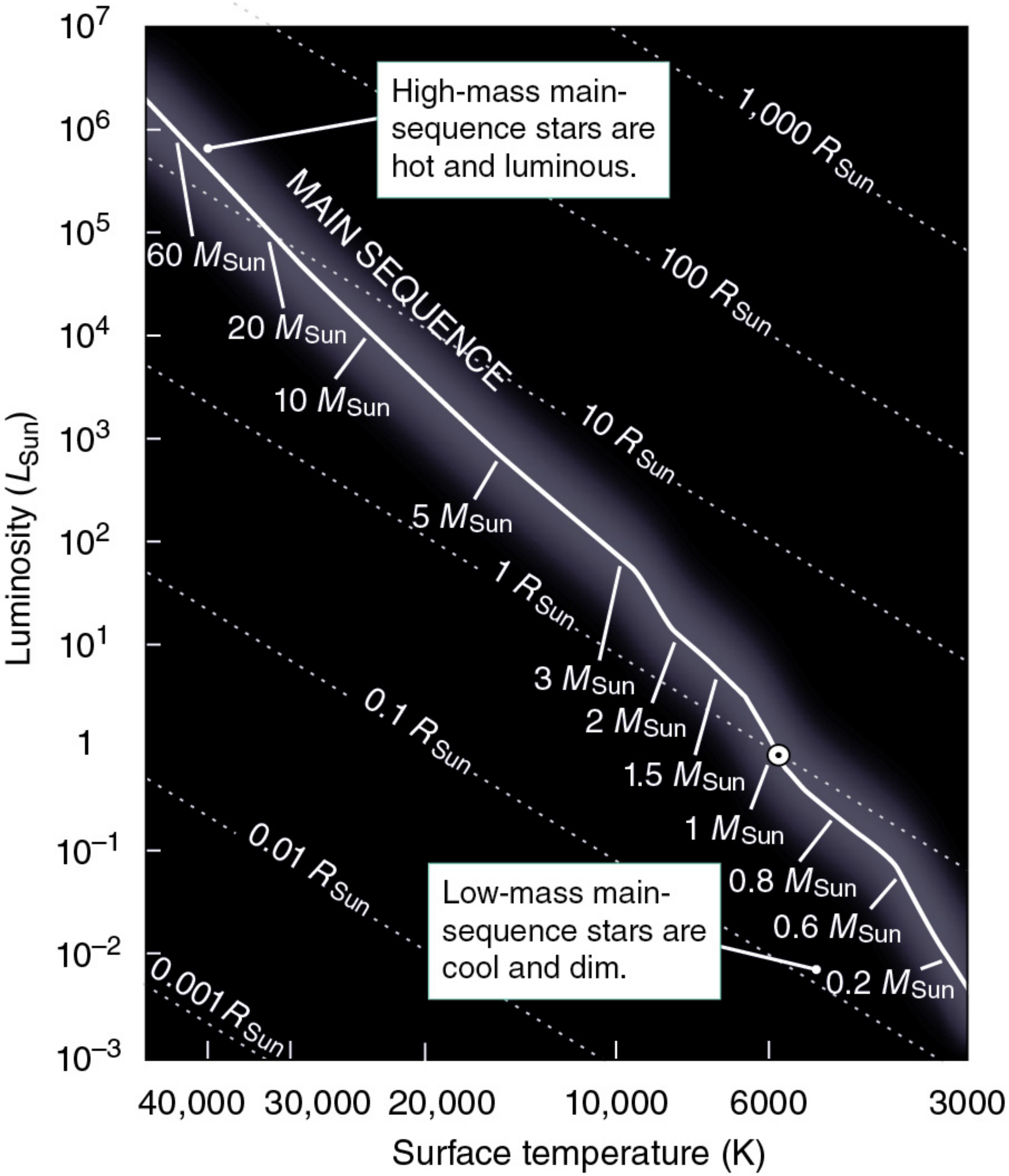


TABLE 13.2 Properties of Main-Sequence Stars

Spectral Type	Temperature (K)	Mass (M_{Sun})	Radius (R_{Sun})	Luminosity (L_{Sun})
O5	42,000	60	13	500,000
B0	30,000	17.5	6.7	32,500
B5	15,200	5.9	3.2	480
A0	9800	2.9	2.0	39
A5	8200	2.0	1.8	12.3
F0	7300	1.6	1.4	5.2
F5	6650	1.4	1.2	2.6
G0	5940	1.05	1.06	1.25
G2 (Sun)	5780	1.00	1.00	1.0
G5	5560	0.92	0.93	0.8
K0	5150	0.79	0.93	0.55
K5	4410	0.67	0.80	0.32
M0	3840	0.51	0.63	0.08
M5	3170	0.21	0.29	0.008



High-mass main-sequence stars are hot and luminous.

Low-mass main-sequence stars are cool and dim.

which led us to conclude that the stars have different MS lifetimes

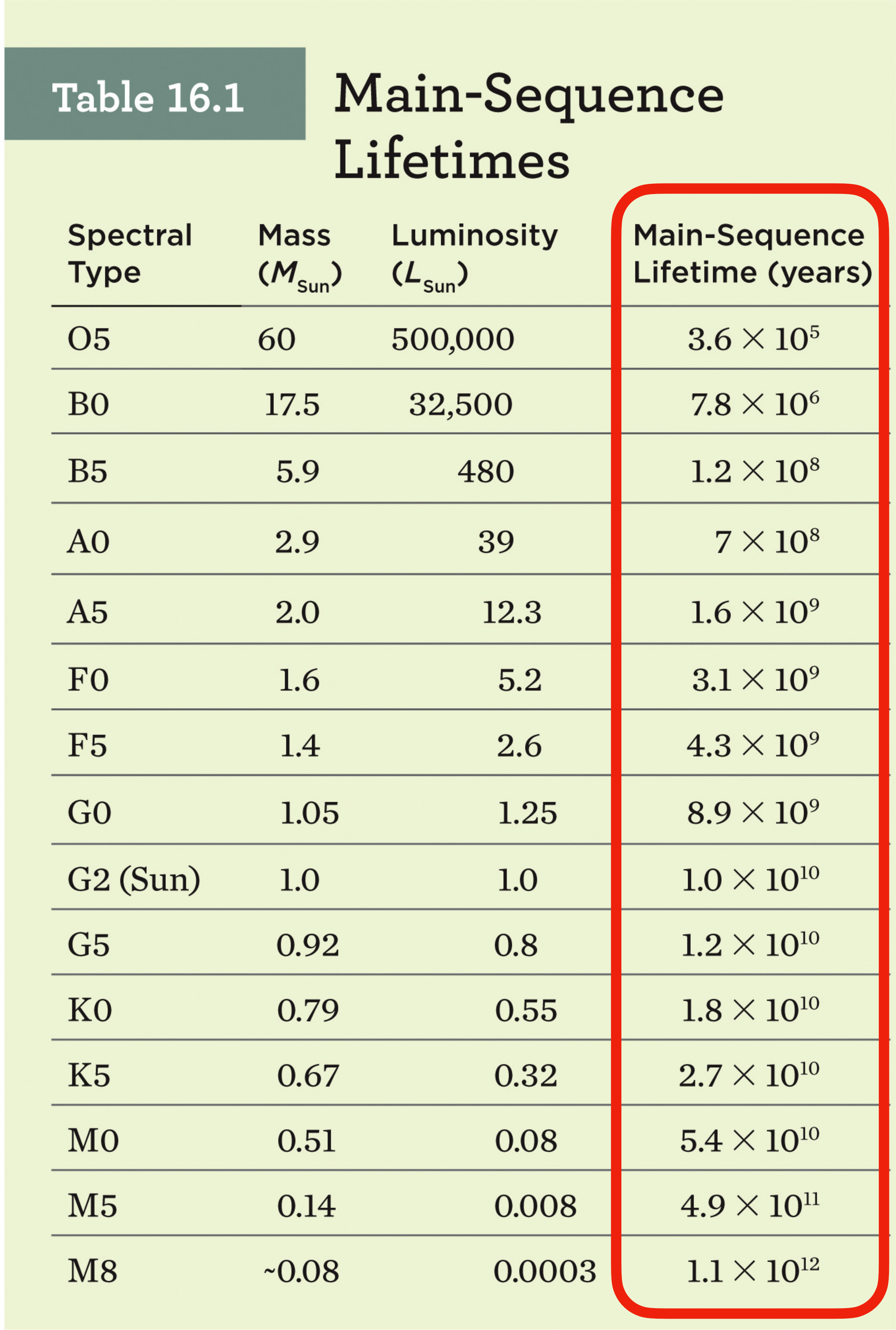
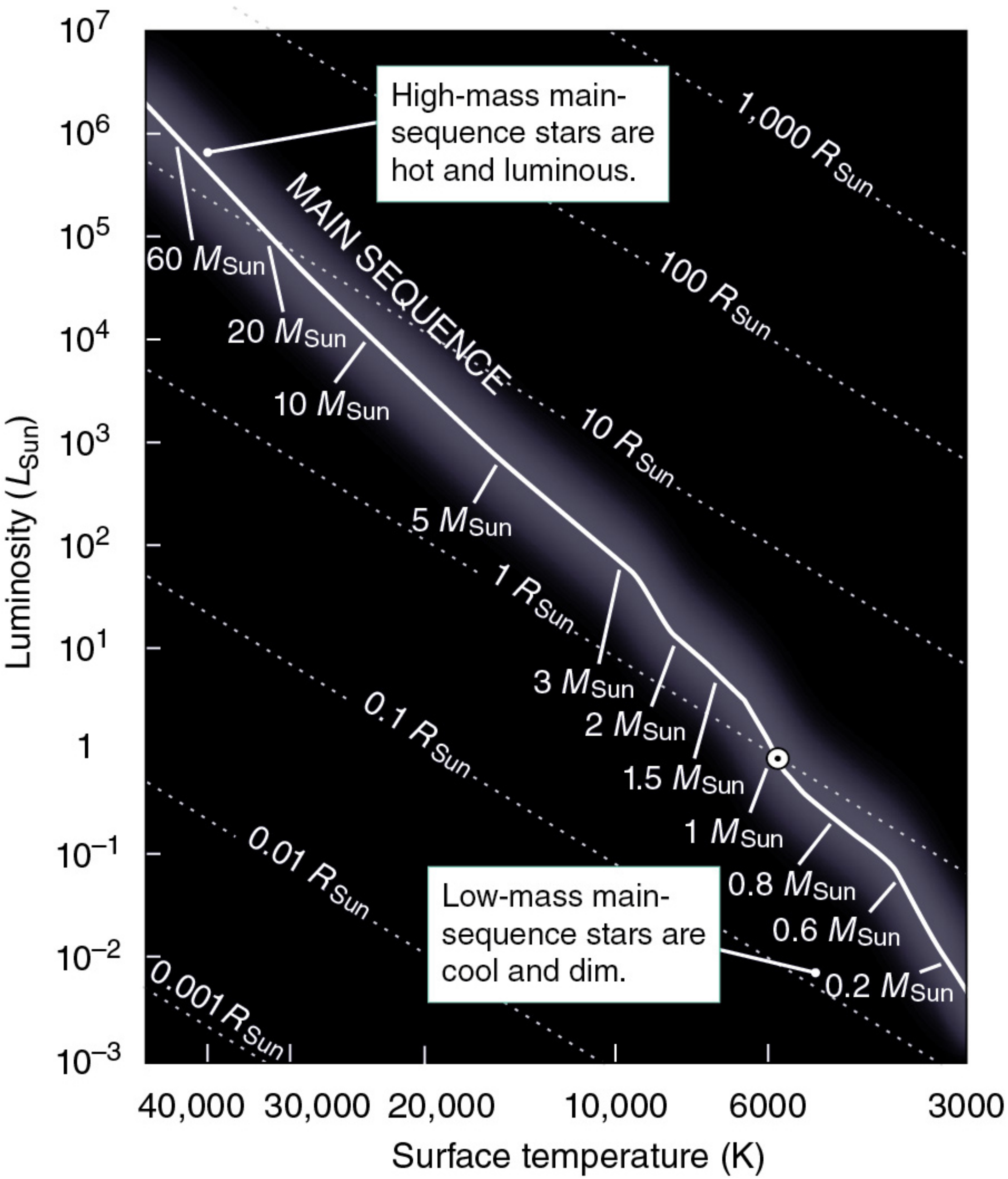


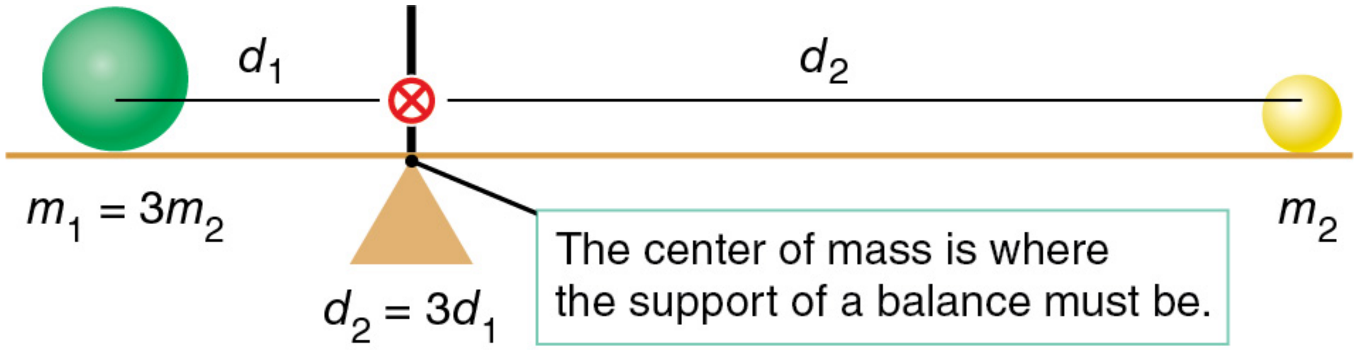
Table 16.1 Main-Sequence Lifetimes

Spectral Type	Mass (M_{Sun})	Luminosity (L_{Sun})	Main-Sequence Lifetime (years)
O5	60	500,000	3.6×10^5
B0	17.5	32,500	7.8×10^6
B5	5.9	480	1.2×10^8
A0	2.9	39	7×10^8
A5	2.0	12.3	1.6×10^9
F0	1.6	5.2	3.1×10^9
F5	1.4	2.6	4.3×10^9
G0	1.05	1.25	8.9×10^9
G2 (Sun)	1.0	1.0	1.0×10^{10}
G5	0.92	0.8	1.2×10^{10}
K0	0.79	0.55	1.8×10^{10}
K5	0.67	0.32	2.7×10^{10}
M0	0.51	0.08	5.4×10^{10}
M5	0.14	0.008	4.9×10^{11}
M8	~0.08	0.0003	1.1×10^{12}



Binary Star - Center of Mass

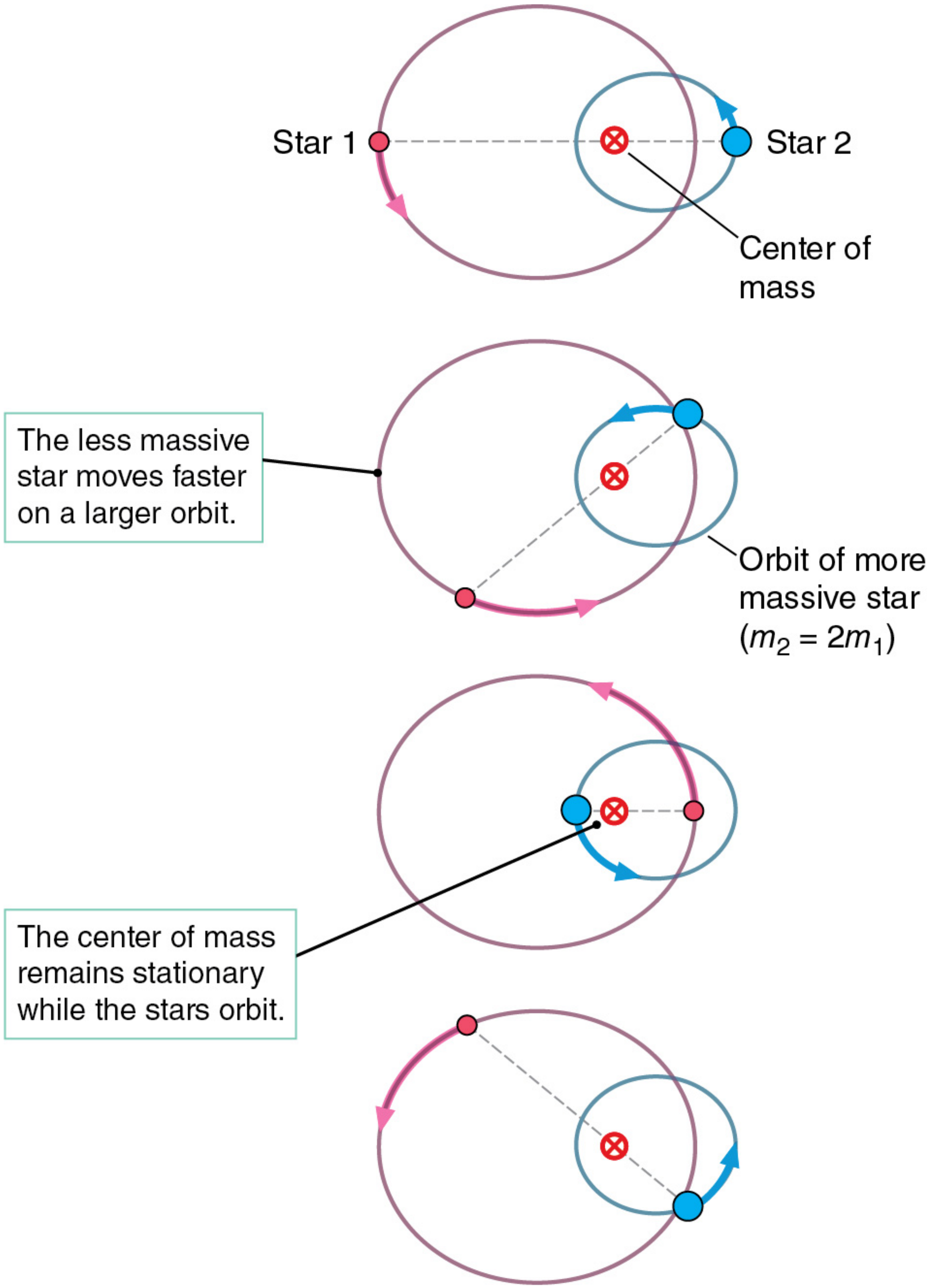
- To measure mass, we must look for the effects of gravity.
- Many stars are **binary stars** orbiting a common **center of mass**.
- A less massive star moves faster on a larger orbit.



The center of mass is where the support of a balance must be.

Center of mass "seesaw" equation:

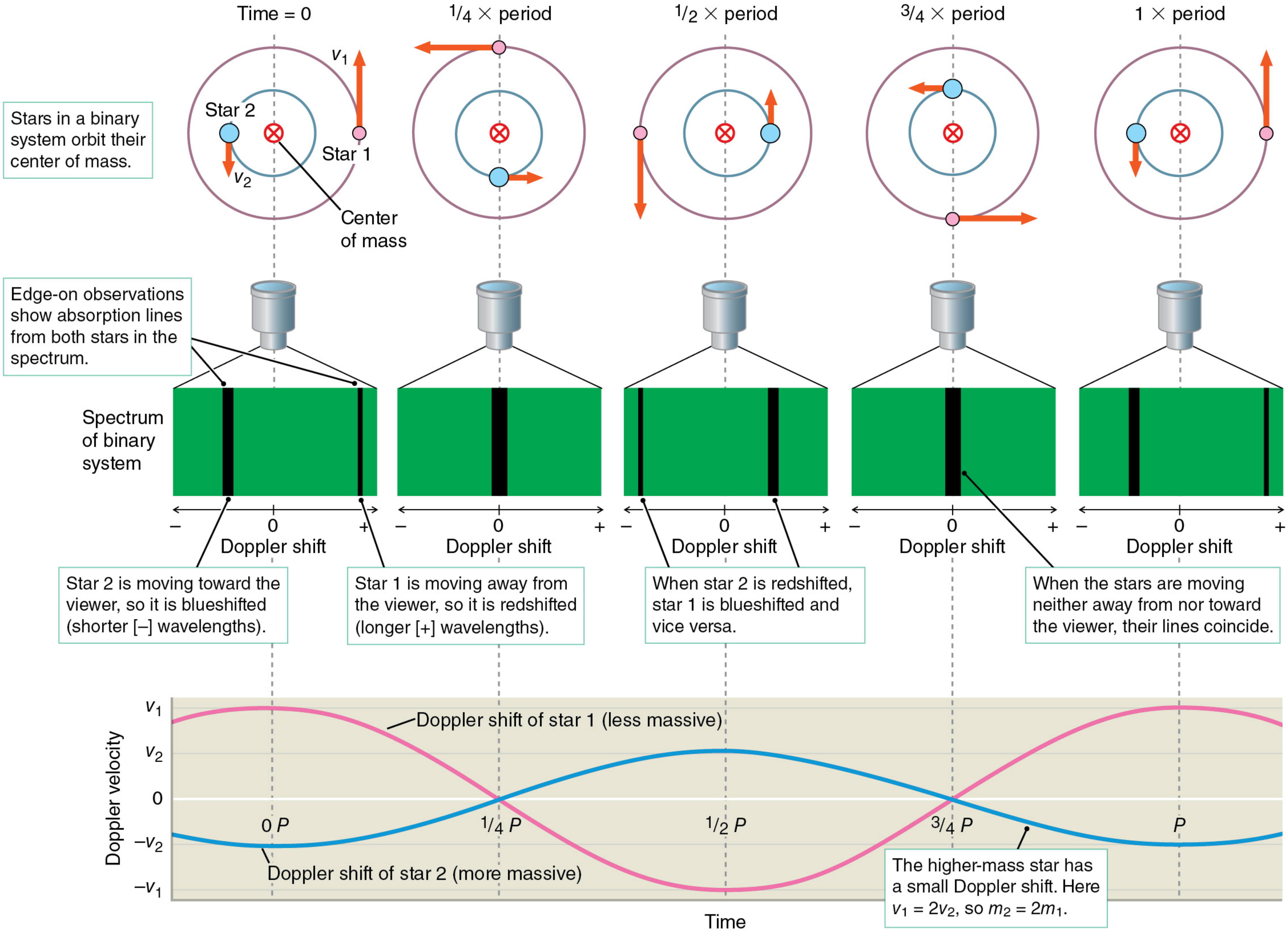
$m_1 d_1 = m_2 d_2$



The less massive star moves faster on a larger orbit.

The center of mass remains stationary while the stars orbit.

Binary Star - Doppler Shift Measurements vs. Time



Stars in a binary system orbit their center of mass.

Edge-on observations show absorption lines from both stars in the spectrum.

Star 2 is moving toward the viewer, so it is blueshifted (shorter [–] wavelengths).

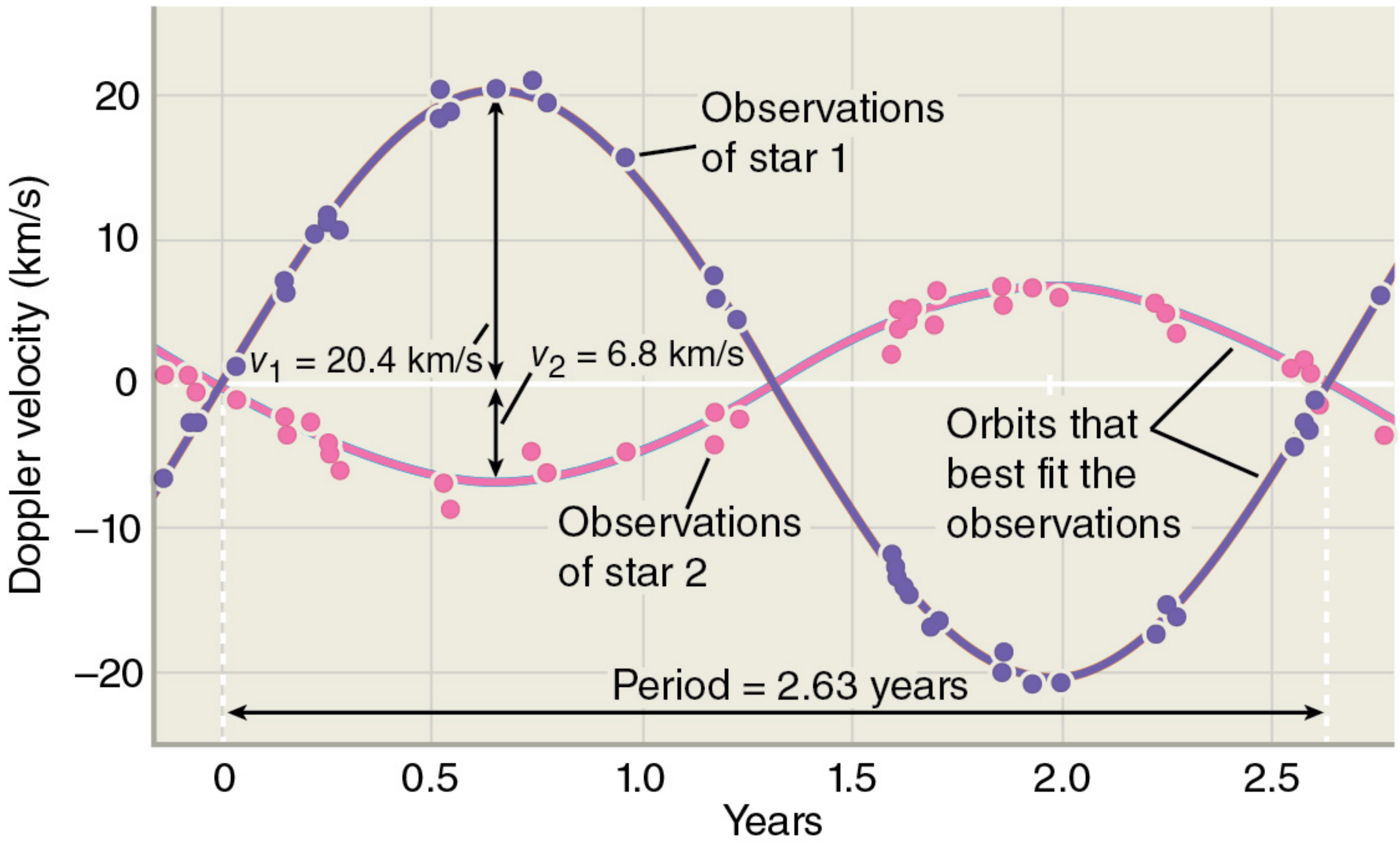
Star 1 is moving away from the viewer, so it is redshifted (longer [+] wavelengths).

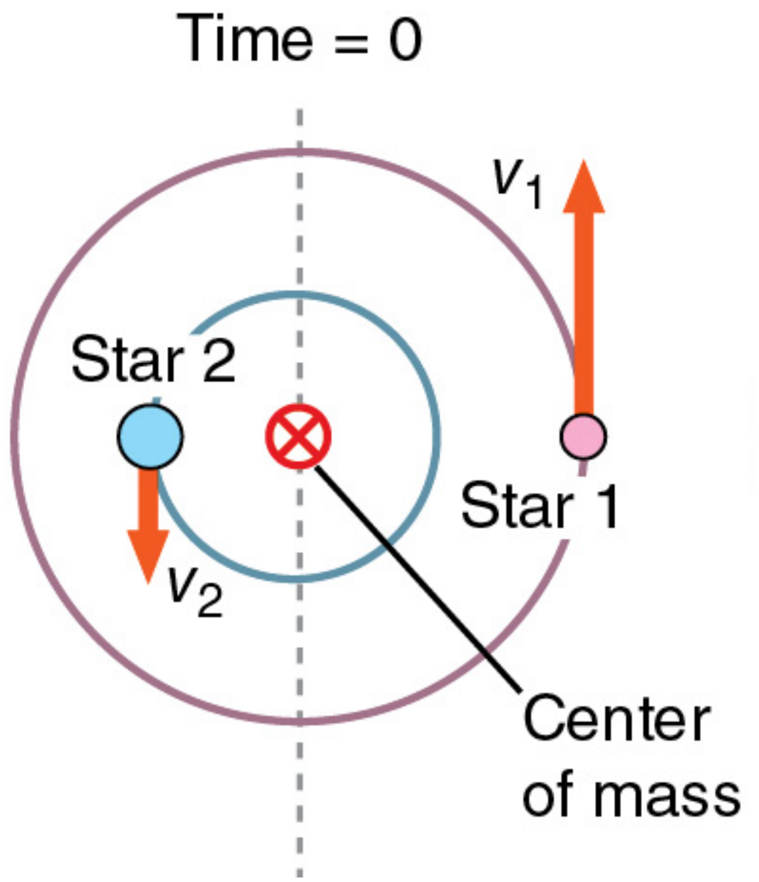
When star 2 is redshifted, star 1 is blueshifted and vice versa.

When the stars are moving neither away from nor toward the viewer, their lines coincide.

The higher-mass star has a small Doppler shift. Here $v_1 = 2v_2$, so $m_2 = 2m_1$.

Measuring the Masses of Stars in Spectroscopic Binaries (viewed edge-on)





- Suppose the system is **viewed edge-on**
- The **Doppler shift** results shown above give key parameters:
  - The period of the binary (***P***)
  - The maximum velocities of star 1 and star 2 ($\boldsymbol{V_{1,max}}$ and $\boldsymbol{V_{2,max}}$)
- From these data we get the circumferences and radii of the two orbits:

$$C_1 = V_{1,max} \times P = 2\pi a_1$$

$$C_2 = V_{2,max} \times P = 2\pi a_2$$

Kepler’s Third Law for One-Body & Two-Body Problems

3rd Law:
period-distance
relation

$$\frac{a^3}{P^2} = \frac{GM}{4\pi^2}$$

But there are two masses (m1 and m2), and two semimajor axes (r1 & r2), how do we use the Kepler’s 3rd law to estimate the total mass?

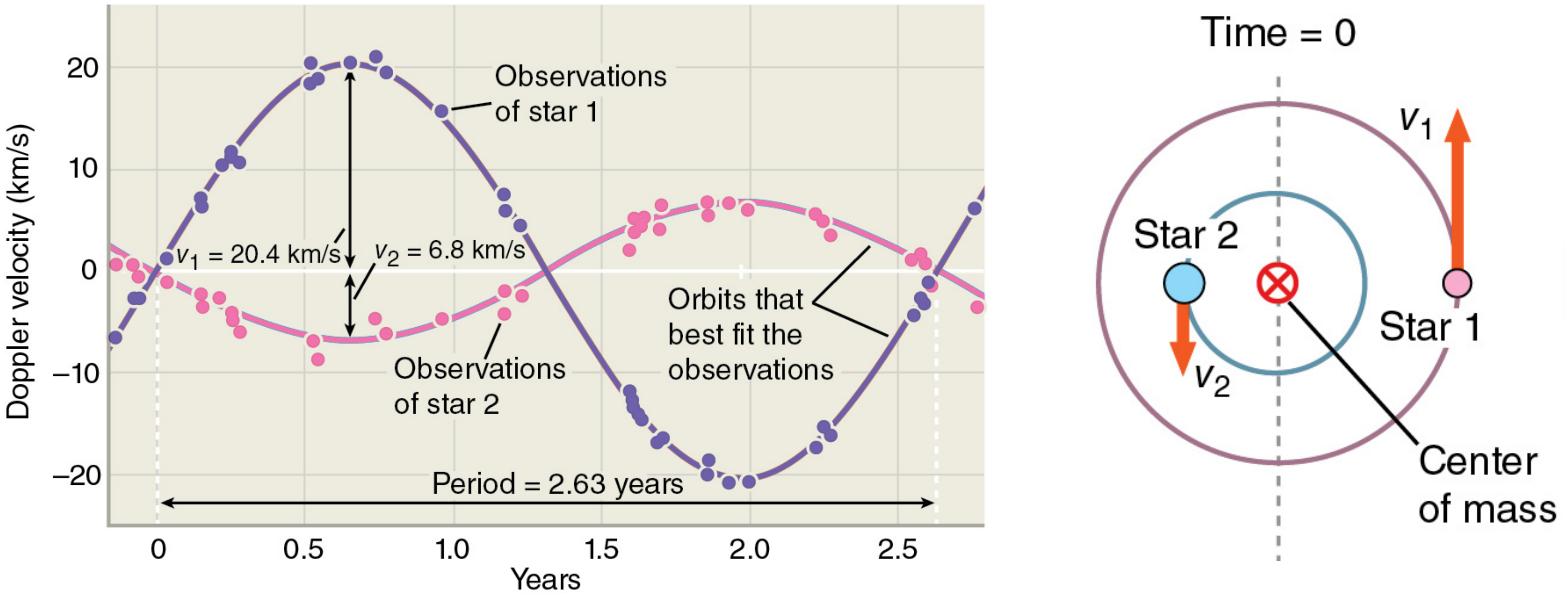
One-body problem:

$$\frac{M}{1\ M_{\text{sun}}} = \left(\frac{a}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

Two-body problem:

$$\frac{M_1 + M_2}{1\ M_{\text{sun}}} = \left(\frac{a_1 + a_2}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

Measuring the Masses of Stars in Spectroscopic Binaries (viewed edge-on)



- Next, we can calculate the total mass using Kepler's third law:

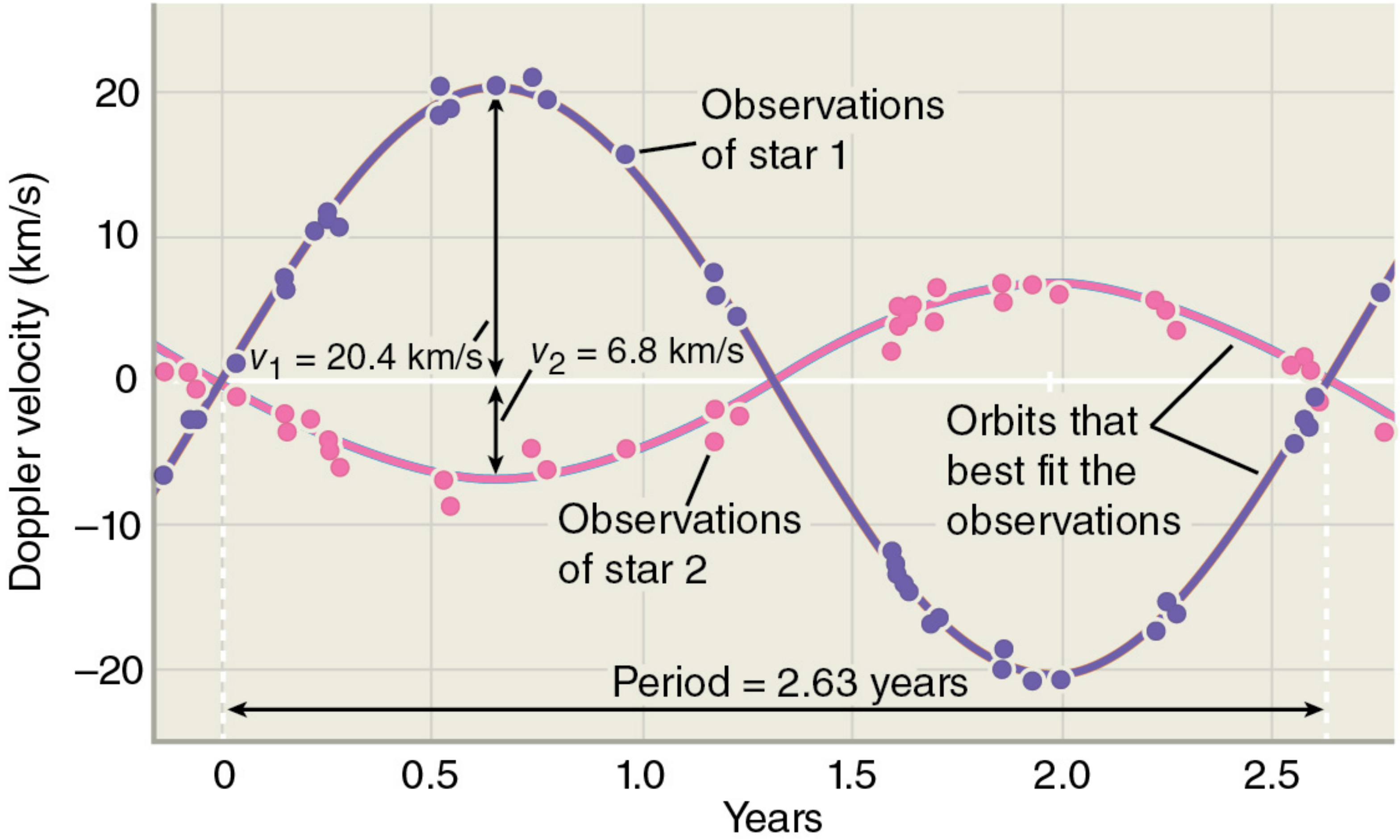
$$\frac{M_1 + M_2}{1\ M_{\text{sun}}} = \left(\frac{a_1 + a_2}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

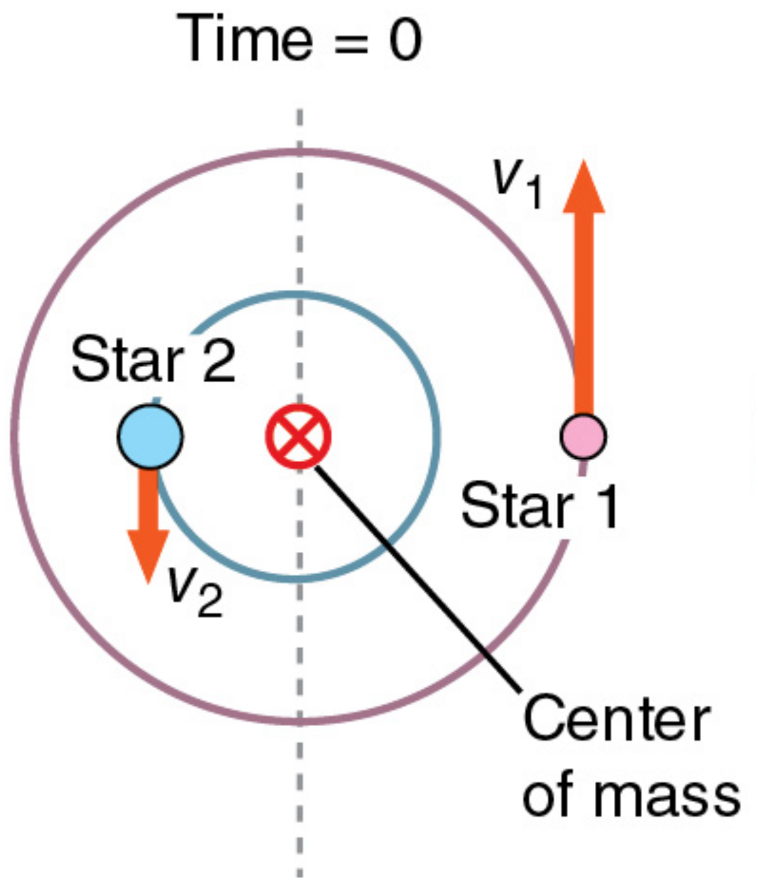
where $a_1 + a_2 = P(V_{1,max} + V_{2,max})/2\pi$

- Finally, we obtain the individual masses based on the velocity ratio (the seesaw equation):

$$\frac{M_1}{M_2} = \frac{a_2}{a_1} = \frac{V_2}{V_1}$$

Inclination Effect on the Masses of Stars in Spectroscopic Binaries





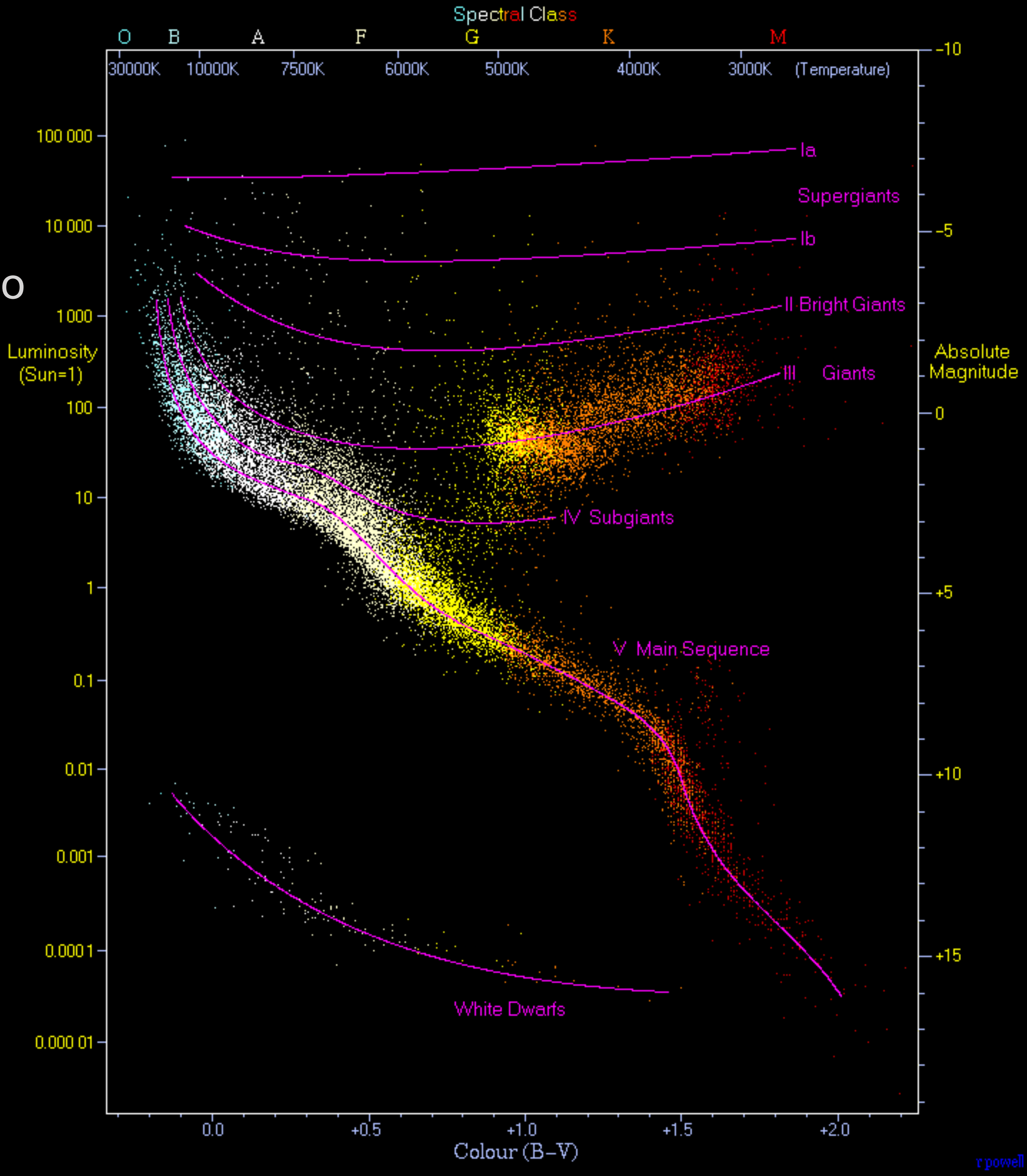
- When we consider the inclination angle of the orbital plane relative to the observer (define *i* = 90 deg when viewed edge-on), we have the total mass of a binary as:

$$(M_1 + M_2) = [(V_{1,max} + V_{2,max})/\sin i]^3 P/8\pi^3$$

- If we assume i=90deg, we end up with a lower limit of the total mass of the system.

Chap 1: Key Concepts

- stellar parallax
- Unit parsec defined by AU
- Pogson’s ratio:
  apparent magnitude and flux ratio
- CCD photometry: count rate to magnitude
- absolute magnitude
- distance modulus (m-M)
- standard candle methods
    - spectroscopic parallax
    - type Ia supernovae
- color index and temperature
- luminosity-temperature-size relation
- HR diagram: the main sequence
- Spectroscopic binaries: Kepler’s 3rd law for binary systems (two-body problem)



Chap 1: Key Equations

$$d = 1 \text{ parsec} \left(\frac{1 \text{ arcsec}}{p}\right)$$

$$m_{\lambda,2} - m_{\lambda,1} = -2.5 \log(f_{\lambda,2}/f_{\lambda,1})$$

$$m_\lambda - M_\lambda = 2.5 \log\left(\frac{d}{10 \text{ parsec}}\right)^2 = 5 \ [\log d(\text{parsec}) - 1]$$

$$d(\text{parsec}) = 10^{1+0.2(m-M)}$$

$$L_\lambda = 4\pi R^2 \, \pi B_\lambda(T)$$

$$L_{\text{bol}} = 4\pi R^2 \, \sigma_{\text{SB}} T^4$$

$$\frac{M_1 + M_2}{1 \ M_{\text{sun}}} = \left(\frac{a_1 + a_2}{1 \text{ AU}}\right)^3 \left(\frac{P}{1 \text{ year}}\right)^{-2}$$

$$\frac{R}{R_\odot} = \sqrt{\frac{L_{\text{bol}}}{L_\odot}} \left(\frac{T}{T_\odot}\right)^{-2}$$

$$(M_1 + M_2)\sin^3 i = \frac{[\max(V_1) + \max(V_2)]^3 P}{8\pi^3}$$

Advanced Topics

- How to measure the size of the Earth's orbit?
- How to measure the ellipticity of the Earth's orbit?
- How spectral lines form in stellar atmosphere and why each element/ion has a preferred temperature range?
- How the Kepler's 3rd law is derived for binary systems?
- What are eclipsing binaries and how their light curves tell us about the stars in the systems?
- How to estimate the sizes of stars?

How to measure Earth's Orbit?

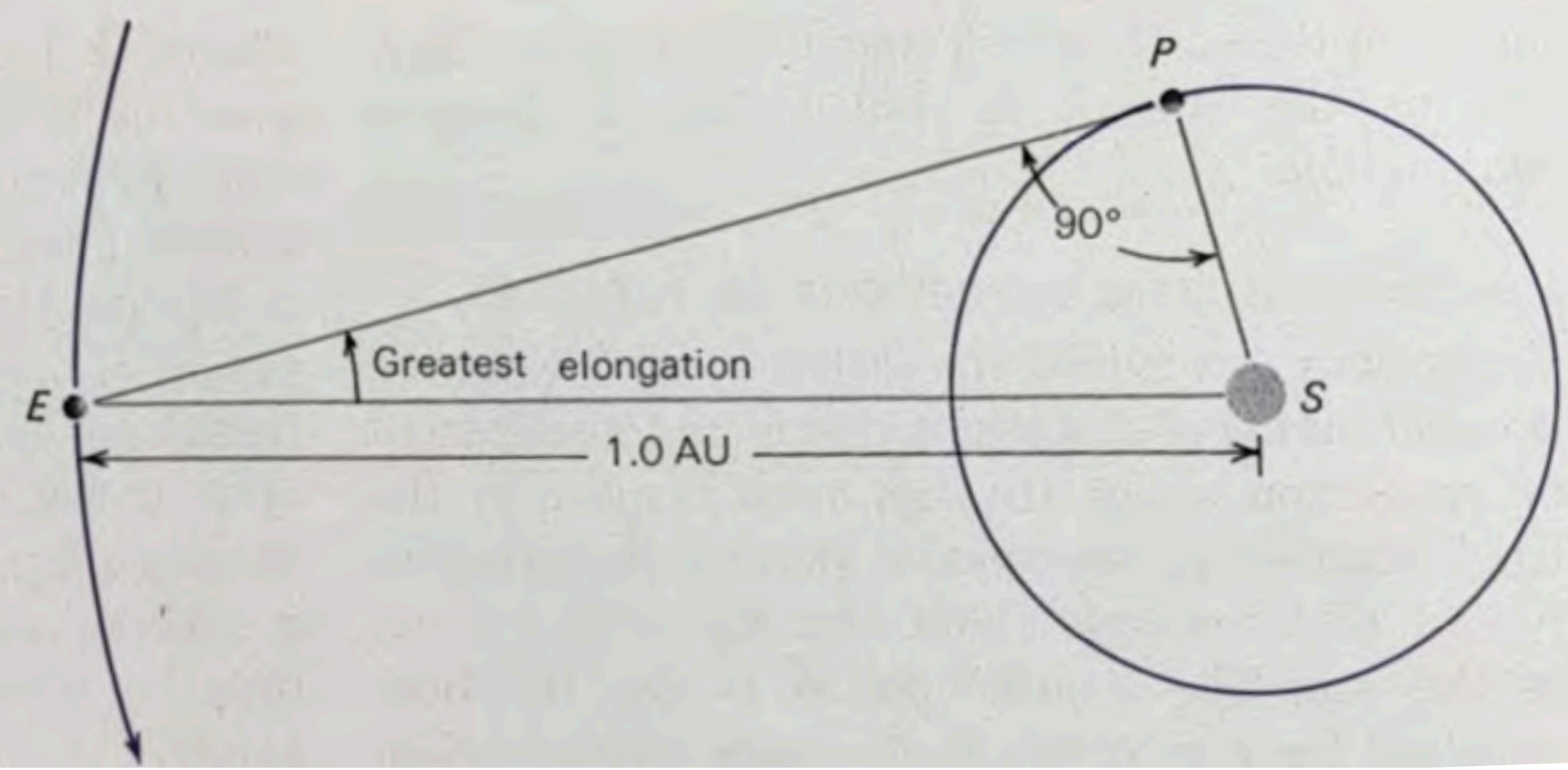
How many km is 1 AU?

Measure Earth’s orbit (AU)

- How do we measure the Astronomical Unit (AU)? Recall that AU is defined as the average heliocentric distance of the Earth, but how long is it exactly?
- The geometric method involves two steps:
  - Measure Venus’ heliocentric distance in AU
  - A parallax measurement of the distance between Earth and Venus during its transit of the Sun

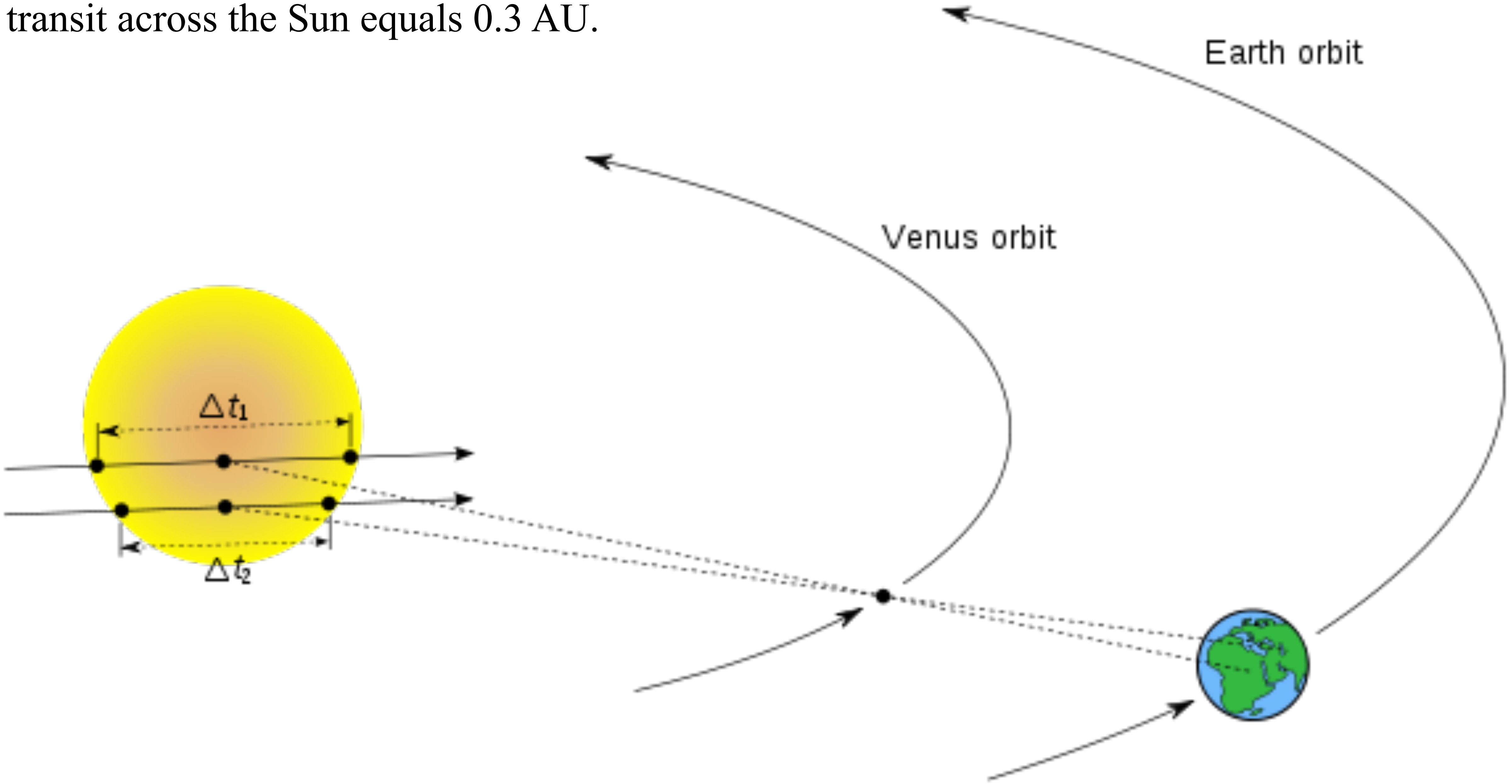
Copernicus' method (1543) of determining the heliocentric distance of inner planets

Inner Planets: R_{AU} = sin(Max Elongation)



Parallax measurement of the AU

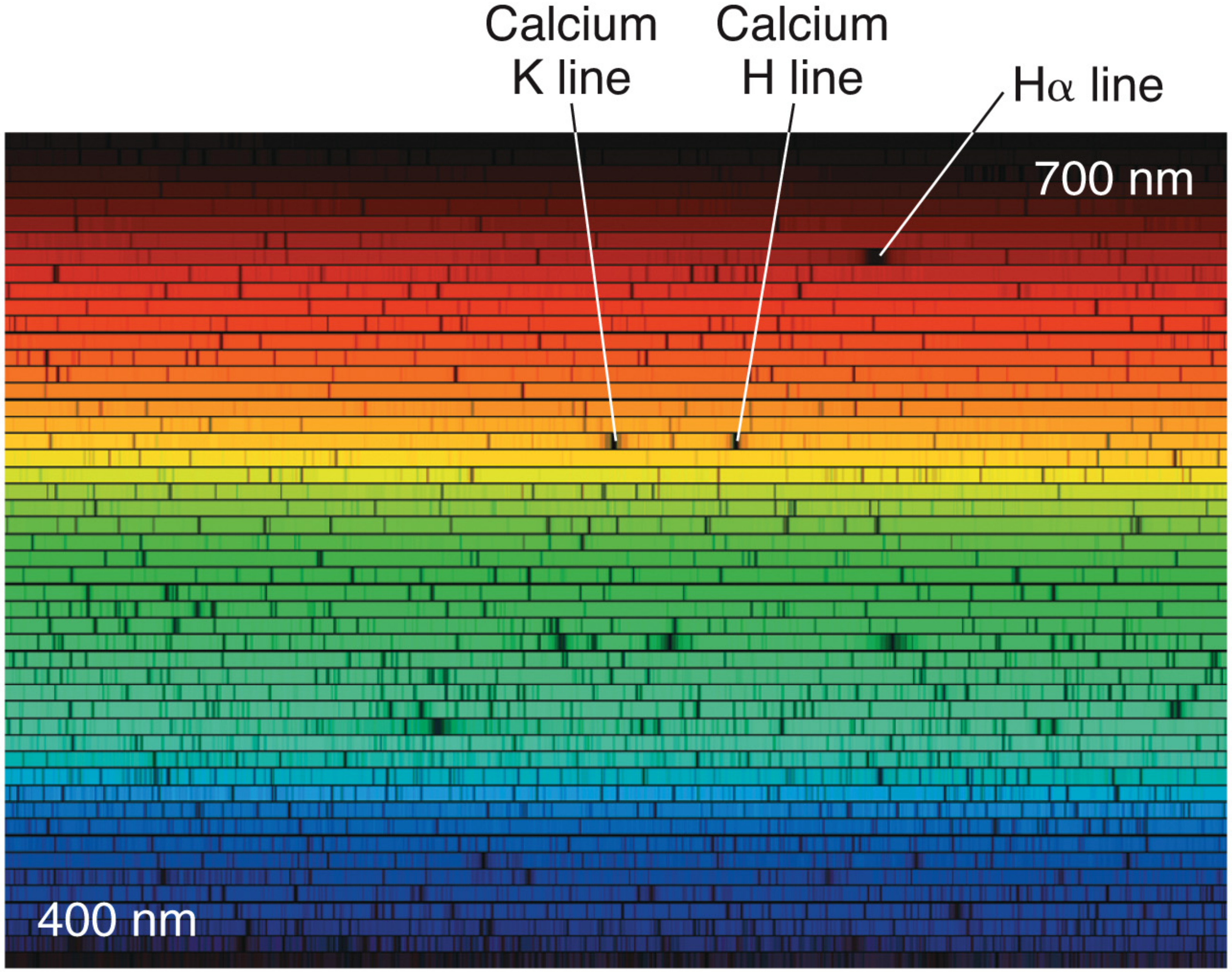
- By observing the transit of Venus at two locations on Earth, one can measure the distance to Venus using the measured **parallax** and the **baseline length**.
- Assuming Venus and Earth are both on circular orbits around the Sun, the greatest elongation angle of Venus tells us its orbit has a radius of 0.7 AU.
- Since the Earth's orbit has a radius of 1 AU, the measured parallax distance of Venus during its transit across the Sun equals 0.3 AU.



Spectral Line Formation

Solar Atmosphere: Understanding the Absorption Line Spectrum

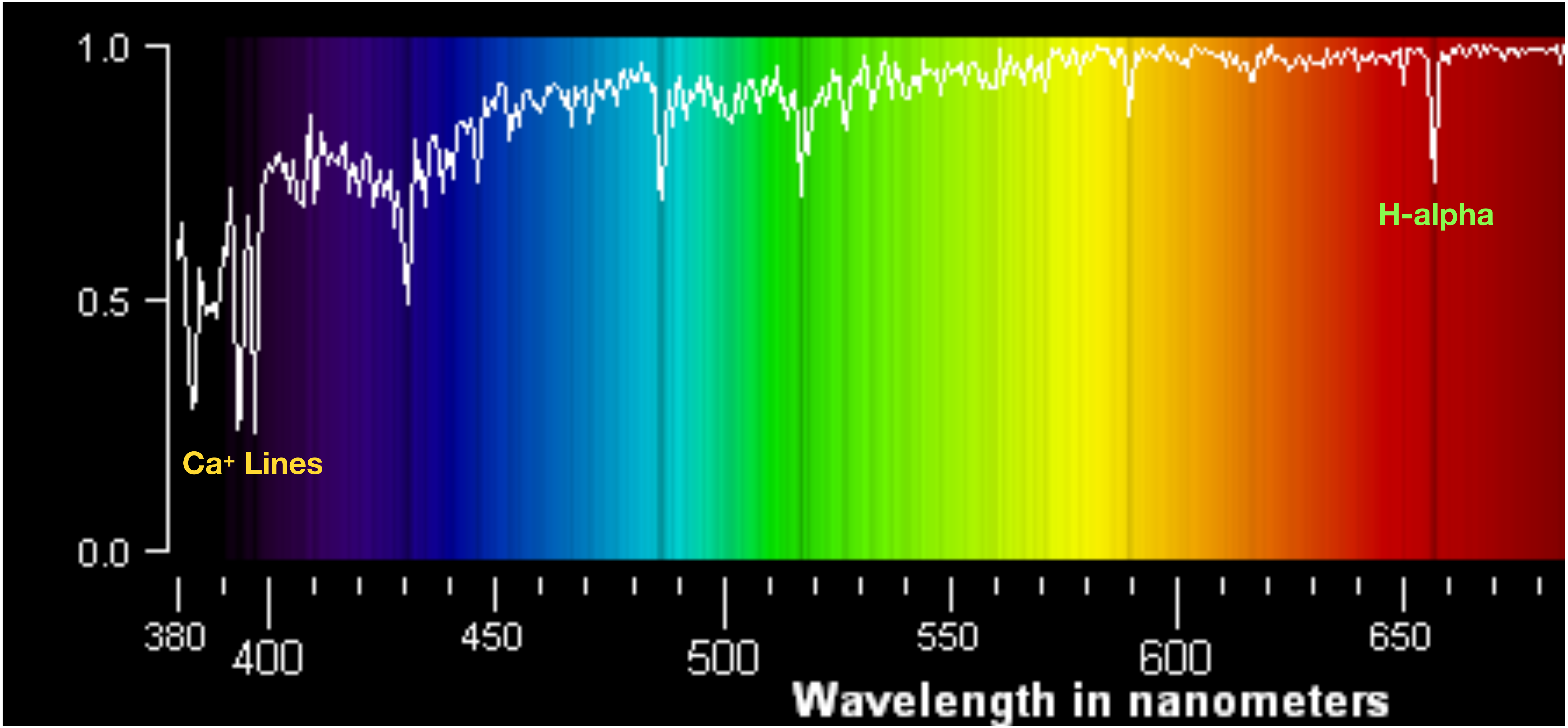
- **The depth of the last scattering surface depends on wavelength**. The depth decreases at wavelengths where there are corresponding atomic or ionic transitions, because the opacity of the gas increases (i.e., cross section increases, thus decreasing the mean free path).
- The lines appear darker because the last scattering surface is at shallower depth compared to other wavelengths, and **shallower depth means lower temperature**.



Nigel Sharp, NOAO/NSO/Kitt Peak FTS/AURA/NSF, https://noirlab.edu/public/images/noao-sun/, https://creativecommons.org/licenses/by/4.0/

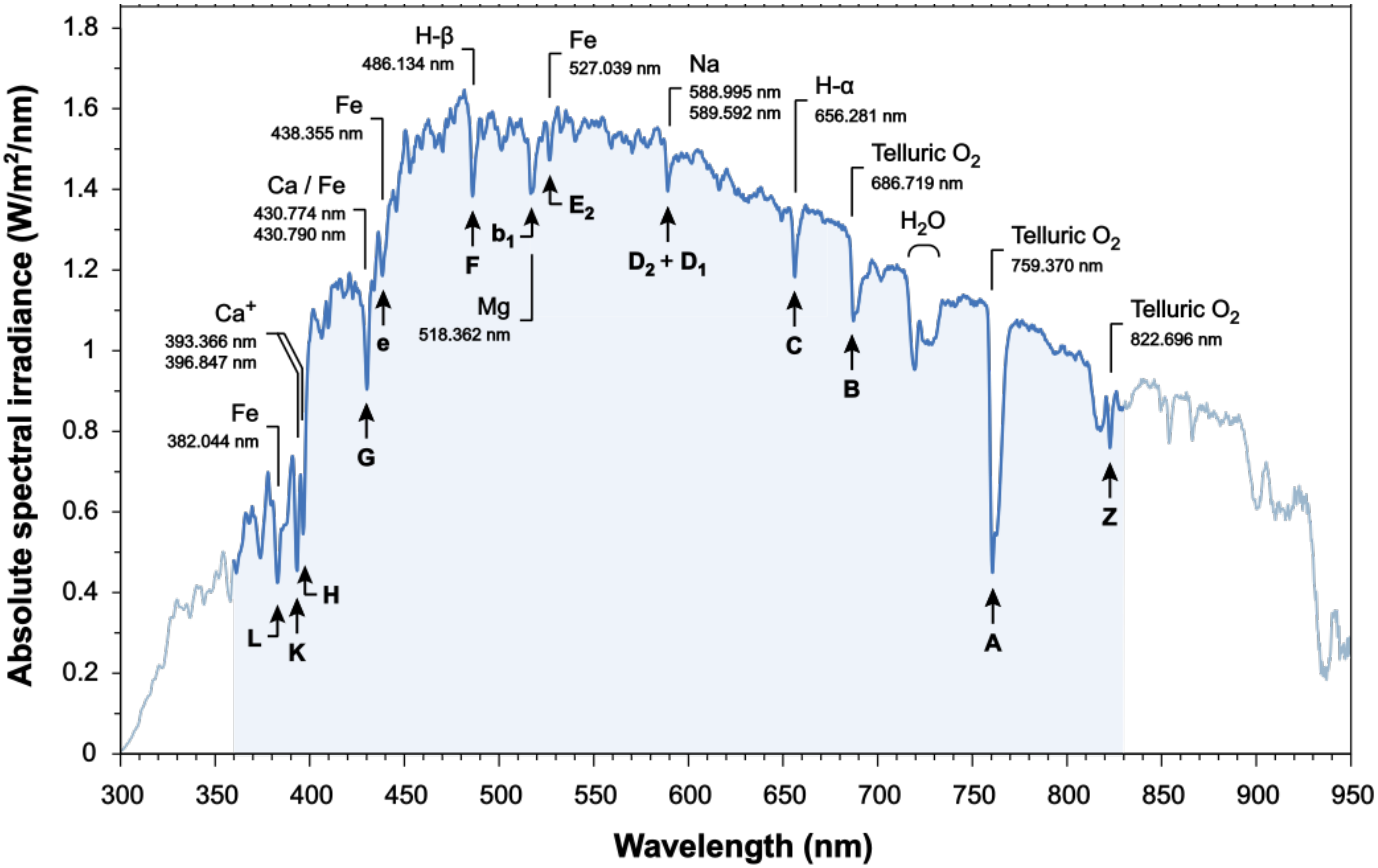
Solar Atmosphere: Understanding the Absorption Line Spectrum

- The Sun displays a complex absorption spectrum from the presence of over 70 elements.
- The strongest lines are from singly-ionized Calcium ions at 3968.5 and 3933.7 Angstroms. This seems quite strange given that Calcium is much more rare than Hydrogen.

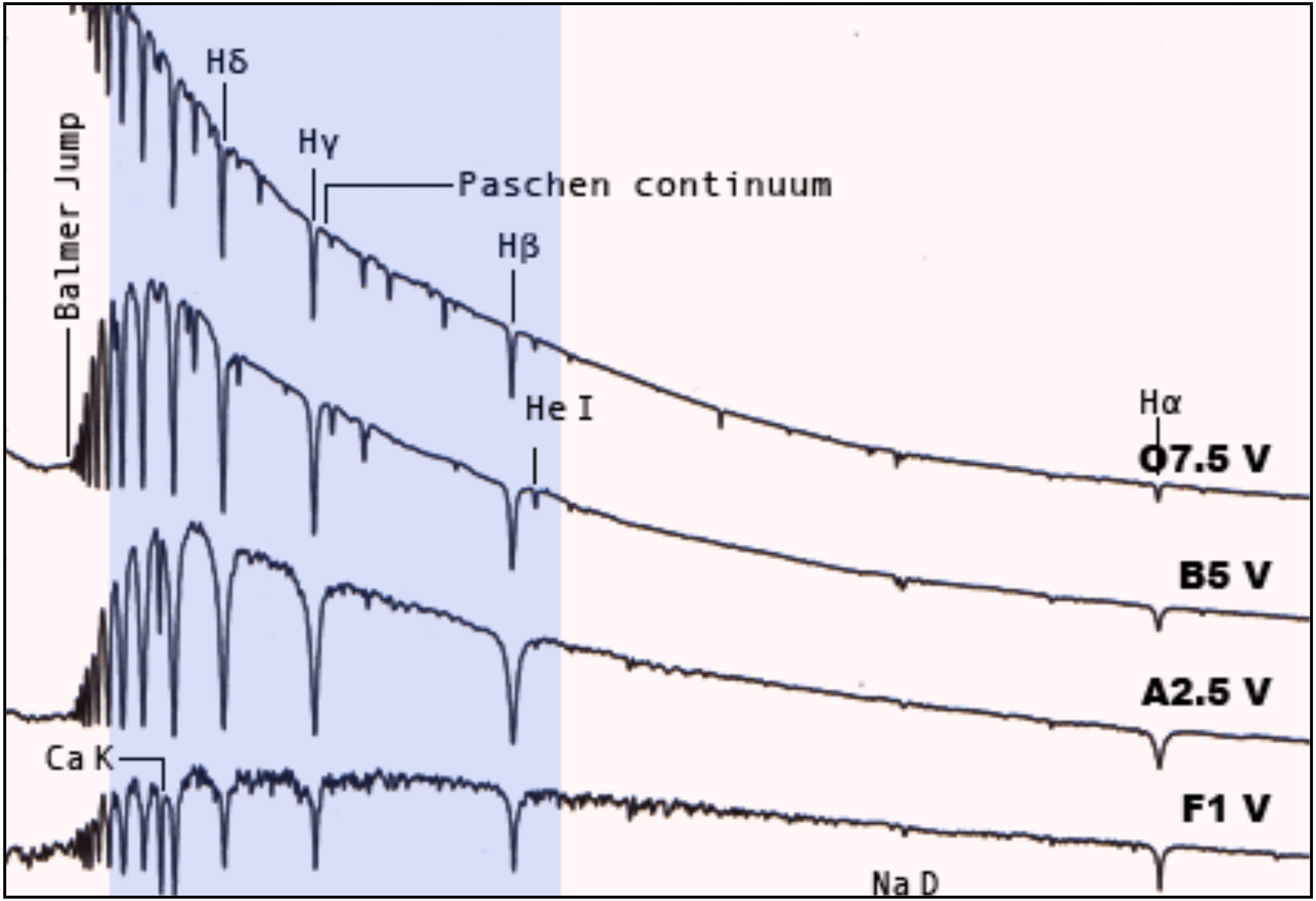


Solar Atmosphere: Understanding the Absorption Line Spectrum

- Why the strongest lines in the Solar spectrum are from singly-ionized Calcium ions in their ground states? Given that for every 1 Ca atom there are 500,000 H atoms.

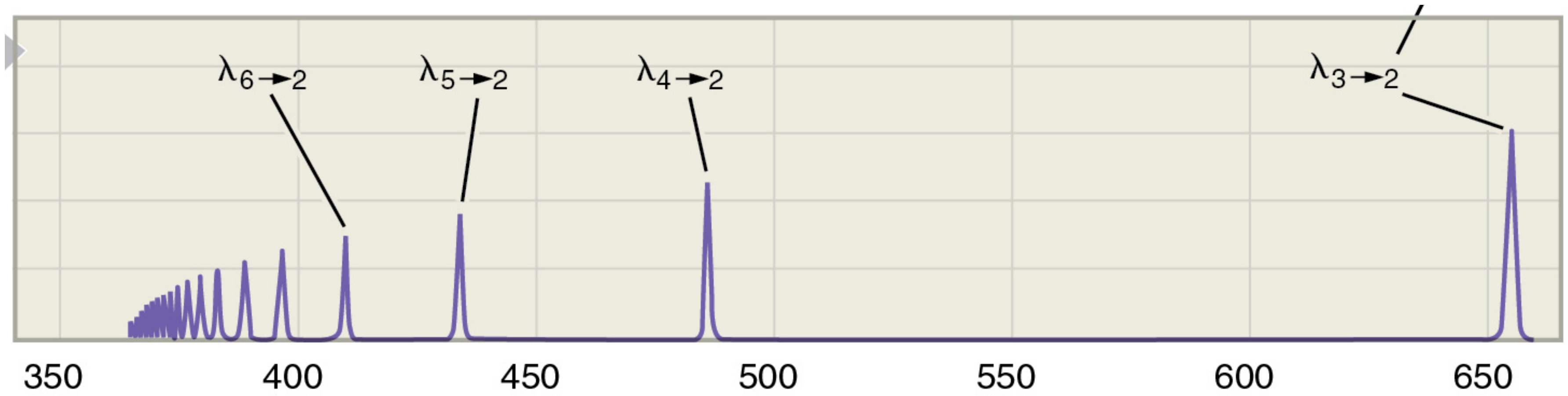


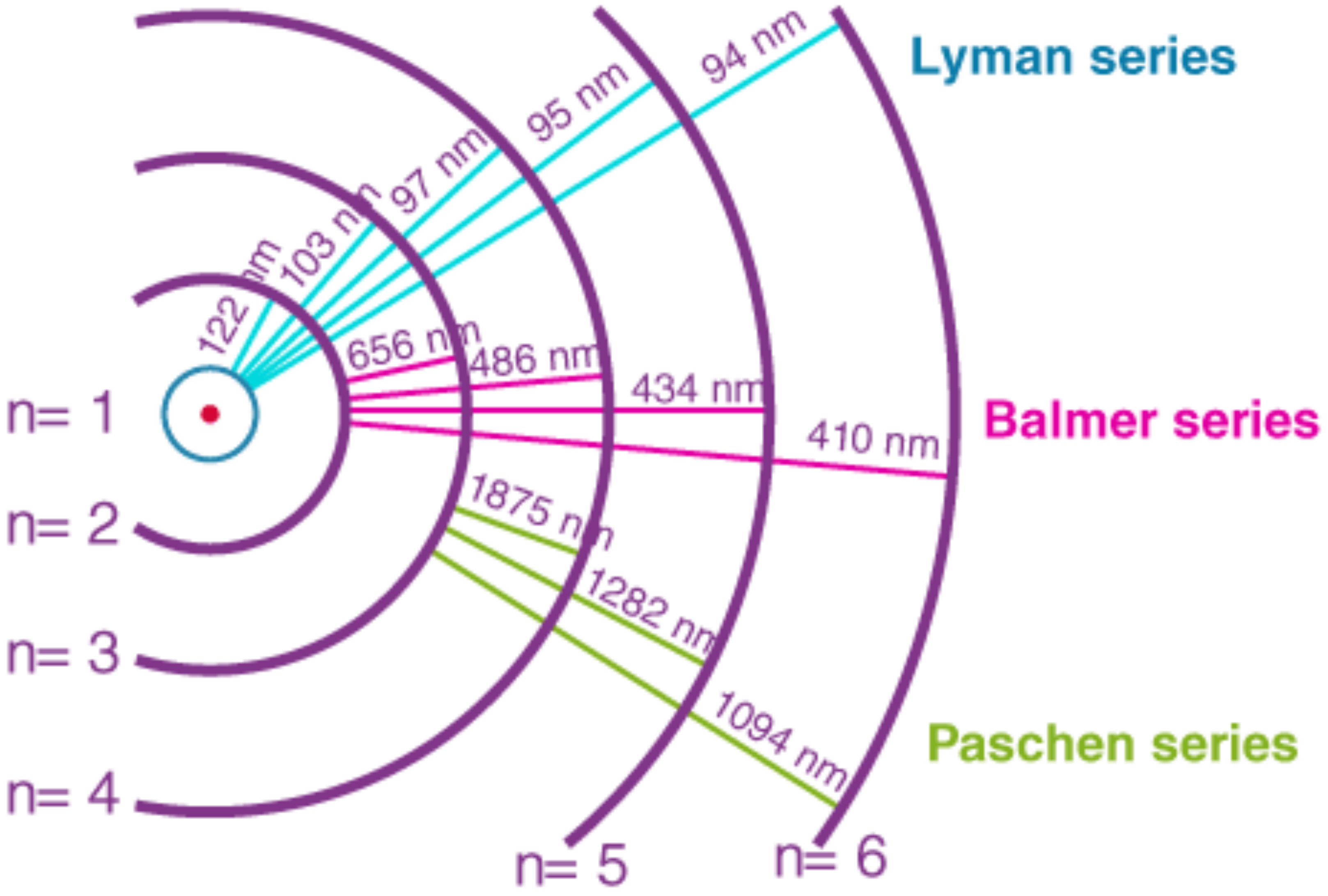
Why Hydrogen Balmer lines are strongest in A-type stars? (H-alpha, H-beta, H-gamma, H-delta, …)



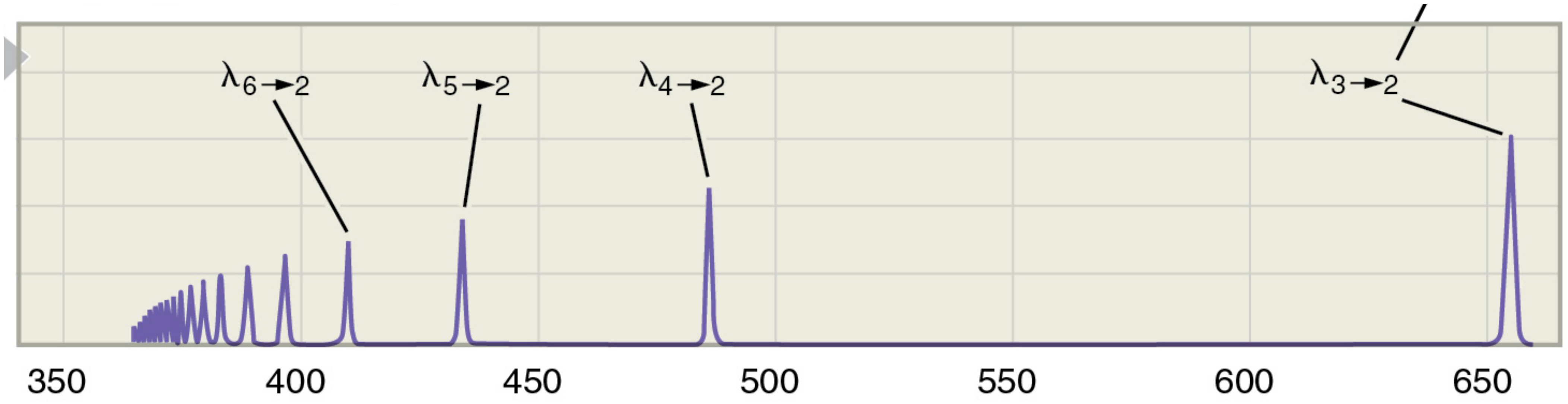
Why Hydrogen Balmer lines are strongest in A-type stars?

- What are needed to produce high opacity in the Hydrogen Balmer lines?
  - **Neutral hydrogen atoms excited to the first excited state (n=2)**





Calculating the Wavelengths of Hydrogen Emission Lines



$$\frac{1}{\lambda} = \left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right)\frac{13.6\text{eV}}{hc}$$

n_{low} = quantum number of lower orbit

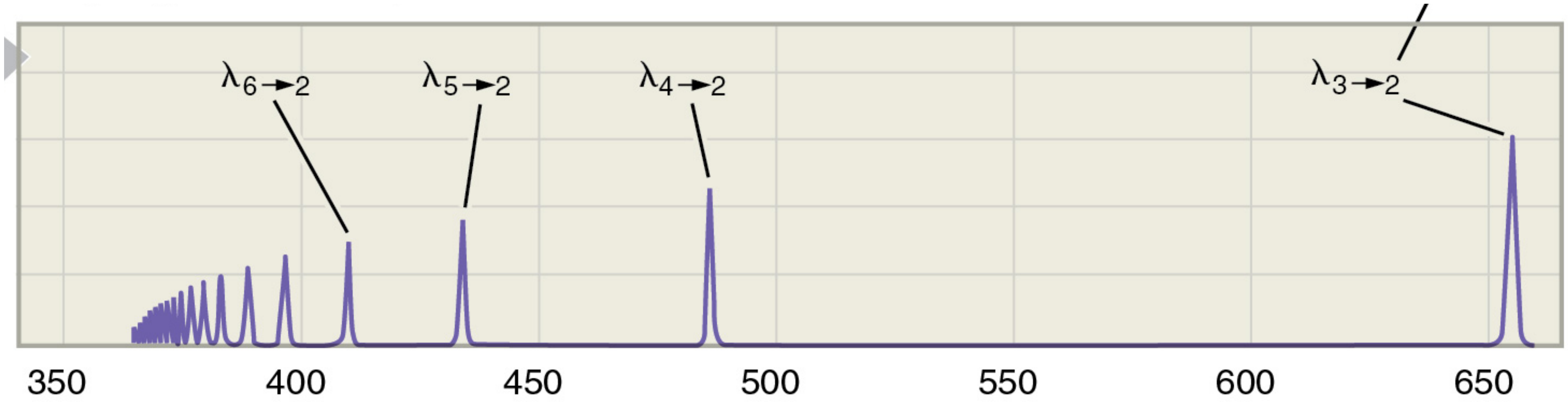
n_{high} = quantum number of higher orbit

λ = wavelength of emitted photon

- Electron-volt (eV) is the amount of kinetic energy gained by a single e- accelerating through an electric potential difference of one volt.

$$T = 11604\ \text{K}\left(\frac{E}{1\ \text{eV}}\right) \qquad \lambda = 1.24\ \mu\text{m}\left(\frac{E}{1\ \text{eV}}\right)$$

Ionization Energy of Hydrogen: 13.6 eV



$$\Delta E = 13.6 \text{ eV} \left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2} \right)$$

n_{low} = quantum number of lower orbit

n_{high} = quantum number of higher orbit

ΔE = energy required for the transition

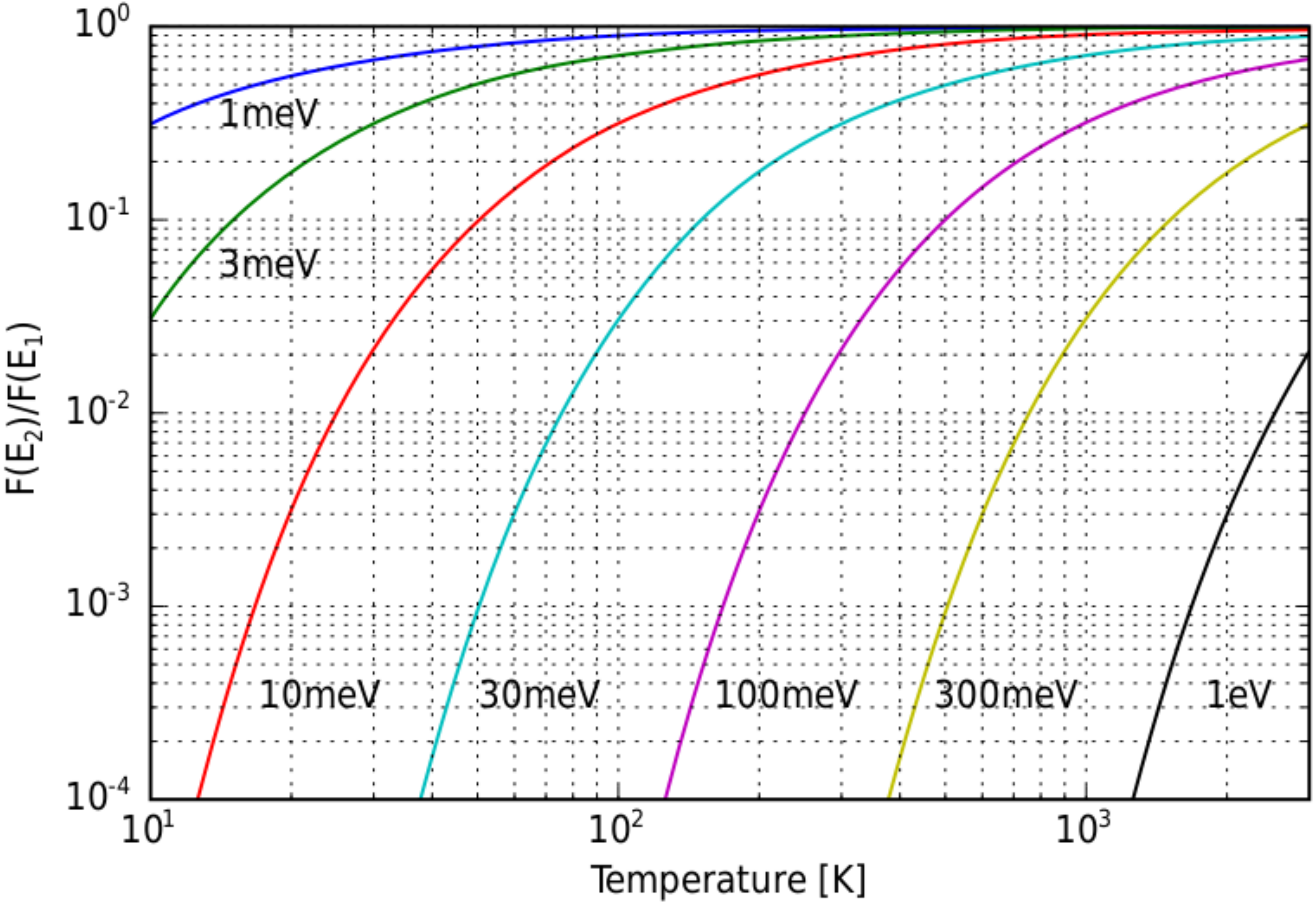
- To calculate the ionization energy, we can plug in $n_{\text{low}} = 1$ and n_{high} = infinity to the equation. What temperature does 13.6 eV correspond to?

$$T = 11604 \text{ K} \left(\frac{E}{1 \text{ eV}} \right)$$

The Excitation of Neutral Hydrogen to Higher Energy Levels

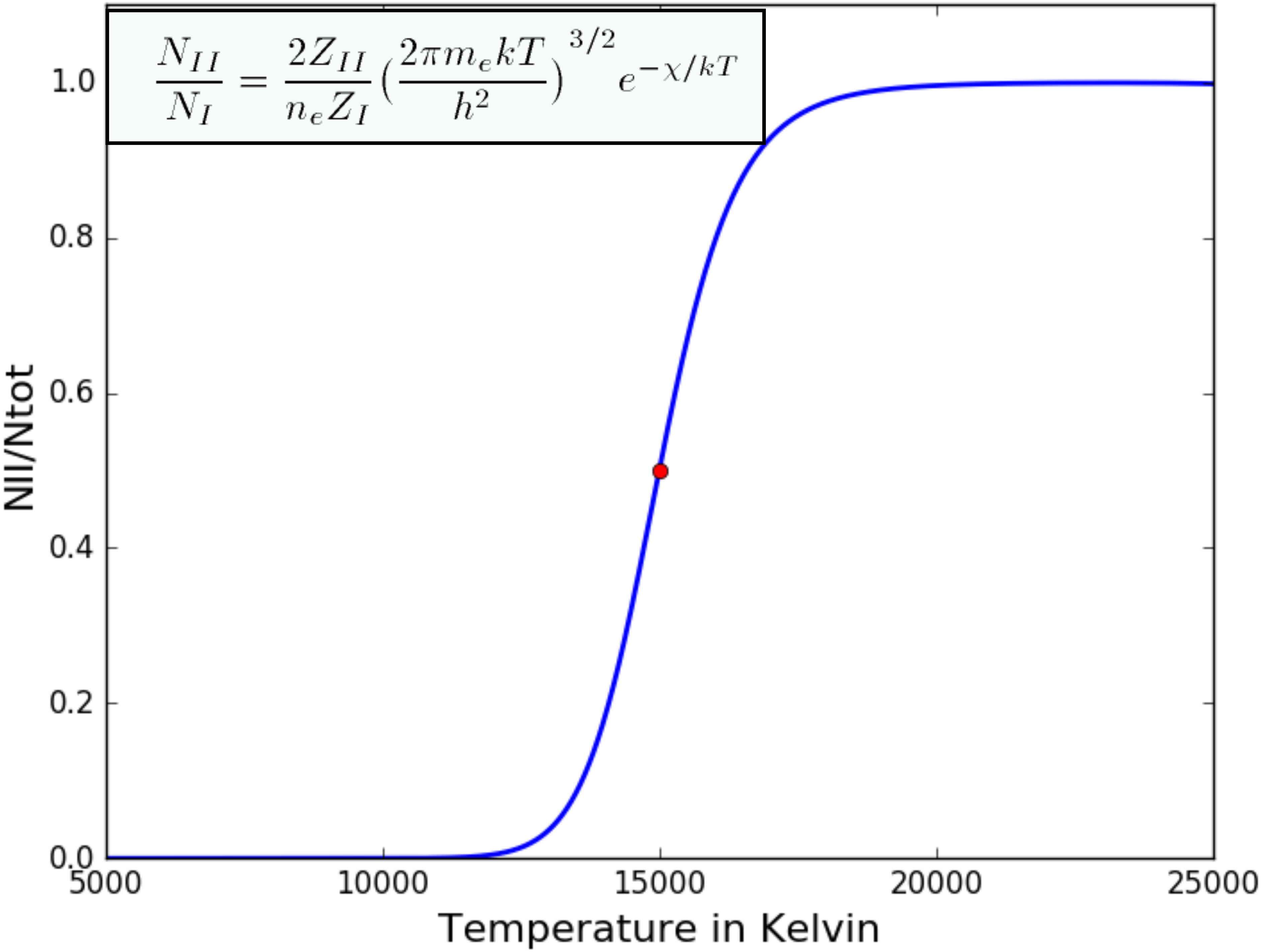
- As temperature increases, more and more remaining neutral hydrogen are excited to the n=2 state (**Excitation Ratio: Boltzmann Distribution**)

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\, e^{-(E_2 - E_1)/kT}$$



Ionization Equilibrium: Balance between ionization and recombination

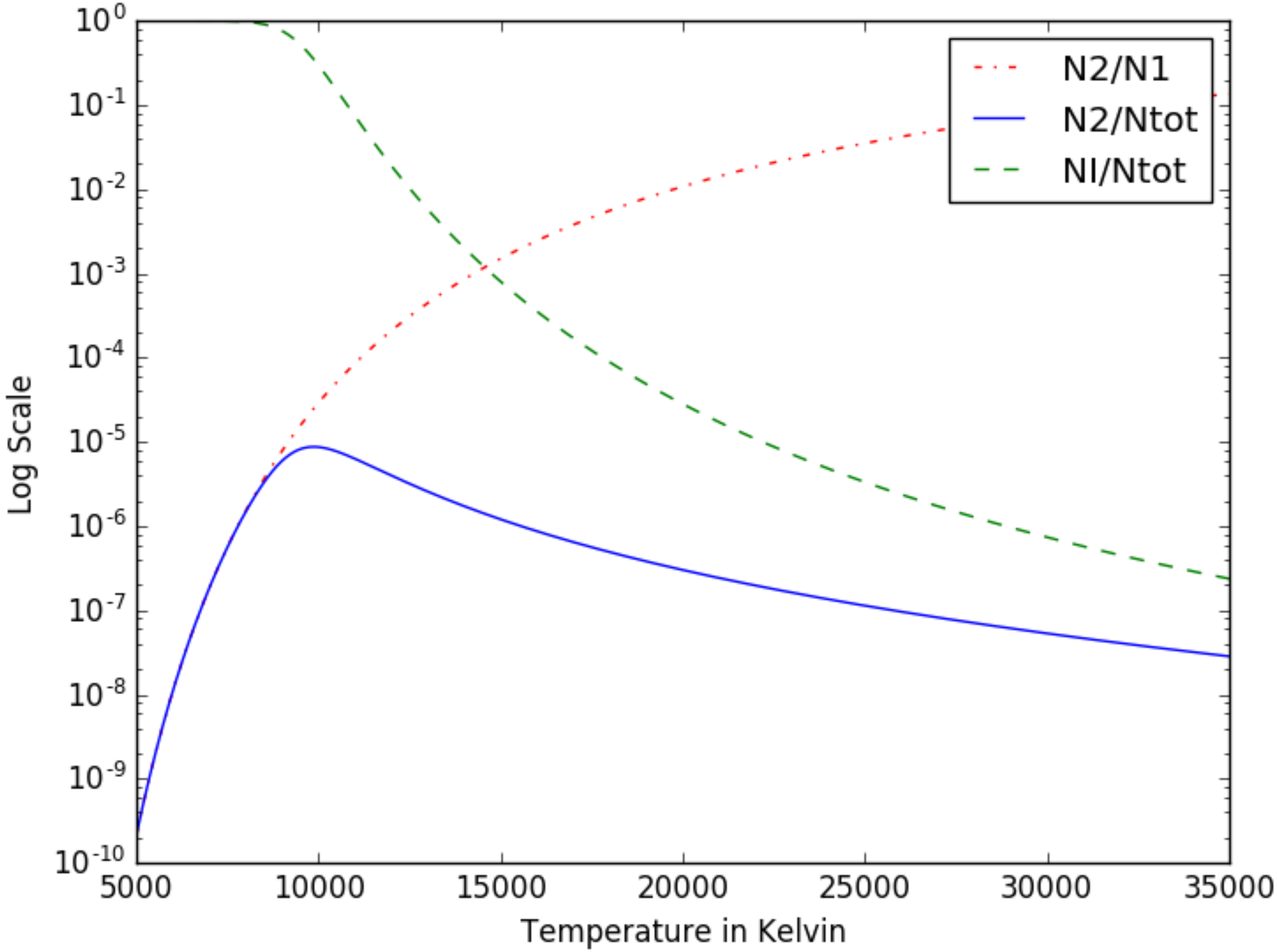
- As temperature increases, more and more hydrogen become ionized, so we are losing neutral hydrogen (Equilibrium calculated with the **Saha Equation**)
- But 15000 K is only 1.3 eV, which is 10x smaller than the ionization energy (13.6 eV)



$$\frac{N_{II}}{N_I} = \frac{2Z_{II}}{n_e Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi/kT}$$

The Combined Effect of Ionization and Excitation

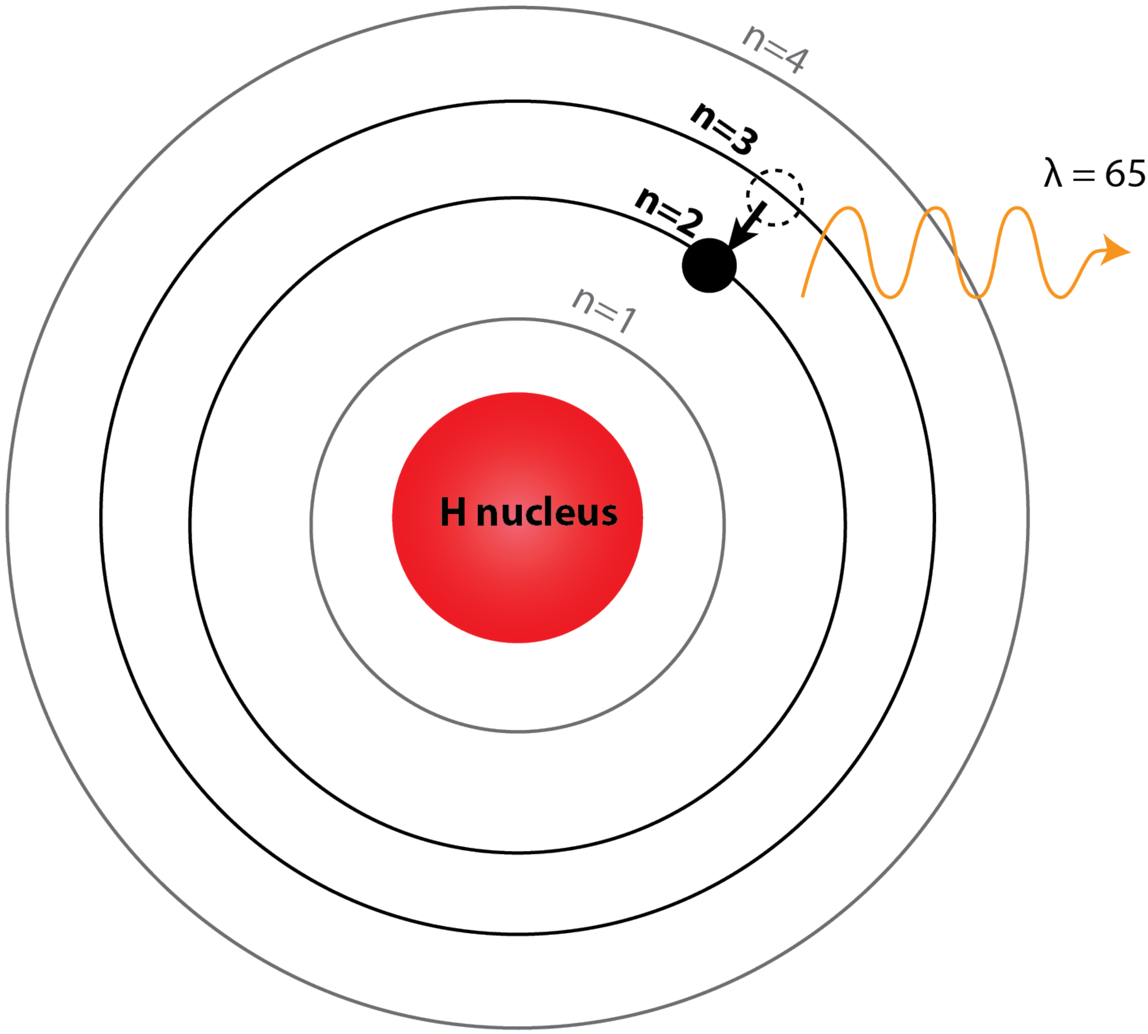
- Causes a peak in the number of desired species that creates the H Balmer lines: neutral hydrogen (H I) at the first excited state (n=2)



Solar Atmosphere: Understanding the Absorption Line Spectrum

- Why the strongest lines in the Solar spectrum are from singly-ionized Calcium ions in their ground states? Given that for every 1 Ca atom there are 500,000 H atoms.

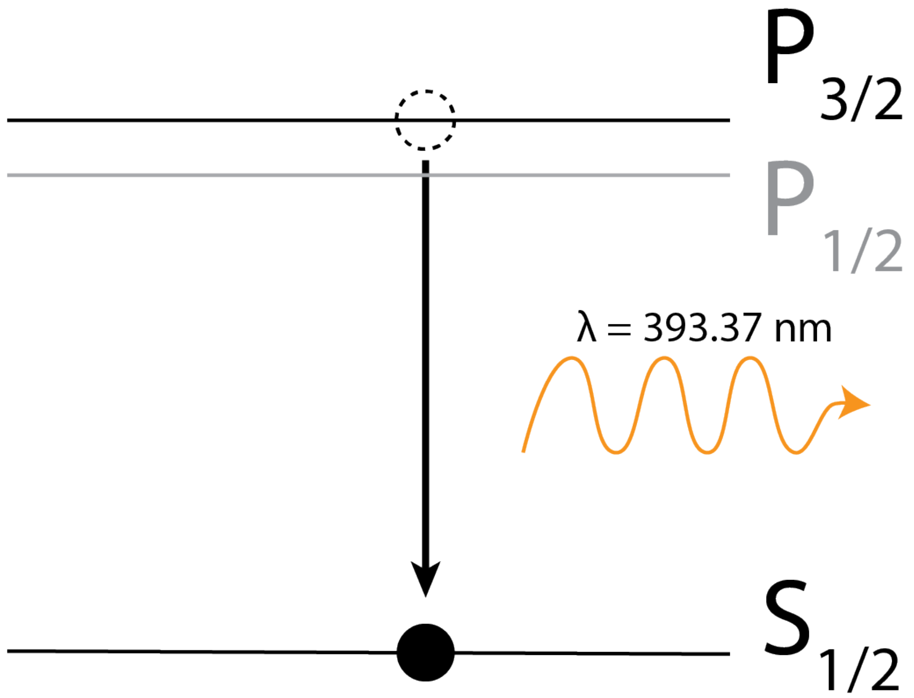
H atom



Hydrogen-alpha
Naming: Balmer lines

n = 4

Ca$^+$ ion



Calcimu II K

Dependence of spectral line strengths on temperature

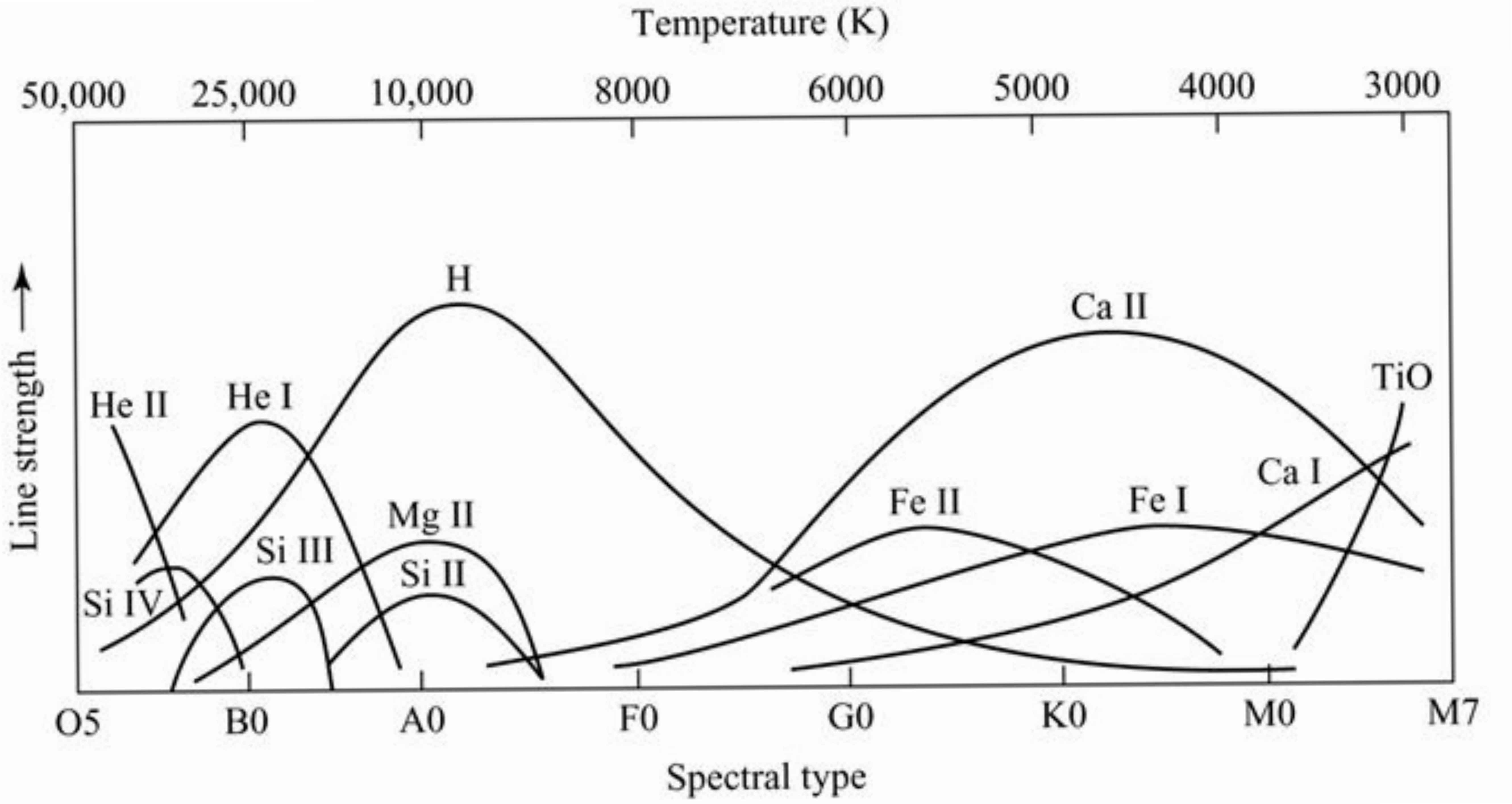


Figure 8.11 Carroll & Ostlie

Two-body Problems

Derivation of K3 Law

Two-body Problem Derivation - The Center-of-Mass Reference Frame

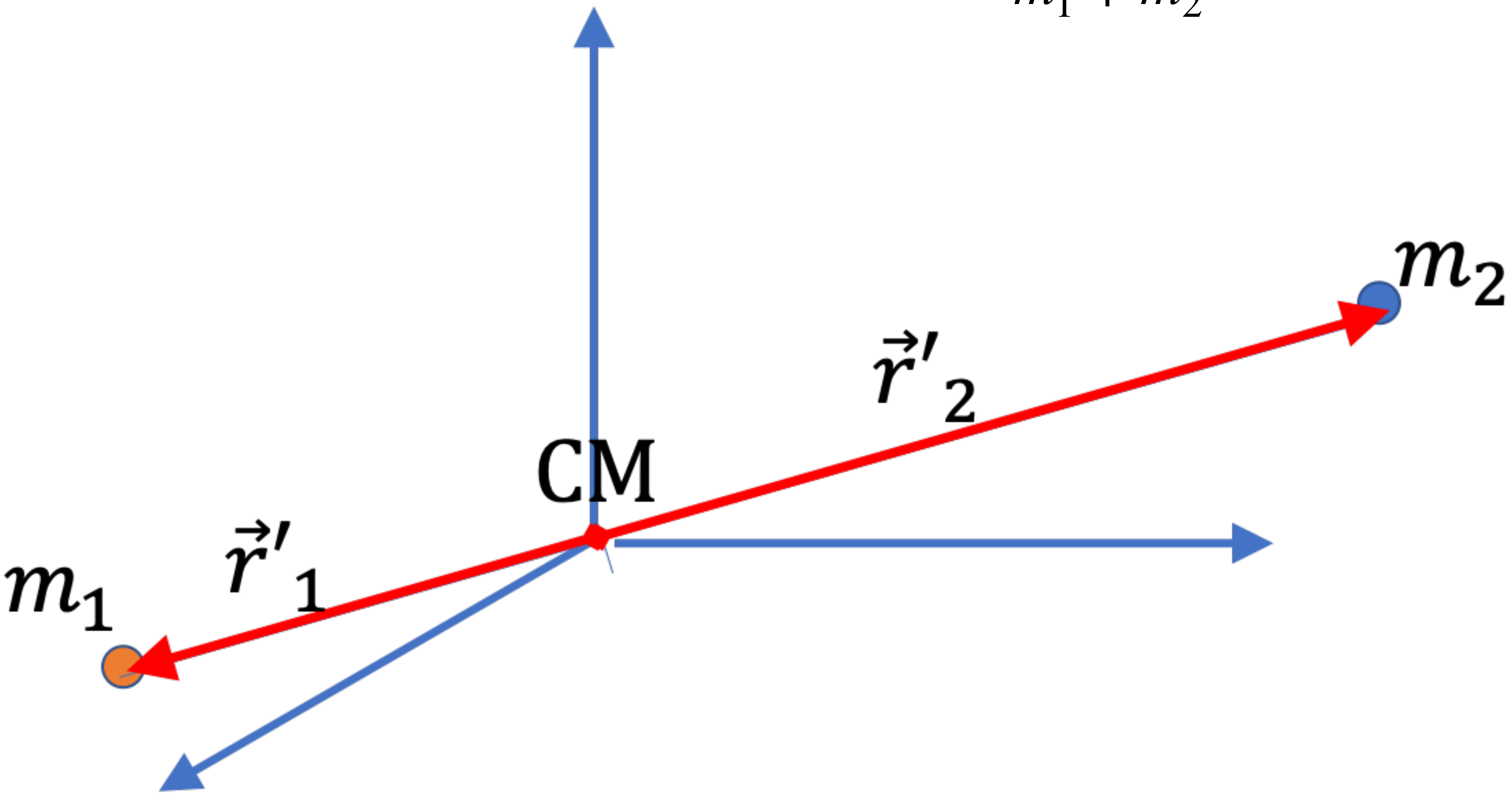
$$m_1\overrightarrow{r_1} + m_2\overrightarrow{r_2} = 0$$

$$\overrightarrow{r_2} - \overrightarrow{r_1} = \vec{r}$$

$$\Rightarrow$$

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r}$$

$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r}$$



Two-Body Problem can be reduced to One-Body Problem

define reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

⇒

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r} = -\frac{\mu}{m_1}\vec{r}$$

$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r} = \frac{\mu}{m_2}\vec{r}$$

⇒

$$\vec{v}_1 = -\frac{\mu}{m_1}\vec{v}$$

$$\vec{v}_2 = \frac{\mu}{m_2}\vec{v}$$

Then write down the total kinetic and gravitational potential energy

$$\boxed{E = \frac{1}{2}m_1|\vec{v}_1|^2 + \frac{1}{2}m_2|\vec{v}_2|^2 - G\frac{m_1 m_2}{|\vec{r}_2 - \vec{r}_1|}}$$

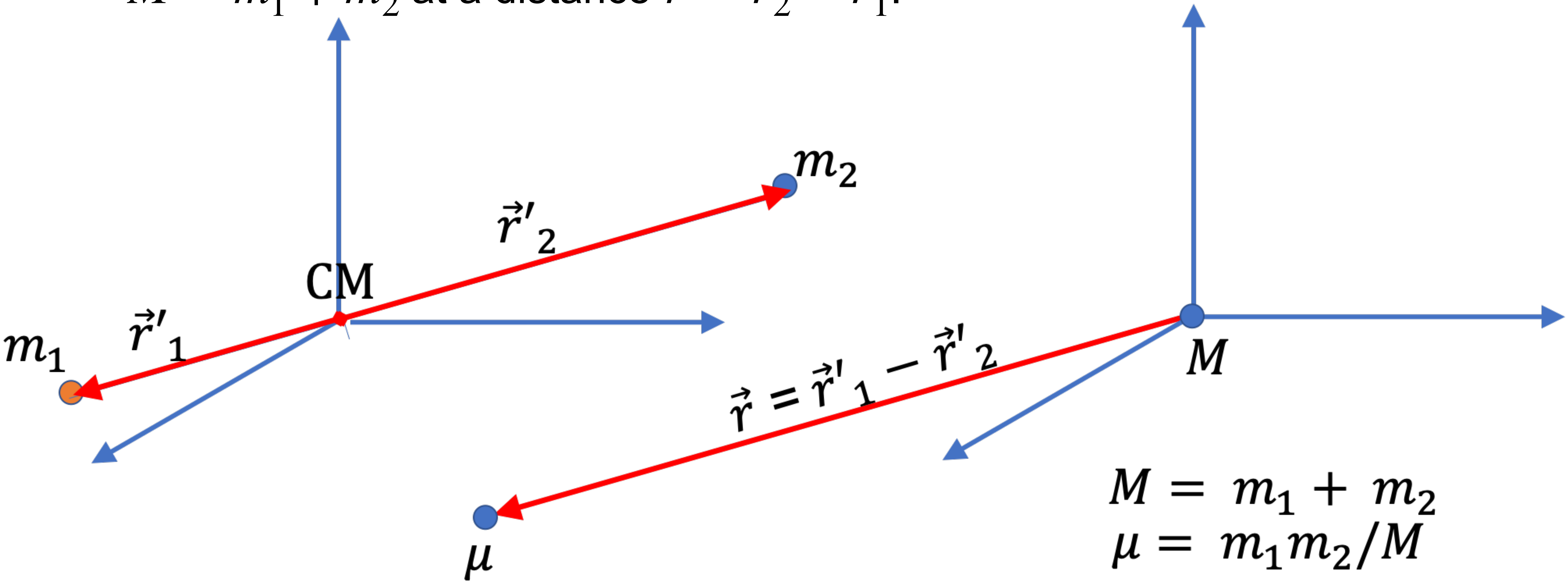
$$= \frac{1}{2}m_1\left(\frac{\mu}{m_1}\right)^2 v^2 + \frac{1}{2}m_2\left(\frac{\mu}{m_2}\right)^2 v^2 - G\frac{(m_1 + m_2)\cdot m_1 m_2/(m_1 + m_2)}{r}$$

$$= \frac{1}{2}\mu\left(\frac{\mu}{m_1} + \frac{\mu}{m_2}\right)v^2 - G\frac{M\mu}{r} \quad \Rightarrow \quad \boxed{E = \frac{1}{2}\mu v^2 - G\frac{M\mu}{r}}$$

- The two-body problem is equivalent to a one-body problem with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving about a **fixed** total mass $M = m_1 + m_2$ at a distance $\vec{r} = \vec{r}_2 - \vec{r}_1$.

Kepler’s 3rd Law for Binary Stars (Two-body Problem)

- The two-body problem is equivalent to a one-body problem with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving about a fixed total mass $M = m_1 + m_2$ at a distance $\vec{r} = \vec{r}_2 - \vec{r}_1$.



$$M = m_1 + m_2$$
$$\mu = m_1 m_2 / M$$

One-body problem:

$$\frac{m}{1\ M_{\text{sun}}} = \left(\frac{a}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

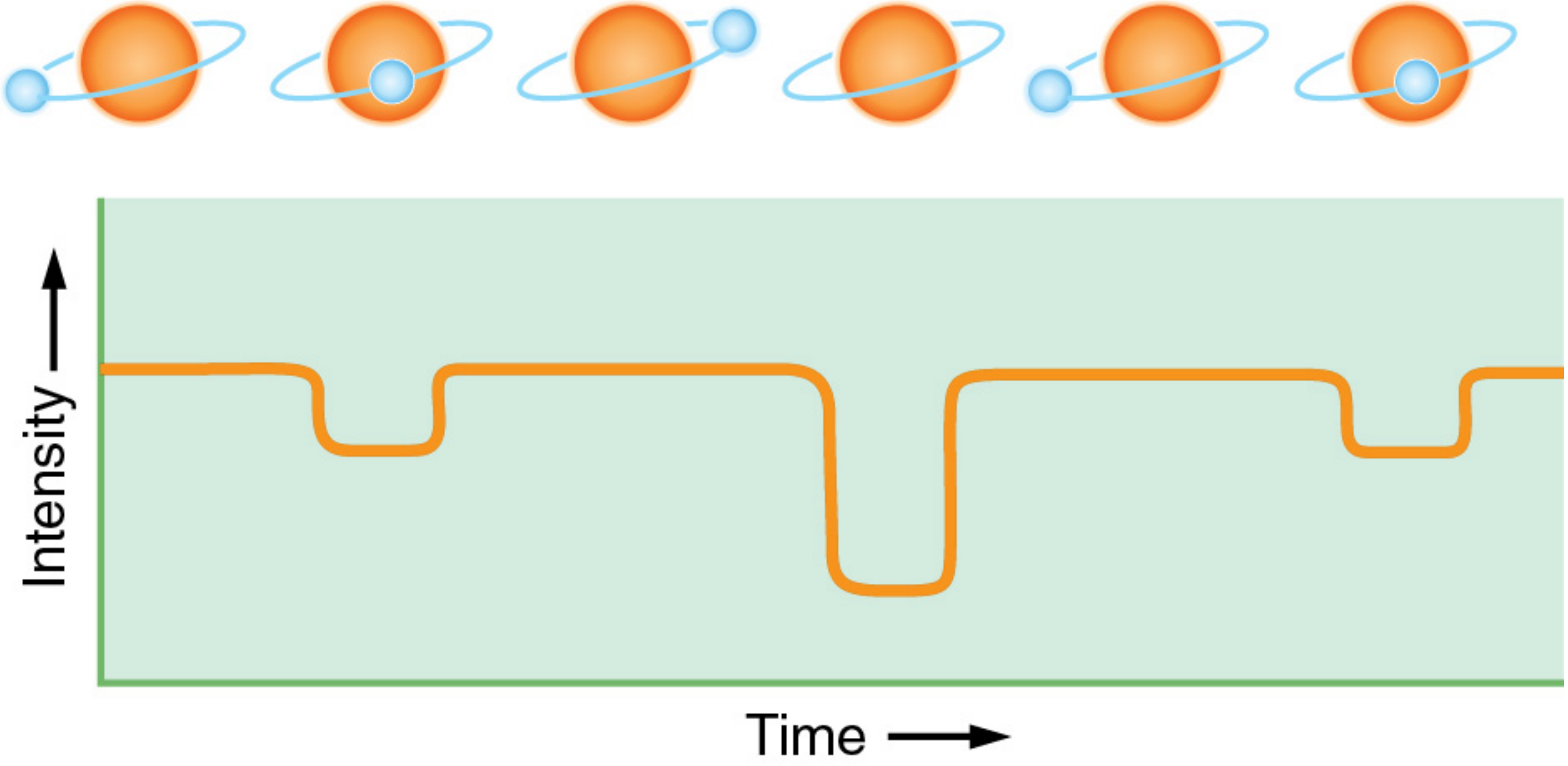
Two-body problem:

$$\frac{m_1 + m_2}{1\ M_{\text{sun}}} = \left(\frac{a_1 + a_2}{1\ \text{AU}}\right)^3 \left(\frac{P}{1\ \text{year}}\right)^{-2}$$

Eclipsing Binaries:

understanding the light curve

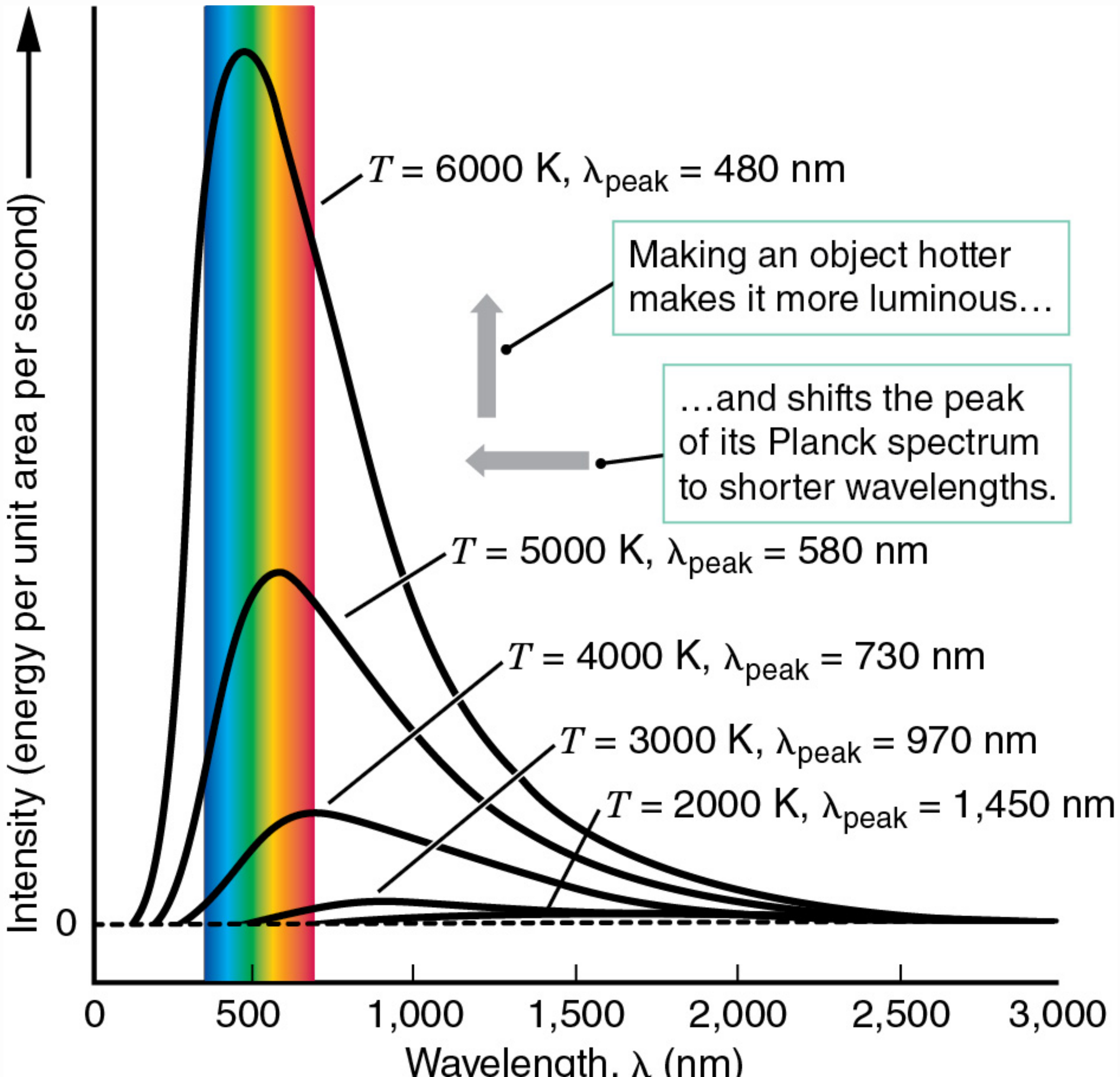
Eclipsing Binary Stars - Light curve from photometry

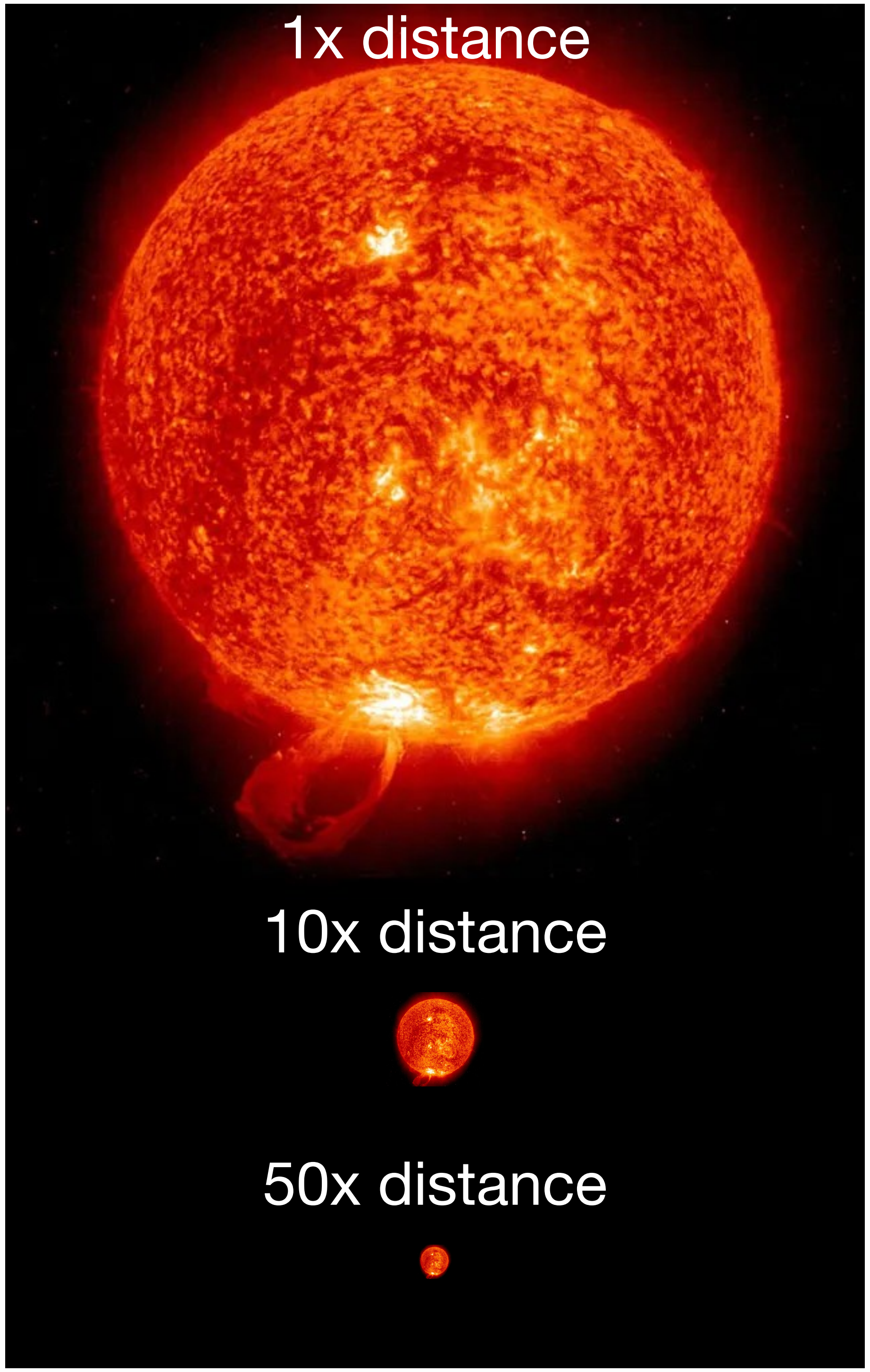


- In an **eclipsing binary** system, the total light coming from the star system decreases when *either* star passes in front of the other.
- But there are two eclipses (A in front of B vs. B in front of A), why one is deeper than the other? What can the eclipse light curve tell us?
- Can we also measure the radii of the stars in these systems?

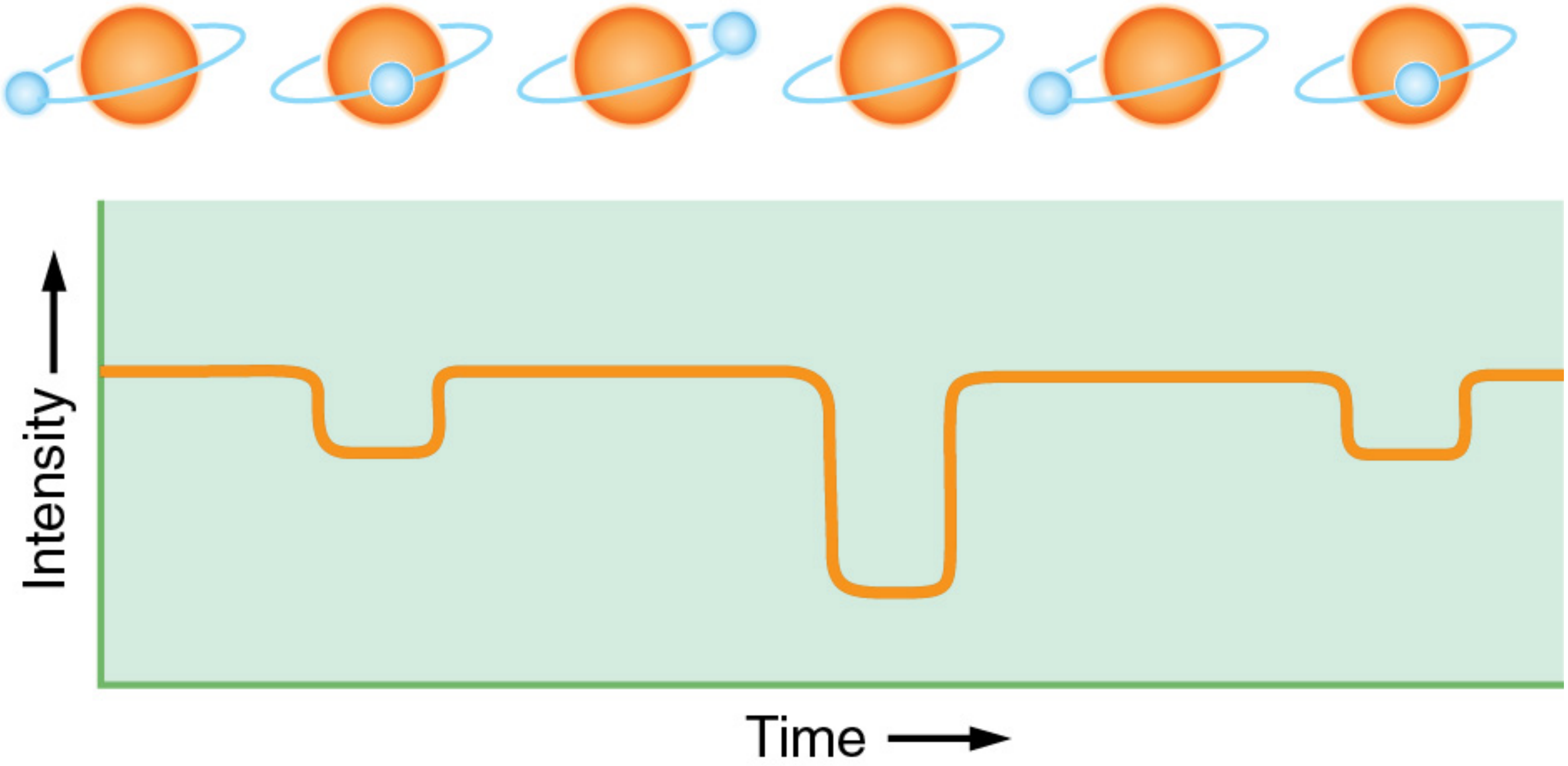
$B_\lambda(T)$ is a Blackbody Emitter's Apparent Surface Brightness

- **Flux density** can be derived from **luminosity density** using the **inverse distance square law**: $F_\lambda = L_\lambda/4\pi d^2 = (4\pi R^2 \cdot \pi B_\lambda(T))/(4\pi d^2) = B_\lambda(T) \cdot \pi R^2/d^2$
- Since $\pi R^2/d^2$ is the **angular area** of the source, hence **Planck function** $B_\lambda(T)$ gives the **surface brightness** of the source *(at λ), which is **distance invariant.***





Eclipsing Binary Stars - Light curve from photometry

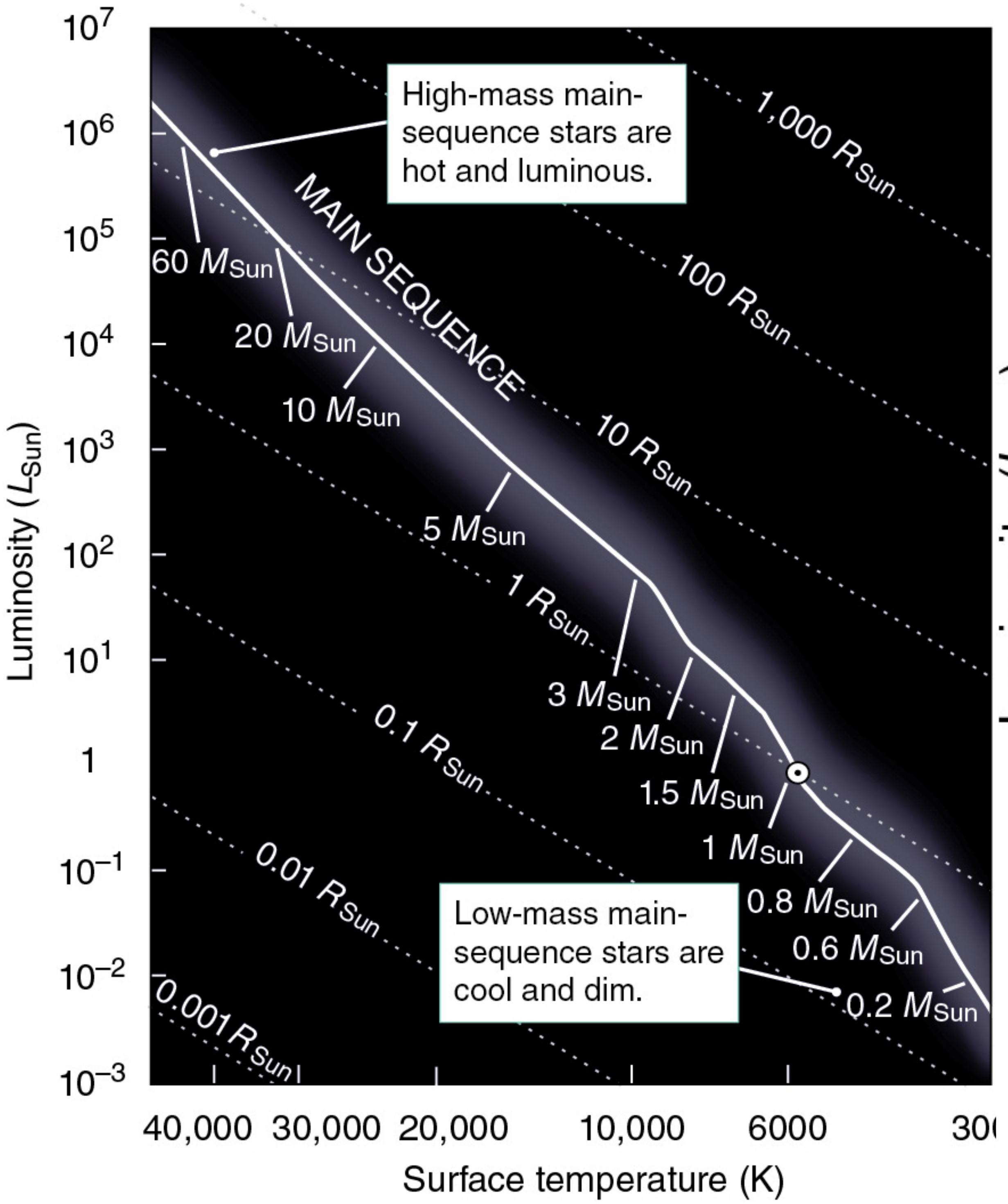


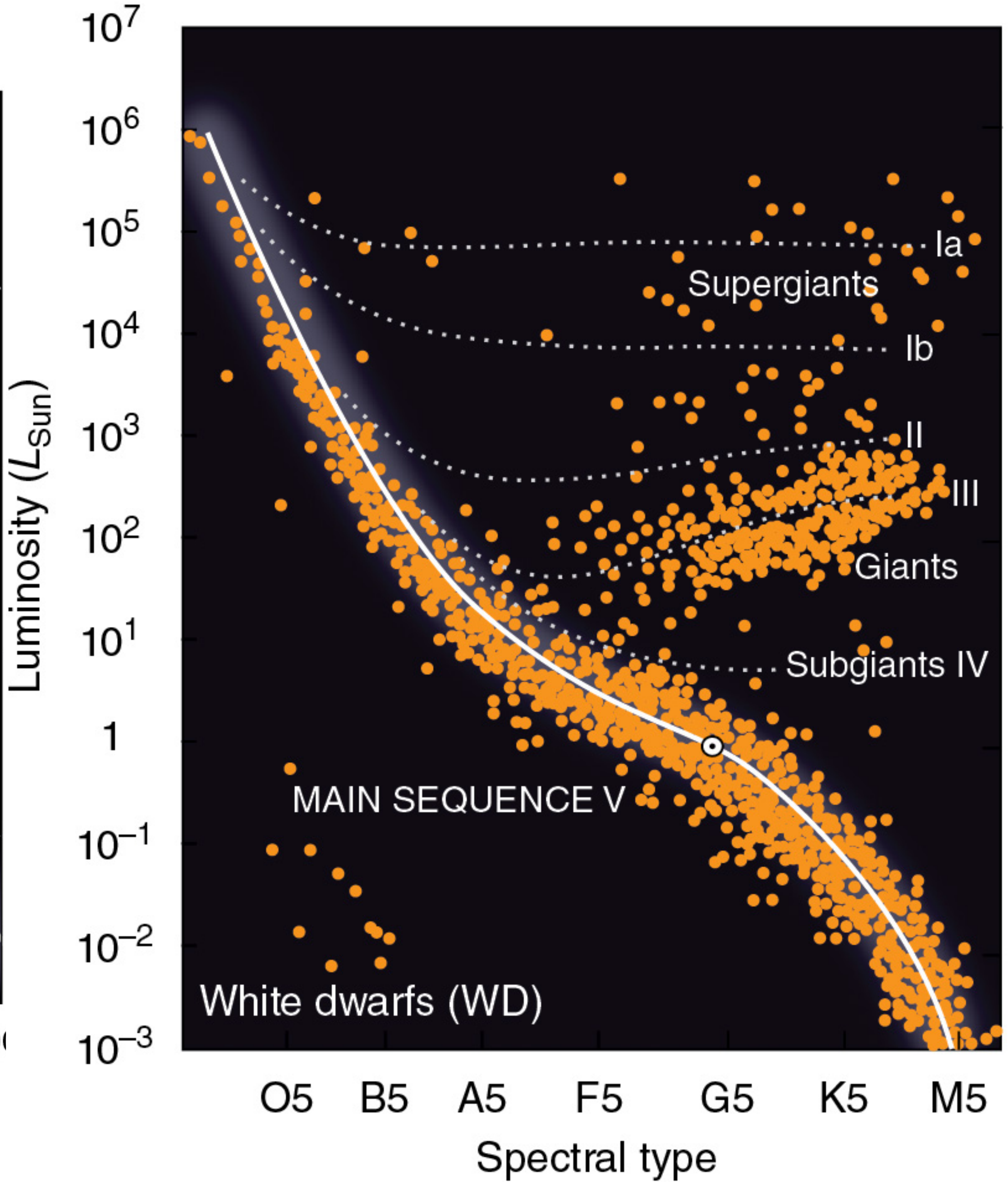
- In an **eclipsing binary** system, the total light coming from the star system decreases when *either* star passes in front of the other.
- But there are two eclipses (A in front of B vs. B in front of A), why one is deeper than the other? What can the eclipse light curve tell us?
- Can we also measure the radii of the stars in these systems?

How to measure the radius of a star?

Size Estimates using Stefan-Boltzmann Law (Req. bolometric luminosity)

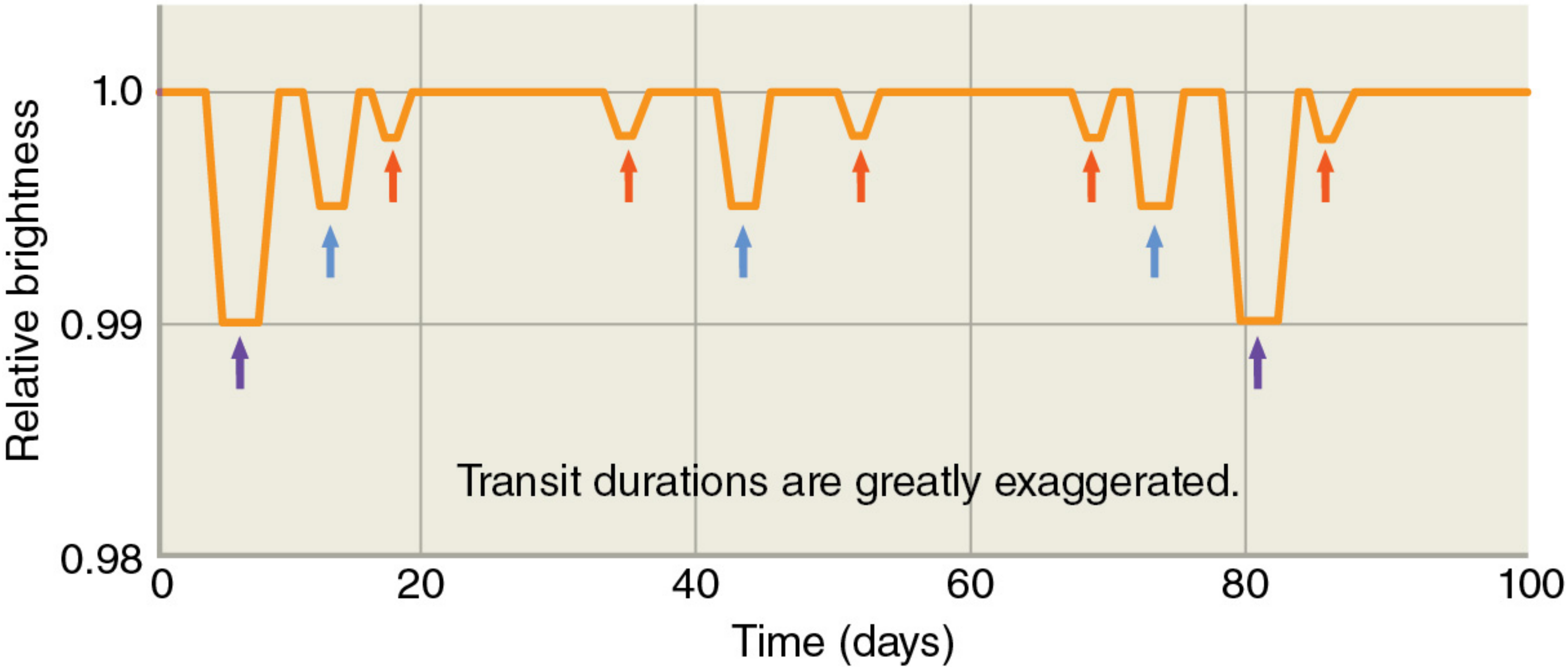
$$L_{\mathrm{bol}} = 4\pi R^2 \times \sigma_{\mathrm{SB}} T^4$$





Planet Size from the Depth of the Transit and Radius of the Star

- Unlike eclipsing binary stars, ***planets only reflects light from the side that faces the star (which is typically too faint to be detectable).***



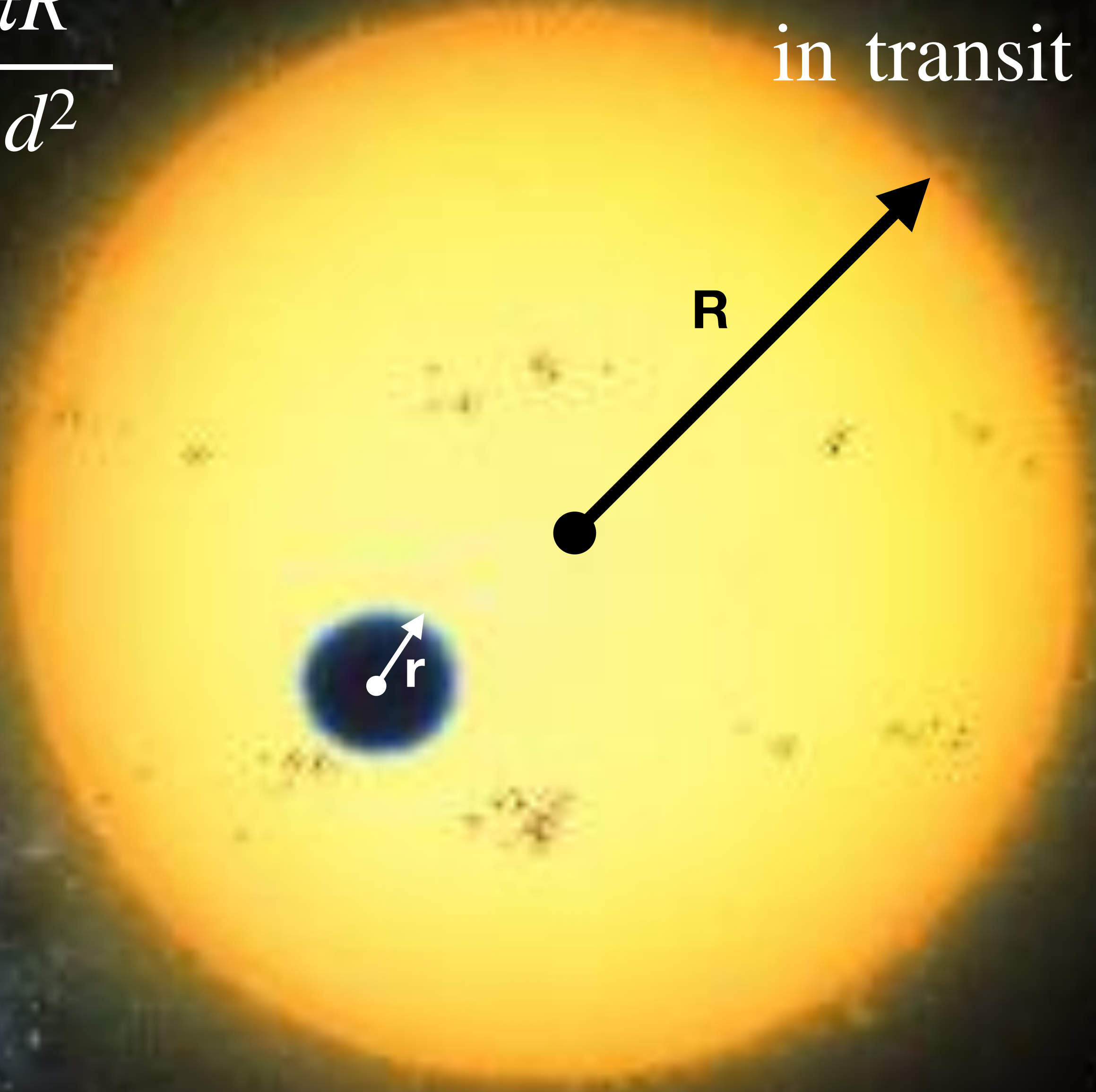
$$\text{fractional reduction in flux} : \frac{F_0 - F_t}{F_0} = \frac{r^2}{R^2}$$

Alternative Method: Radius of the Star from Transit Depth

Once we know the planet radius from transit ingress/egress time,
we can use the ***transit depth*** *to estimate the radius of the star.*

$$\text{no transit}: F_0 = \frac{F_S}{\pi}\frac{\pi R^2}{d^2}$$

$$\text{in transit}: F_t = \frac{F_S}{\pi}\frac{\pi(R^2 - r^2)}{d^2}$$



$$\text{fractional reduction in flux}: \frac{F_0 - F_t}{F_0} = \frac{r^2}{R^2}$$

© 1999 Lynette

Planet Size from Ingress & Egress time (requires Mass of the Star)

$$r = v_{\mathrm{circ}}(t_2 - t_1)/2$$

$$a_{AU} = (M_{\mathrm{solar-mass}} P^2_{\mathrm{year}})^{1/3}$$

$$v_{\mathrm{circ}} = \frac{2\pi a}{P_{\mathrm{orbit}}}$$